Static Program Analysis

Lecture 21: Shape Analysis & Final Remarks

Winter Semester 2016/17

Thomas Noll
Software Modeling and Verification Group
RWTH Aachen University

https://moves.rwth-aachen.de/teaching/ws-1617/spa/

Outline of Lecture 21

Recap: Pointer Analysis

Shape Analysis

Further Topic in Program Analysis

Final Remarks

Recap: Pointer Analysis

The Shape Analysis Approach

- **Goal:** determine the possible shapes of a dynamically allocated data structure at given program point
- **Interesting information:**
  - data types (to avoid type errors, such as dereferencing `nil`)
  - aliasing (different pointer variables having same value)
  - sharing (different heap pointers referencing same location)
  - reachability of nodes (garbage collection)
  - disjointness of heap regions (parallelisability)
  - shapes (lists, trees, absence of cycles, ...)
- **Concrete questions:**
  - Does `x.next` point to a shared element?
  - Does a variable `p` point to an allocated element every time `p` is dereferenced?
  - Does a variable point to an acyclic list?
  - Does a variable point to a doubly-linked list?
  - Can a loop or procedure cause a memory leak?
- **Here:** basic outline; details in [Nielson/Nielson/Hankin 2005, Sct. 2.6]

Recap: Pointer Analysis

Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Arithmetic expressions	*AExp*	a
Boolean expressions	*BExp*	b
Selector names	*Sel*	sel
Pointer expressions	*PExp*	p
Commands (statements)	*Cmd*	c

Context-free grammar:

$$
\begin{aligned}
a &::= z \mid x \mid a_1 + a_2 \mid \ldots \mid p \mid \texttt{nil} \in AExp \\
b &::= t \mid a_1 = a_2 \mid b_1 \wedge b_2 \mid \ldots \mid \texttt{is-nil}(p) \in BExp \\
p &::= x \mid x\,.\,sel \\
c &::= [\texttt{skip}]^l \mid [p := a]^l \mid c_1 ; c_2 \mid \texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \texttt{ end} \mid \\
&\qquad \texttt{while } [b]^l \texttt{ do } c \texttt{ end} \mid [\texttt{malloc } p]^l \in Cmd
\end{aligned}
$$

Shape Graphs I

Approach: representation of (infinitely many) concrete heap states by (finitely many) abstract shape graphs

- abstract nodes X = sets of variables
- interpretation: $x \in X$ iff x points to concrete node represented by X
- $\emptyset$ represents all concrete nodes that are not directly addressed by pointer variables
- $x, y \in X$ (with $x \neq y$) indicate aliasing (as x and y point to the same concrete node)
- if $x.sel$ and y refer to the same heap address and if X, Y are abstract nodes with $x \in X$ and $y \in Y$, this yields abstract edge $X \stackrel{sel}{\Longrightarrow} Y$ (similarly for $X = \emptyset$ or $Y = \emptyset$)
- transfer functions transform (sets of) shape graphs

Shape Graphs II

Definition (Shape graph)

A shape graph

$$G = (Abs, \Longrightarrow)$$

consists of

- a set $Abs \subseteq 2^{Var}$ of abstract locations and
- an abstract heap $\Longrightarrow \subseteq Abs \times Sel \times Abs$
  - notation: $X \overset{sel}{\Longrightarrow} Y$ for $(X, sel, Y) \in \Longrightarrow$

with the following properties:

Disjointness: $X, Y \in Abs \implies X = Y$ or $X \cap Y = \emptyset$

(a variable can refer to at most one heap location)

Determinacy: $X \neq \emptyset$ and $X \overset{sel}{\Longrightarrow} Y$ and $X \overset{sel}{\Longrightarrow} Z \implies Y = Z$

(target location is unique if source node is unique)

SG denotes the set of all shape graphs.

From Heap Configurations to Shape Graphs

Definition

Given a heap configuration $H = (Nod, Sel, Var, \sigma, \longrightarrow)$, the corresponding shape graph $G = (Abs, \Longrightarrow)$ is defined by

- $Abs := \{\sigma^{-1}(n) \mid n \in Nod\}$
  $\quad = \{\{x \in Var \mid \sigma(x) = n\} \mid n \in Nod\}$
- For all $X, Y \in Abs$ and $sel \in Sel$:

$$X \overset{sel}{\Longrightarrow} Y \iff \exists n_X, n_y \in Nod : \sigma^{-1}(n_X) = X, \sigma^{-1}(n_Y) = Y, n_X \xrightarrow{sel} n_Y$$

Remark: yields Galois connection between sets of heap configurations and sets of shape graphs, both ordered by $\subseteq$

Shape Graphs and Concrete Heap Properties

Example

Let $G = (Abs, \Longrightarrow)$ be a shape graph. Then the following concrete heap properties can be expressed as conditions on G:

- $\mathtt{x} \neq \mathtt{nil}$
  $\iff \exists X \in Abs : \mathtt{x} \in X$
- $\mathtt{x} = \mathtt{y} \neq \mathtt{nil}$ (aliasing)
  $\iff \exists Z \in Abs : \mathtt{x}, \mathtt{y} \in Z$
- $\mathtt{x.sel1} = \mathtt{y.sel2} \neq \mathtt{nil}$ (sharing)
  $\Longrightarrow \exists X, Y, Z \in Abs : \mathtt{x} \in X, \mathtt{y} \in Y, X \stackrel{\mathtt{sel1}}{\Longrightarrow} Z \stackrel{\mathtt{sel2}}{\Longleftarrow} Y$
  ("$\Longleftarrow$" only valid if $Z \neq \emptyset$)

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- Domain: $(D, \sqsubseteq) := (2^{SG}, \subseteq)$ $\quad$ (Var, Sel finite $\Longrightarrow$ SG finite $\Longrightarrow$ 2^{SG} finite $\Longrightarrow$ ACC)

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- Extremal value: $\iota :=$ {shape graphs for possible initial values of Var}

Example 21.1 (List reversal; cf. Example 20.5)

- Variables: $Var = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$
- Assumption: $\mathtt{x}$ points to any (finite, non-cyclic) list, $\mathtt{y} = \mathtt{z} = \mathtt{nil}$

$$\Rightarrow \iota = \left\{ \underbrace{(\emptyset, \emptyset)}_{\text{empty}}, \underbrace{\boxed{\{\mathtt{x}\}}}_{\text{1 elem.}}, \underbrace{\boxed{\{\mathtt{x}\}} \overset{\mathtt{next}}{\Longrightarrow} \boxed{\emptyset}}_{\text{2 elem.}}, \underbrace{\boxed{\{\mathtt{x}\}} \overset{\mathtt{next}}{\Longrightarrow} \overset{\overset{\mathtt{next}}{\curvearrowright}}{\boxed{\emptyset}}}_{\geq \text{3 elem.}} \right\}$$

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The Transfer Functions

Transform each single shape graph into a set of shape graphs: for each $l \in Lab$,

$$\varphi_l : 2^{SG} \to 2^{SG} : \{G_1, \ldots, G_n\}) \mapsto \bigcup_{i=1}^{n} \varphi_l(G_i)$$

Definition 21.2 (Transfer functions for shape analysis)

$\varphi_l(G) \subseteq SG$ is determined by B^l (where $G = (Abs, \Longrightarrow)$):

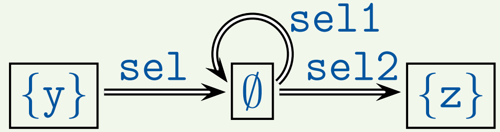
- $[\mathtt{skip}]^l$: $\varphi_l(G) := \{G\}$
- $[b]^l$: $\varphi_l(G) := \{G\}$
- $[p := a]^l$: case-by-case analysis w.r.t. p and a
  - [Nielson/Nielson/Hankin 2005, Sct. 2.6.3]: 12 cases on 11 p.
  - may involve (high degree of) non-determinism
  - see example on following slide
- $[\mathtt{malloc}\ x]^l$: $\varphi_l(G) := \{(Abs' \cup \{\{x\}\}, \Longrightarrow')\}$ with
  - $Abs' := \{X \setminus \{x\} \mid X \in Abs\}$
  - $\forall X, Y \in Abs, sel \in Sel$:

    $X \setminus \{x\} \overset{sel}{\Longrightarrow}{}' Y \setminus \{x\}$ iff $X \overset{sel}{\Longrightarrow} Y$
- $[\mathtt{malloc}\ x.sel]^l$: equivalent to $[\mathtt{malloc}\ t]^{l_1}; [x.sel := t]^{l_2}; [t := \mathtt{nil}]^{l_3}$ (with fresh $t \in Var$ and $l_1, l_2, l_3 \in Lab$)
- Fixpoint solution yields $SG_l \subseteq SG$ for each $l \in Lab$

Shape Analysis

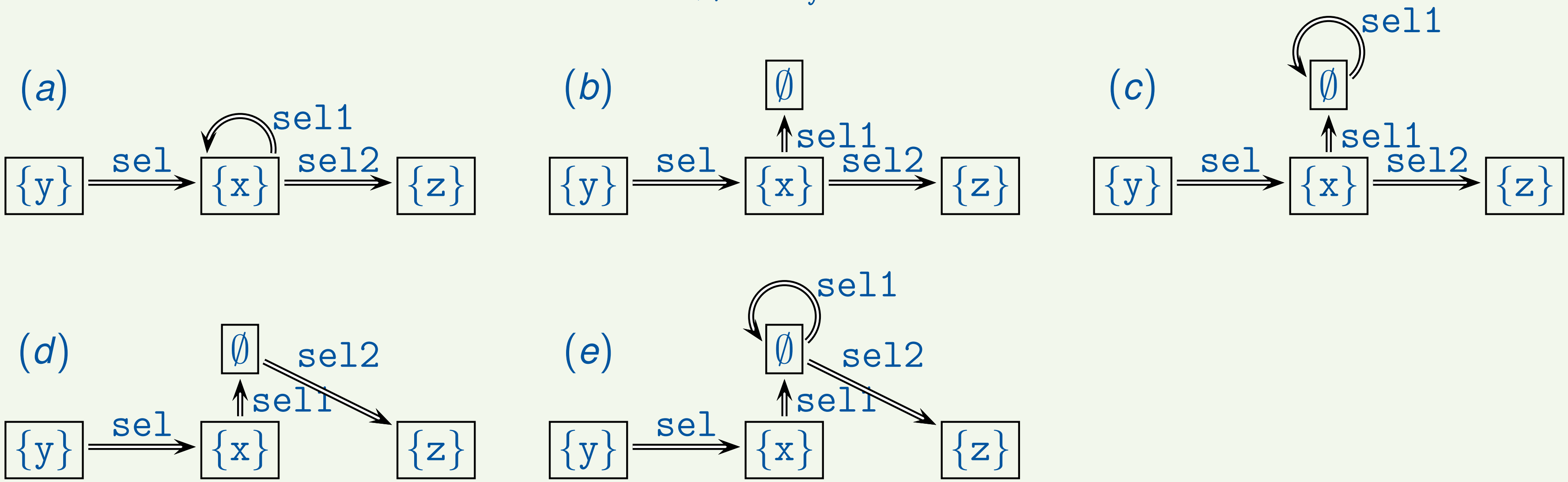
An Example

Example 21.3 (Transfer function for pointer assignment)



(justification: on the board)

$\downarrow \varphi_{\texttt{x := y.sel}}$



Soundness of Abstraction

Theorem 21.4 (Safety of approximation)

Let H be a heap configuration with corresponding shape graph G (according to Definition 20.7), and let $l \in Lab$. If B^l maps H to heap configuration H', then there exists a shape graph $G' \in \varphi_l(G)$ that corresponds to H'.

Proof.

omitted □

Application to List Reversal

Example 21.5 (List reversal; cf. Example 20.5)

Shape analysis of list reversal program yields final result

$$\left\{ \underbrace{(\emptyset, \emptyset)}_{\text{empty}} \;,\; \underbrace{\boxed{\{y\}}}_{\text{1 elem.}} \;,\; \underbrace{\boxed{\{y\}} \overset{\texttt{next}}{\Longrightarrow} \boxed{\emptyset}}_{\text{2 elem.}} \;,\; \underbrace{\boxed{\{y\}} \overset{\texttt{next}}{\Longrightarrow} \overset{\overset{\texttt{next}}{\curvearrowright}}{\boxed{\emptyset}}}_{\geq \text{ 3 elem.}} \right\}$$

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Interpretation:

+ Result again a finite list
– but potentially cyclic (may be a “lasso”, but not a ring)
– also “reversal” property not guaranteed
  – result could be in wrong order or have more/less entries

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Dedicated Algorithms for Pointer Analysis

- `nil` Pointer Analysis: checks whether dereferencing operations possibly involve `nil` pointers
  - with shape analysis: $x = \texttt{nil}$ possible for $x \in Var$ at $l \in Lab$ if there exists $G = (Abs, \Longrightarrow) \in SG_l$ such that $x \notin \bigcup_{X \in Abs} X$
- Points-To Analysis: yields function pt that for each $x \in Var$ returns set $pt(x) \subseteq Nod$ of possible pointer targets
  - x and y may be aliases if $pt(x) \cap pt(y) \neq \emptyset$
  - with shape analysis: there exists $G = (Abs, \Longrightarrow) \in SG_l$ and $Z \in Abs$ such that $x, y \in Z$
- Usually faster and sometimes more precise than shape analysis, but less general (only "shallow" properties)
- Fastest algorithms are flow-insensitive (points-to edges only added but never removed)

Graph Grammar Approaches to Pointer Analysis

- E.g., J. Heinen, C. Jansen, J.-P. Katoen, T. Noll: *Verifying Pointer Programs using Graph Grammars*. Science of Computer Programming 97, 157–162, 2015
- Idea: specify data structures by graph production rules
- Concretisation by forward application
- Abstraction by backward application
- All pointer operations remain concrete

$\Rightarrow$ Avoids involved definition of transfer functions

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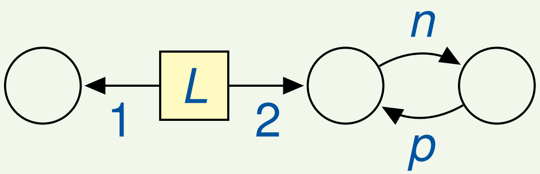

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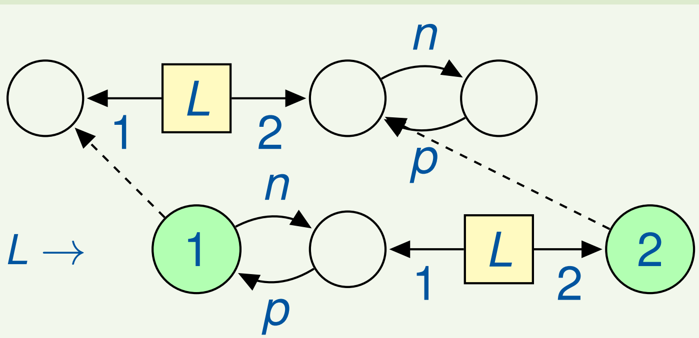

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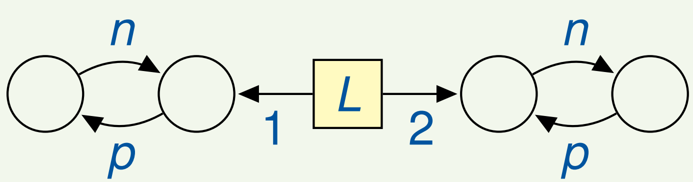

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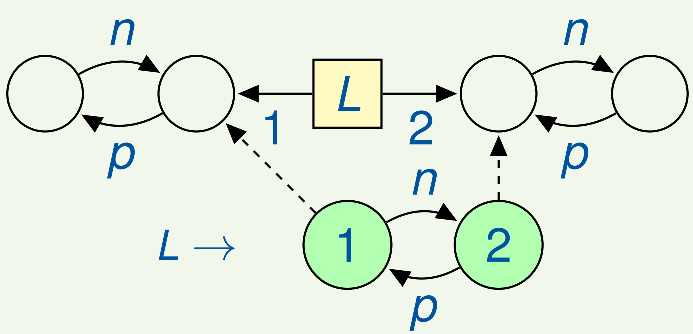

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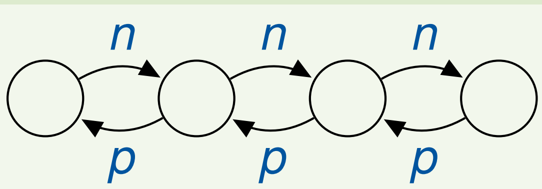
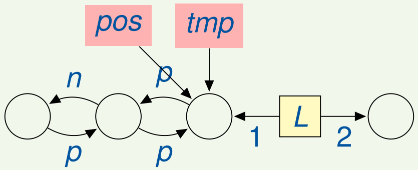


Abstract Execution Using Graph Grammars

Example 21.7 (`tmp := pos.next;`)



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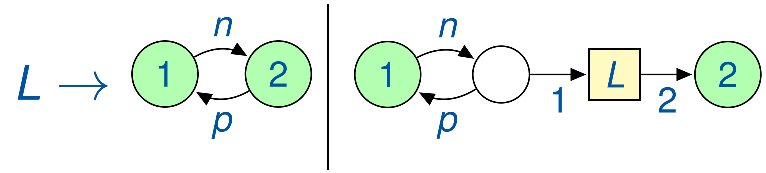

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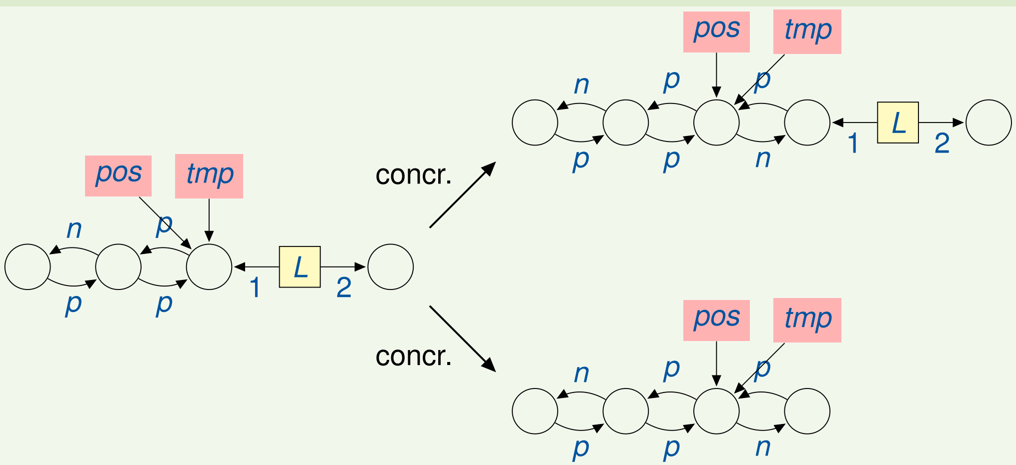

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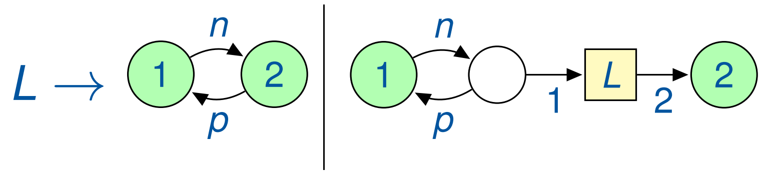

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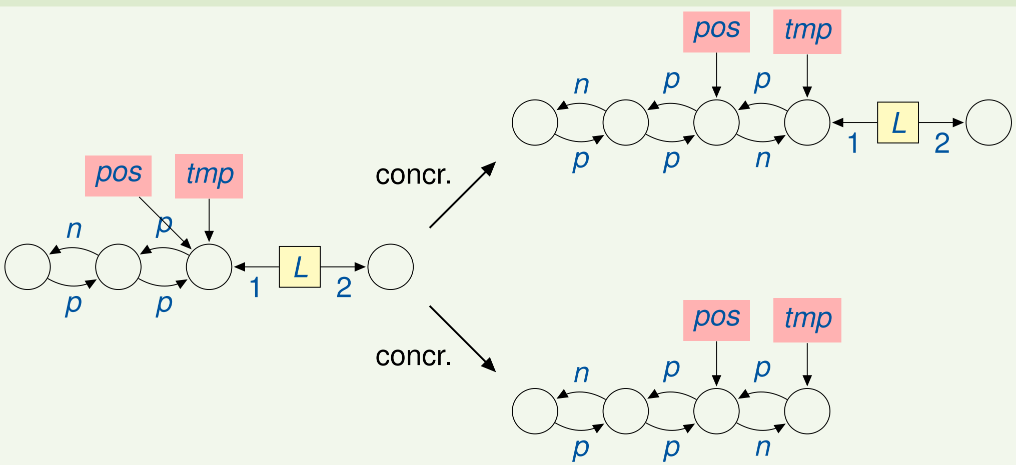

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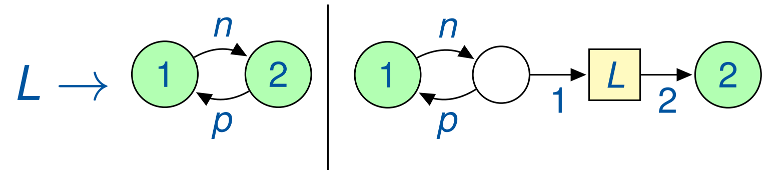

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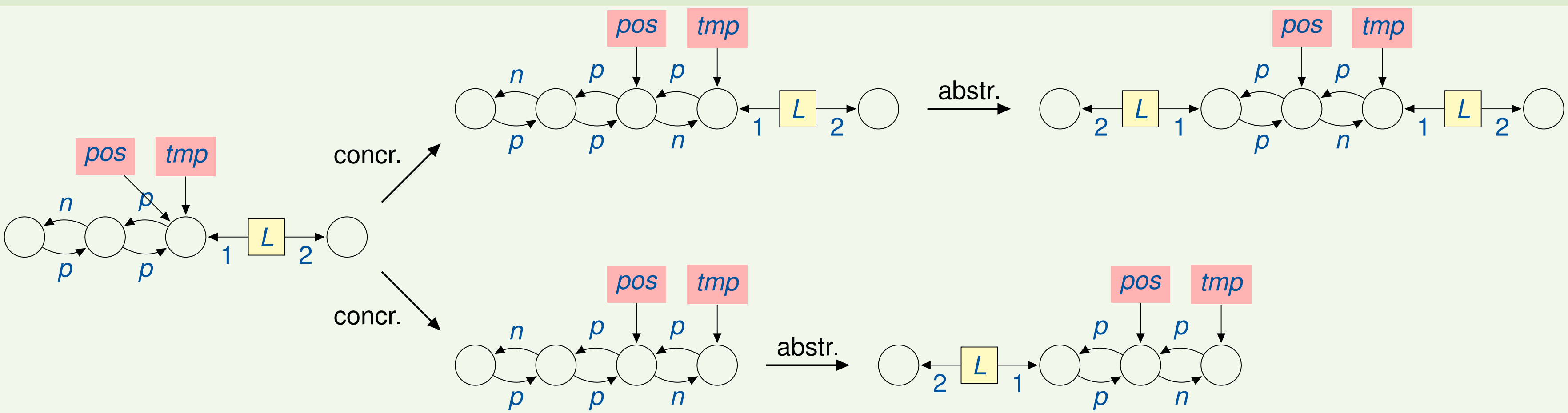

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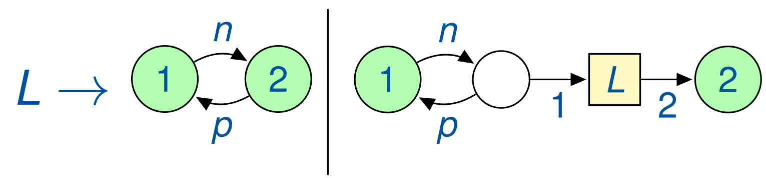

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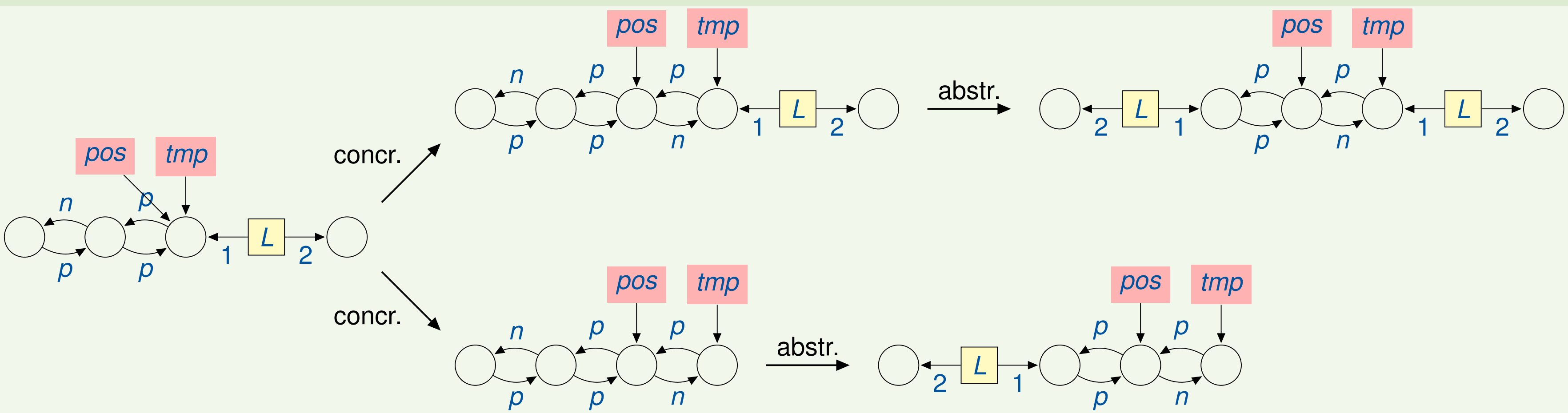

Principle

Concretise whenever necessary; abstract whenever possible.

Correctness of Dataflow Analyses

- **So far:** semantics and dataflow analysis of programs considered independently (formal soundness proofs only for abstract interpretation; cf. Lecture 12/13)

Further Topic in Program Analysis

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Correctness of Dataflow Analyses

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- Of course both are (and should be) related!
- To this aim: compare results of concrete semantics (Definition 11.9) with outcome of analysis
- See [Nielson/Nielson/Hankin 2005, Sct. 2.2] for details

Example 21.8 (Correctness of Constant Propagation)

Let $c \in Cmd$, $l \in Lab_c$, $x \in Var$, and $z \in \mathbb{Z}$ such that $\mathrm{CP}_l(x) = z$.
Then for all $\sigma_0, \sigma \in \Sigma$ such that $\langle \mathrm{init}(c), \sigma_0 \rangle \to^* \langle l, \sigma \rangle$, $\sigma(x) = z$.

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Written Exam

- Dates:
  - Tue 21 Feb, 15:00–17:00, AH 2/3
  - Thu 23 Mar, 10:00–12:00, AH 2
- Q&A session on Wed 08 Feb (12:00, AH 3)
  - please submit questions beforehand to Christina Jansen or Christoph Matheja

Forthcoming Course in SS 2017

Compiler Construction [Noll; V3 Ü2]

1. Lexical analysis of programs (Scanner)
2. Syntactic analysis of programs (Parser)
3. Semantic analysis of programs
4. Code generation
5. Tools for compiler construction