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Concurrency Theory WS 2017/2018

— Series 3 —

Hand in until November 3 before the exercise class.

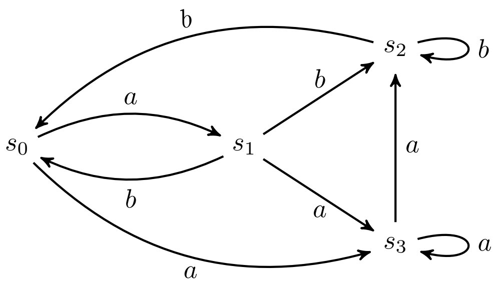
Exercise 1 (Semantics of HML with recursion) (5 Points)

Let $(S, Act, \rightarrow)$ be an LTS, the semantics of a HML formula $F \in HMF_X$ is defined in lecture 5 as a function

$$[\![F]\!] : 2^S \rightarrow 2^S.$$

1) Show that $[\![F]\!]$ is monotonic over the complete lattice $(2^S, \subseteq)$.

2) An LTS $(S, Act, \rightarrow)$ is given as follows:



Consider following questions:

- compute $[\![\langle b\rangle[a]\texttt{tt} \wedge \langle b\rangle[b]X]\!](\{s_0, s_2\})$.
- compute the set of processes satisfying following property

$$X \stackrel{\min}{=} \langle b\rangle\langle a\rangle\texttt{tt} \vee \langle b\rangle[b]X$$

- compute the sets of processes satisfying following equational systems

$$\begin{aligned} A &\stackrel{\max}{=} [a]B \\ B &\stackrel{\max}{=} \langle a\rangle C \wedge [b]B \\ C &\stackrel{\max}{=} [b]B \end{aligned}$$

Exercise 2 (Complete Lattices) (5 Points)

Let $(L, \sqsubseteq)$ and $(M, \sqsubseteq)$ be complete lattices, and M finite.

1. $\gamma\colon M \to L$ is monotone,
2. $\gamma(\top) = \top$, and
3. for each $m_1, m_2 \in M$ with $m_1 \not\sqsubseteq m_2$ and $m_2 \not\sqsubseteq m_1$ it holds that $\gamma(\bigsqcap\{m_1, m_2\}) = \bigsqcap\{\gamma(m_1), \gamma(m_2)\}$

For each of the following, give an M, L and γ s.t.:

- (i) and (ii) hold, but (iii) not.
- (i) and (iii) hold, but (ii) not.
- (ii) and (iii) hold, but (i) not.

Show that (i)—(iii) are jointly equivalent to $\gamma\colon M \to L$ satisfying

$$\gamma\left(\bigsqcap Y\right) = \bigsqcap\{\gamma(l) \mid l \in Y\}$$

for each $Y \subseteq M$.

Exercise 3 (Tarski's Fixed Point Theorem) (0 Points)

Show the second part of Theorem 5.12, i.e. for a complete lattice $(D, \sqsubseteq)$ and $f : D \to D$ monotonic prove that the greatest fixed point of f exists and is given by

$$\mathrm{FIX}(f) \;=\; \bigsqcup\{d \in D \mid d \sqsubseteq f(d)\}.$$