Static Program Analysis

Lecture 9: Dataflow Analysis VIII (Conditional Interval Analysis & Java Virtual Machine)

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Recap: Taking Conditional Branches into Account

Extending the Syntax of WHILE Programs

Definition (Labelled WHILE programs with assertions)

The syntax of labelled WHILE programs with assertions is defined by the following context-free grammar:

$$a ::= z \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \in \mathit{AExp}$$
$$b ::= t \mid a_1 = a_2 \mid a_1 > a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in \mathit{BExp}$$
$$c ::= [\texttt{skip}]^l \mid [x := a]^l \mid c_1 ; c_2 \mid$$
$$\texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \texttt{ end} \mid \texttt{while } [b]^l \texttt{ do } c \texttt{ end} \mid [\texttt{assert } b]^l \in \mathit{Cmd}$$

To be done:

- Definition of transfer functions for `assert` blocks (depending on analysis problem)
- Idea: assertions as filters that let only “compatible” information pass

Original Interval Analysis

So far:

- The domain $(\mathit{Int}, \subseteq)$ of intervals over $\mathbb{Z}$ is defined by

$$\mathit{Int} := \{[z_1, z_2] \mid z_1 \in \mathbb{Z} \cup \{-\infty\}, z_2 \in \mathbb{Z} \cup \{+\infty\}\}, z_1 \leq z_2\} \cup \{\emptyset\}$$

  where
  - $-\infty \leq z$ and $z \leq +\infty$ (for all $z \in \mathbb{Z}$)
  - $\emptyset \subseteq J$ (for all $J \in \mathit{Int}$)
  - $[y_1, y_2] \subseteq [z_1, z_2]$ iff $y_1 \geq z_1$ and $y_2 \leq z_2$
- Transfer functions $\{\varphi_l \mid l \in \mathit{Lab}\}$ are defined by

$$\varphi_l(\delta) := \begin{cases} \delta & \text{if } B^l = \texttt{skip} \text{ or } B^l \in \mathit{BExp} \\ \delta[x \mapsto \mathit{val}_\delta(a)] & \text{if } B^l = (x := a) \end{cases}$$

  where

$$\mathit{val}_\delta(x) := \delta(x) \qquad \mathit{val}_\delta(a_1 + a_2) := \mathit{val}_\delta(a_1) \oplus \mathit{val}_\delta(a_2)$$
$$\mathit{val}_\delta(z) := [z, z] \qquad \mathit{val}_\delta(a_1 - a_2) := \mathit{val}_\delta(a_1) \ominus \mathit{val}_\delta(a_2)$$
$$\mathit{val}_\delta(a_1 * a_2) := \mathit{val}_\delta(a_1) \odot \mathit{val}_\delta(a_2)$$

Transfer Functions of Assertions I

Additionally for $B^l = (\texttt{assert}\ b)$, $\delta : Var_c \to Int$ and $x \in Var_c$:

$$\varphi_l(\delta)(x) := \begin{cases} \emptyset & \text{if } Z = \emptyset \\ \left[\bigsqcap_{\mathbb{Z}\cup\{-\infty\}} Z, \bigsqcup_{\mathbb{Z}\cup\{+\infty\}} Z\right] & \text{otherwise} \end{cases}$$

where

- $Z := \{\sigma(x) \mid \sigma \in \Sigma_\delta, val_\sigma(b) = \text{true}\}$
- $\Sigma_\delta := \{\sigma : Var_c \to \mathbb{Z} \mid \forall y \in Var_c : \sigma(y) \in \delta(y)\}$
  (and thus $\Sigma_\delta = \emptyset$ iff $\delta(y) = \emptyset$ for some $y \in Var_c$)
- $val_\sigma : BExp \to \mathbb{B}$: see Slide 8.25

Transfer Functions of Assertions II

Example 9.1

$$Var_c = \{\mathtt{x}, \mathtt{y}\}, \delta = (\underbrace{[-\infty, 2]}_{\mathtt{x}}, \underbrace{[0, +\infty]}_{\mathtt{y}})$$

$$\begin{aligned}
\Longrightarrow \quad & \varphi_{\mathtt{assert\ x>0}}(\delta) = ([1, 2], [0, +\infty]) \\
& \varphi_{\mathtt{assert\ x=y}}(\delta) = ([0, 2], [0, 2]) \\
& \varphi_{\mathtt{assert\ x>y}}(\delta) = ([1, 2], [0, 1]) \\
& \varphi_{\mathtt{assert\ x<y}}(\delta) = ([-\infty, 2], [0, +\infty])
\end{aligned}$$

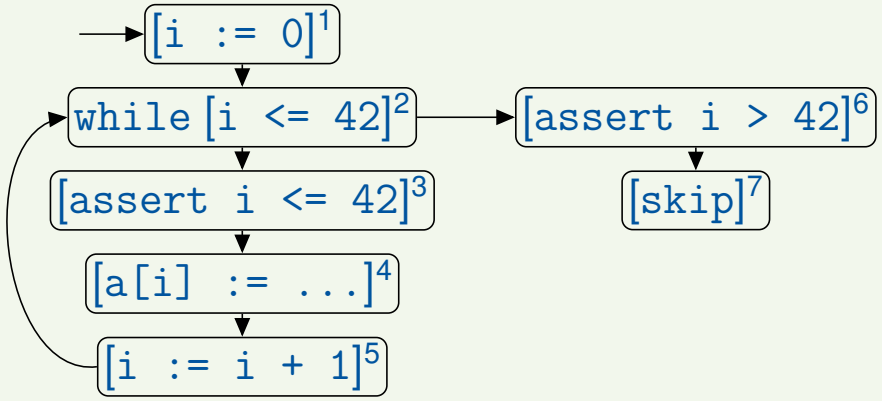
Remarks:

- Again for $B^l = (\mathtt{assert}\ b)$ and $\delta : Var_c \to Int$, $\varphi_l(\delta) \sqsubseteq \delta$ and hence $\Sigma_{\varphi_l(\delta)} \subseteq \Sigma_\delta$ ("filter")
- Again if $\mathsf{AI}_l(x) = \emptyset$ for some $l \in Lab_c$ and $x \in Var_c$, then l is unreachable (and $\mathsf{AI}_l(y) = \emptyset$ for all $y \in Var_c$)

Interval Analysis with Assertions

An Example

Example 9.2 (Interval analysis for array index; cf. Example 7.2)

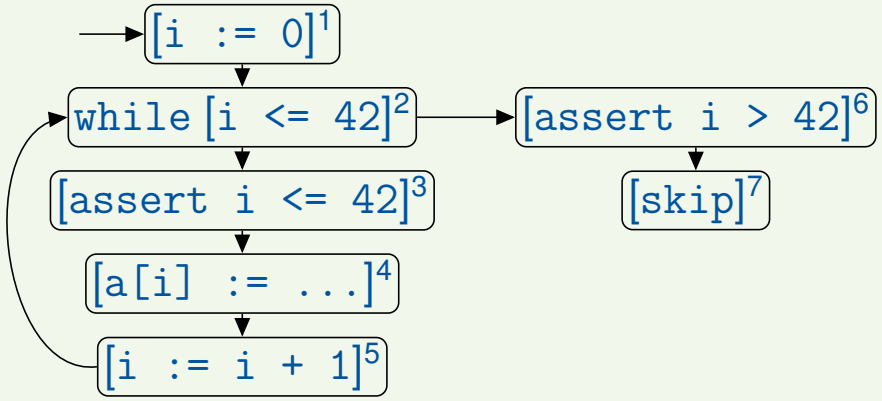


$$\varphi_1(J) = [0,0]$$
$$\varphi_2(J) = J$$
$$\varphi_3(J) = J \cap [-\infty, 42]$$
$$\varphi_4(J) = J$$
$$\varphi_5(\emptyset) = \emptyset$$
$$\varphi_5([i_1, i_2]) = [i_1 + 1, i_2 + 1]$$
$$\varphi_6(J) = J \cap [43, +\infty]$$

Worklist w. widening	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$12, 23, 34, 45, 52, 26, 67$	$[-\infty, +\infty]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$23, 34, 45, 52, 26, 67$	$[-\infty, +\infty]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$34, 45, 52, 26, 67$	$[-\infty, +\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$45, 52, 26, 67$	$[-\infty, +\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$
$52, 26, 67$	$[-\infty, +\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
$23, 26, 67$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
$34, 26, 67$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
$45, 26, 67$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0,0]$	$\emptyset$	$\emptyset$
$52, 26, 67$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$\emptyset$	$\emptyset$
$26, 67$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$\emptyset$	$\emptyset$
67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$\emptyset$

An Example (continued)

Example 9.2 (Interval analysis for array index; continued)



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$$\varphi_5(\emptyset) = \emptyset$$
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$$\varphi_6(J) = J \cap [43, +\infty]$$

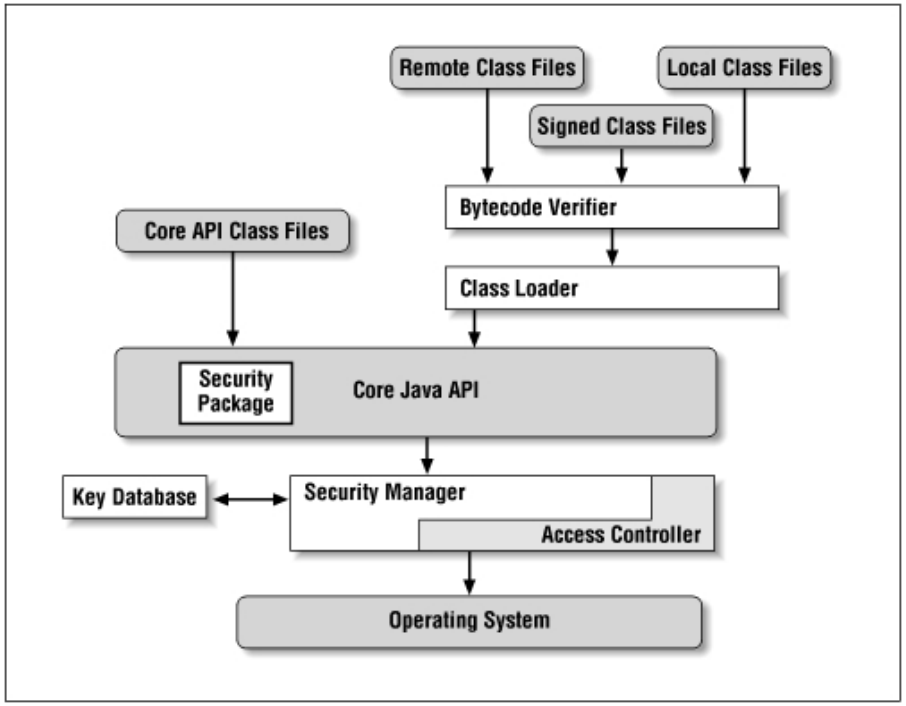
Narrowing	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$\text{fix}^{\nabla}(\Phi_S)$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^2(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^3(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^4(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,+\infty]$
$\Phi_S^5(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,43]$
$\Phi_S^6(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,43]$

Java Bytecode

- Intermediate language between high-level language and machine code
- Execution on Java Virtual Machine (JVM)
- **Advantages:**
  - architecture independency (especially for web applications)
  - faster than pure (i.e., source code) interpretation
- **Problem:** security issues
  - destruction of data
  - modification of data
  - disclosure of personal information
  - modification of other programs

Java Security: the Sandbox

- Insulation layer providing indirect access to system resources
- Hardware access via API classes and methods
- Bytecode verification upon loading
  - well-typedness
  - proper object referencing
  - proper control flow



The Java Virtual Machine

- Conventional stack-based abstract machine
- Supports object-oriented features: classes, methods, etc.
- Stack for method parameters and intermediate results of expression evaluations
- Registers for source-level local variables
- Both part of method activation record (and thus preserved across method calls)
- Method entry point specifies required number of
  - registers (m_r)
  - stack slots (m_s; for memory allocation)
- (Most) instructions are typed

Example: Factorial Function

Example 9.3 (Factorial function)

Java source code:

```
static int factorial(int n)
{ int res;
  for (res = 1; n > 0; n--) res = res * n;
  return res; }
```

Corresponding JVM bytecode:

```
method static int factorial(int), 2 registers, 2 stack slots
   1: istore 0    // store n in register 0
   2: iconst_1    // push constant 1
   3: istore 1    // store res in register 1
   4: iload 0     // push n
   5: ifle 12     // if <= 0, go to end
   6: iload 1     // push res
   7: iload 0     // push n
   8: imul        // res * n on top of stack
   9: istore 1    // store in res
  10: iinc 0, -1  // decrement n
  11: goto 4      // go to loop header
  12: iload 1     // push res
  13: ireturn     // return res to caller
```

JVM Instruction Set (excerpt; $\approx$ 200 instructions in total)

`iload` n: push integer from register n

`istore` n: pop integer into register n

`iconst_`z: push integer z

`aconst_null`: push null reference

`iadd`: add two topmost integers on stack and push sum

`getfield` C f τ: pop reference to object o (of class C) and push value of field f of o (of type τ)

`putfield` C f τ: pop value v (of type τ) and reference to object o (of class C) and assign v to field f of o

`new` C: create new object (of class C) and push reference

`invoke` C M $\tau_0(\tau_1, \ldots, \tau_n)$: pop values $v_1, \ldots, v_n$ (of type $\tau_1, \ldots, \tau_n$) and reference to object o (of class C), call method M of o with parameters $v_1, \ldots, v_n$, and push return value (of type τ_0)

`if_icmpeq` l: pop two topmost integers from stack and jump to line l if equal

`ireturn`: return to caller with integer result on top of stack

Malicious Bytecode

Example 9.4 (Malicious bytecode)

```
1: iconst_5
2: iconst_1
3: putfield A f int
```

interprets second stack entry (5) as reference to object of class `A` and assigns topmost stack entry (1) to field `f` of this object

Correctness of Bytecode

Conditions to ensure proper operation:

Type correctness: arguments of instructions always of expected type

No stack over-/underflow: never push to full stack or pop from empty stack

Code containment: PC must always point into the method code

Register initialization: load from non-parameter register only after store

Object initialization: constructor must be invoked before using class instance

Access control: operations must respect visibility modifiers (`private`/`protected`/`public`)

Options:

- dynamic checking at execution time ("defensive JVM approach")
  - expensive, slows down execution
- static checking at loading time (here)
  - verified code executable at full speed without extra dynamic checks

The Java Bytecode Verifier

Summary: dataflow analysis applied to type-level abstract interpretation of JVM

1. Association of type information with register and stack contents
   - set of types forms a complete lattice
2. Simulation of execution of instructions at type level (“symbolic execution”)
3. Use dataflow analysis to cover all concrete executions
4. Modular analysis: proceeds method per method

(see X. Leroy: *Java Bytecode Verification: Algorithms and Formalizations*, Journal of Automated Reasoning 30(3-4), 2003, 235–269)

The Type-Level Abstract Interpreter

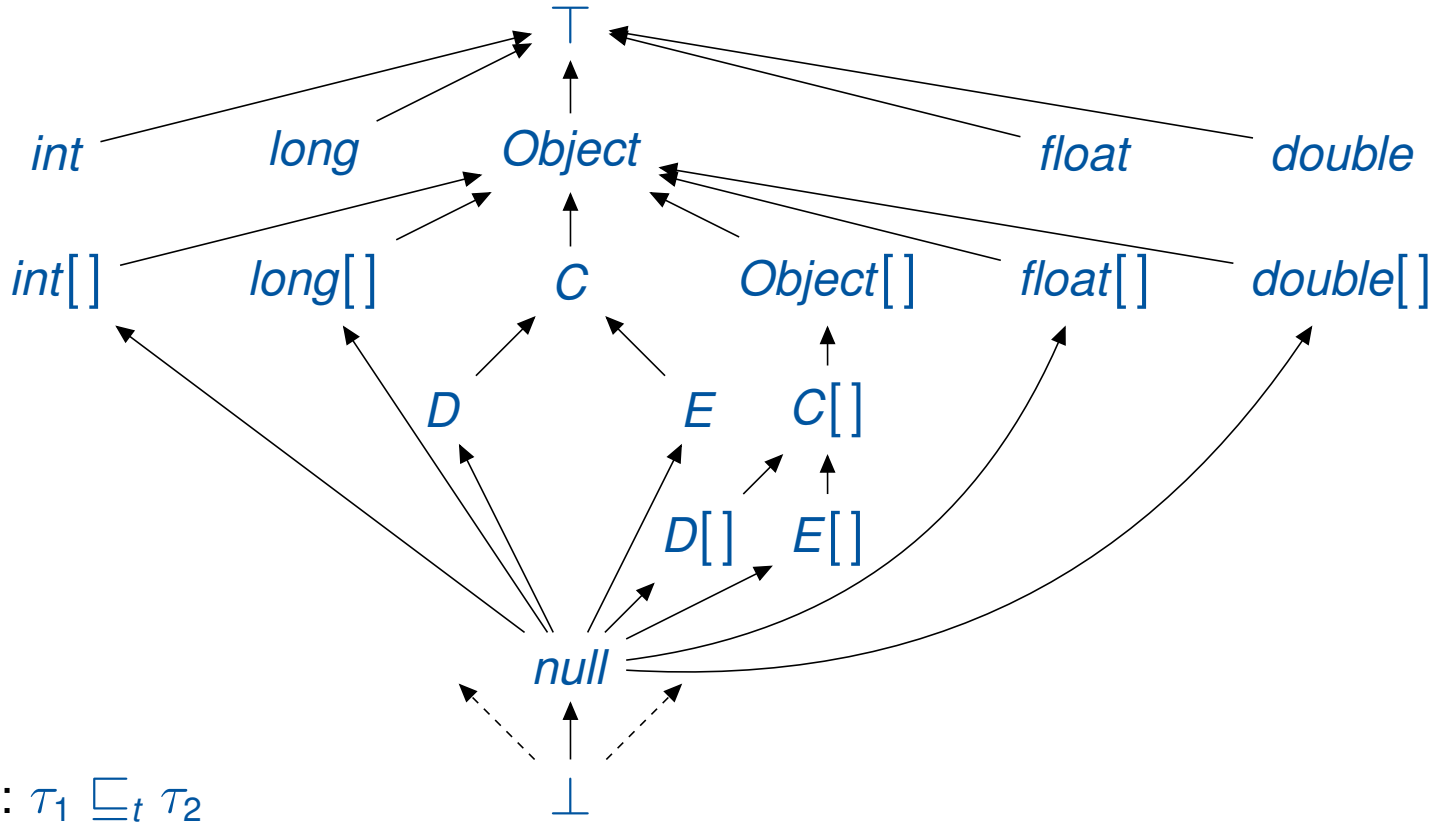
Types

The set of types, *Typ*, is composed of

- Primitive types:
  - *int* (covering *boolean*, *byte*, *char*, *short*)
  - *long*
  - *float*
  - *double*
- Object reference types: C for every class name C
- Array types: $\tau[\,]$ for every primitive or object reference type τ
- Method types: $\tau_0(\tau_1, \ldots, \tau_n)$ for $n \in \mathbb{N}$, $\tau_i \in \mathit{Typ}$
- Special types:
  - *null* (null reference)
  - *Object* (any object)
  - $\top$ (contents of uninitialised registers, i.e., any value)
  - $\bot$ (absence of any value)

The Subtyping Relation (excerpt)

(C, D, E user-defined classes; D, E extending C)



Notation: $\tau_1 \sqsubseteq_t \tau_2$

The Type-Level Abstract Interpreter

- **Idea:** execute JVM instructions on types (rather than concrete values)
  - stack type $S \in Typ^{\leq m_s}$ (top to the left)
  - register type $R : \{0, \ldots, m_r - 1\} \to Typ$
- Represented as transition relation

$$i : (S, R) \to (S', R')$$

  where
  - i: current instruction
  - (S, R): stack/register type before execution
  - (S', R'): stack/register type after execution
- Errors (type mismatch, stack over-/underflow, ...) denoted by absence of transition

The Type-Level Abstract Interpreter

Some Transition Rules

$$\texttt{iconst\_}z: \quad (S,R) \to (\mathit{int}.S,R) \quad \text{if } |S| < m_s$$

$$\texttt{aconst\_null}: \quad (S,R) \to (\mathit{null}.S,R) \quad \text{if } |S| < m_s$$

$$\texttt{iadd}: \quad (\mathit{int}.\mathit{int}.S,R) \to (\mathit{int}.S,R)$$

$$\texttt{if\_icmpeq}\ l: \quad (\mathit{int}.\mathit{int}.S,R) \to (S,R)$$

$$\texttt{iload}\ n: \quad (S,R) \to (\mathit{int}.S,R) \quad \text{if } 0 \le n < m_r, R(n) = \mathit{int}, |S| < m_s$$

$$\texttt{aload}\ n: \quad (S,R) \to (R(n).S,R) \quad \text{if } 0 \le n < m_r, R(n) \sqsubseteq_t \mathit{Object}, |S| < m_s$$

$$\texttt{istore}\ n: \quad (\mathit{int}.S,R) \to (S,R[n \mapsto \mathit{int}]) \quad \text{if } 0 \le n < m_r$$

$$\texttt{astore}\ n: \quad (\tau.S,R) \to (S,R[n \mapsto \tau]) \quad \text{if } 0 \le n < m_r, \tau \sqsubseteq_t \mathit{Object}$$

$$\texttt{getfield}\ C\ f\ \tau: \quad (D.S,R) \to (\tau.S,R) \quad \text{if } D \sqsubseteq_t C$$

$$\texttt{putfield}\ C\ f\ \tau: \quad (\tau'.D.S,R) \to (S,R) \quad \text{if } \tau' \sqsubseteq_t \tau, D \sqsubseteq_t C$$

$$\texttt{invoke}\ C\ M\ \sigma: \quad (\tau'_n \ldots \tau'_1.\tau'.S,R) \to (\tau_0.S,R) \quad \text{if } \sigma = \tau_0(\tau_1,\ldots,\tau_n), \tau'_i \sqsubseteq_t \tau_i \text{ for } 1 \le i \le n, \tau' \sqsubseteq_t C$$

Some Theoretical Properties

Lemma 9.5

1. $(Typ, \sqsubseteq_t)$ *is a complete lattice satisfying ACC.*
2. *(Determinacy) The transitions of the abstract interpreter define a partial function: If* $i : (S, R) \to (S_1, R_1)$ *and* $i : (S, R) \to (S_2, R_2)$*, then* $S_1 = S_2$ *and* $R_1 = R_2$*.*
3. *(Soundness) If* $i : (S, R) \to (S', R')$*, then for all concrete states* (s, r) *matching* (S, R)*, the defensive JVM will not stop with a run-time type exception when applying* i *to* (s, r) *(but rather change to some* (s', r') *matching* (S', R')*).*

Proof.

see X. Leroy: *Java Bytecode Verification: Algorithms and Formalizations* □