Static Program Analysis

Lecture 18: Interprocedural Dataflow Analysis I (MVP Solution)

Winter Semester 2016/17

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https://moves.rwth-aachen.de/teaching/ws-1617/spa/

Online Registration for Seminars and Practical Courses (Praktika) in Summer Term 2017

Who?

Students of:
- Master Courses
- Bachelor Informatik (~~Pro~~Seminar!)

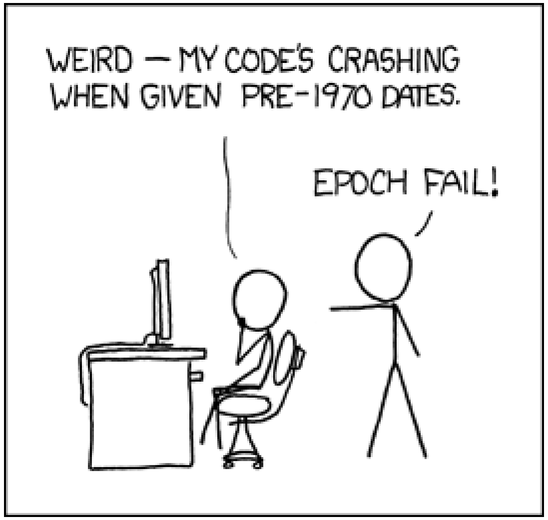
Where?

www.graphics.rwth-aachen.de/apse

When?

13.01.2017 – 29.01.2017

Seminar *Verification and Static Analysis of Software* (SS 2017)



https://xkcd.com/376

Topics

- Pointer and shape analysis
- Advanced model checking techniques
- Analysis of probabilistic programs
- ...

More information

`https://moves.rwth-aachen.de/teaching/ss-17/vsas/`

Registration

between January 13 and 29 via
`https://www.graphics.rwth-aachen.de/apse/`

Outline of Lecture 18

Interprocedural Dataflow Analysis

Intraprocedural vs. Interprocedural Analysis

The MVP Solution

Overview

- **So far:** only intraprocedural analyses (i.e., without user-defined functions or procedures or just within their bodies)

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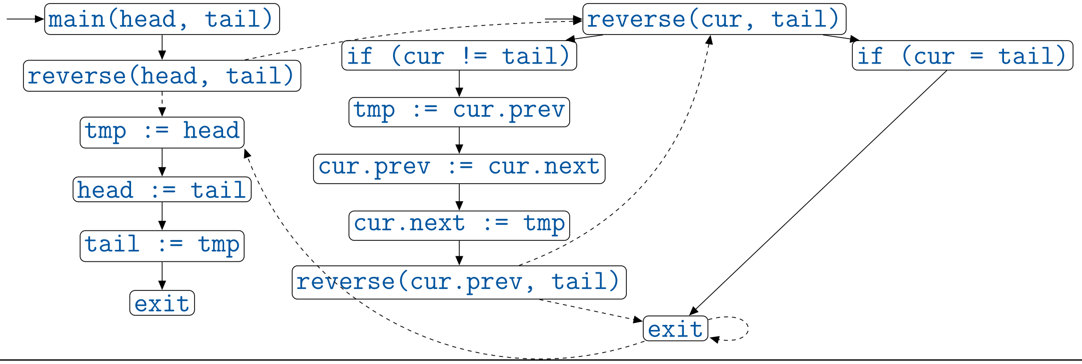
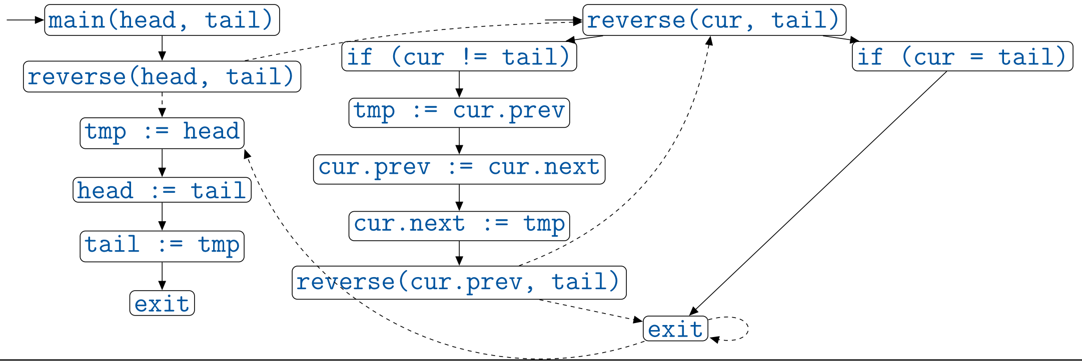
- **Now:** interprocedural dataflow analysis
- **Complications:**
  - correct matching between calls and returns
  - parameter passing (aliasing effects)



Interprocedural Dataflow Analysis

Overview

- **So far:** only intraprocedural analyses (i.e., without user-defined functions or procedures or just within their bodies)
- **Now:** interprocedural dataflow analysis
- **Complications:**
  - correct matching between calls and returns
  - parameter passing (aliasing effects)
- **Here:** simple setting
  - only top-level declarations, no blocks or nested declarations
  - mutual recursion
  - one call-by-value and one call-by-result parameter
    (extension to multiple and call-by-value-result parameters straightforward)



Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Procedure identifiers	$Pid = \{\mathtt{P}, \mathtt{Q}, \ldots\}$	P
Procedure declarations	$PDec$	p
Commands (statements)	Cmd	c

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Context-free grammar:

$$p ::= \mathtt{proc}\ [P\mathtt{(val}\ x\mathtt{,res}\ y\mathtt{)}]^{l_n}\ \mathtt{is}\ c\ [\mathtt{end}]^{l_x}\mathtt{;}p \mid \varepsilon \in PDec$$

$$c ::= [\mathtt{skip}]^{l} \mid [x \mathtt{ := } a]^{l} \mid c_1\mathtt{;}c_2 \mid \mathtt{if}\ [b]^{l}\ \mathtt{then}\ c_1\ \mathtt{else}\ c_2\ \mathtt{end} \mid \mathtt{while}\ [b]^{l}\ \mathtt{do}\ c\ \mathtt{end} \mid [\mathtt{call}\ P\mathtt{(}a\mathtt{,}x\mathtt{)}]^{l_c}_{l_r} \in Cmd$$

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$$c ::= [\mathtt{skip}]^l \mid [x := a]^l \mid c_1 ; c_2 \mid \mathtt{if}\ [b]^l\ \mathtt{then}\ c_1\ \mathtt{else}\ c_2\ \mathtt{end} \mid$$
$$\mathtt{while}\ [b]^l\ \mathtt{do}\ c\ \mathtt{end} \mid [\mathtt{call}\ P(a, x)]^{l_c}_{l_r} \in Cmd$$

- All labels and procedure names in program $p\,c$ distinct
- In $\mathtt{proc}\ [P(\mathtt{val}\ x, \mathtt{res}\ y)]^{l_n}\ \mathtt{is}\ c\ [\mathtt{end}]^{l_x}$, l_n / l_x refers to the entry / exit of P
- In $[\mathtt{call}\ P(a, x)]^{l_c}_{l_r}$, l_c/l_r refers to the call of / return from P
- First parameter call-by-value (input), second call-by-result (output)

An Example

Example 18.1 (Fibonacci numbers)

(with extension by multiple call-by-value parameters)

$$
\begin{array}{l}
\texttt{proc}\;[\texttt{Fib(val x, y, res z)}]^1\;\texttt{is} \\
\quad \texttt{if}\;[\texttt{x < 2}]^2\;\texttt{then} \\
\qquad [\texttt{z := y + 1}]^3 \\
\quad \texttt{else} \\
\qquad [\texttt{call Fib(x-1, y, z)}]^4_5\texttt{;} \\
\qquad [\texttt{call Fib(x-2, z, z)}]^6_7 \\
\quad \texttt{end} \\
[\texttt{end}]^8\texttt{;} \\
[\texttt{call Fib(5, 0, v)}]^9_{10}
\end{array}
$$

Procedure Flow Graphs I

Definition 18.2 (Procedure flow graphs; extends Def. 2.3 and 2.4)

The auxiliary functions init, final, and flow are extended as follows:

$$
\begin{aligned}
\mathrm{init}(\texttt{proc}\ [P(\texttt{val}\ x, \texttt{res}\ y)]^{l_n}\ \texttt{is}\ c\ [\texttt{end}]^{l_x}) &:= l_n \\
\mathrm{final}(\texttt{proc}\ [P(\texttt{val}\ x, \texttt{res}\ y)]^{l_n}\ \texttt{is}\ c\ [\texttt{end}]^{l_x}) &:= \{l_x\} \\
\mathrm{flow}(\texttt{proc}\ [P(\texttt{val}\ x, \texttt{res}\ y)]^{l_n}\ \texttt{is}\ c\ [\texttt{end}]^{l_x}) &:= \{(l_n, \mathrm{init}(c))\} \cup \mathrm{flow}(c) \\
&\quad\ \cup\ \{(l, l_x) \mid l \in \mathrm{final}(c)\} \\
\mathrm{init}([\texttt{call}\ P(a, x)]^{l_c}_{l_r}) &:= l_c \\
\mathrm{final}([\texttt{call}\ P(a, x)]^{l_c}_{l_r}) &:= \{l_r\} \\
\mathrm{flow}([\texttt{call}\ P(a, x)]^{l_c}_{l_r}) &:= \{(l_c; l_n), (l_x; l_r)\}
\end{aligned}
$$

Procedure Flow Graphs I

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\text{final}([\texttt{call}\ P(a,x)]^{l_c}_{l_r}) &:= \{l_r\} \\
\text{flow}([\texttt{call}\ P(a,x)]^{l_c}_{l_r}) &:= \{(l_c; l_n), (l_x; l_r)\}
\end{aligned}$$

Moreover the interprocedural flow of a program $p\ c$ is defined by

$$\begin{aligned}
\text{iflow} := \{(l_c, l_n, l_x, l_r) \mid\ & p \text{ contains } \texttt{proc}\ [P(\texttt{val}\ x,\texttt{res}\ y)]^{l_n}\ \texttt{is}\ c\ [\texttt{end}]^{l_x} \text{ and} \\
& c \text{ contains } [\texttt{call}\ P(a,x)]^{l_c}_{l_r}\} \\
\subseteq Lab^4 &
\end{aligned}$$

Procedure Flow Graphs II

Example 18.3 (Fibonacci numbers)

Flow graph of

$$
\begin{array}{l}
\texttt{proc}\ [\texttt{Fib(val x, y, res z)}]^1\ \texttt{is} \\
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(on the board)

Procedure Flow Graphs II

Example 18.3 (Fibonacci numbers)

Flow graph of

```
proc [Fib(val x, y, res z)]^1 is
  if [x < 2]^2 then
    [z := y + 1]^3
  else
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  end
[end]^8;
[call Fib(5, 0, v)]^9_10
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(on the board)

Here iflow $= \{(9,1,8,10),(4,1,8,5),(6,1,8,7)\}$

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The MVP Solution

Intraprocedural vs. Interprocedural Analysis

Naive Formulation I

- **Attempt:** directly transfer techniques from intraprocedural analysis

  $\Longrightarrow$ treat $(l_c; l_n)$ like (l_c, l_n) and $(l_x; l_r)$ like (l_x, l_r)

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- **Attempt:** directly transfer techniques from intraprocedural analysis
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- Given: dataflow system $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$
- For each procedure call $[\texttt{call } P(a, x)]^{l_c}_{l_r}$:
  transfer functions $\varphi_{l_c}, \varphi_{l_r} : D \to D$ (definition later)
- For each procedure declaration $\texttt{proc } [P(\texttt{val } x, \texttt{res } y)]^{l_n} \texttt{ is } c \ [\texttt{end}]^{l_x}$:
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  transfer functions $\varphi_{l_n}, \varphi_{l_x} : D \to D$ (definition later)
- Induces equation system

$$\mathrm{AI}_l = \begin{cases} \iota & \text{if } l \in E \\ \bigsqcup\{\varphi_{l'}(\mathrm{AI}_{l'}) \mid (l', l) \in F \text{ or } (l'; l) \in F\} & \text{otherwise} \end{cases}$$

Intraprocedural vs. Interprocedural Analysis

Naive Formulation I

- **Attempt:** directly transfer techniques from intraprocedural analysis
  $\Longrightarrow$ treat $(l_c; l_n)$ like (l_c, l_n) and $(l_x; l_r)$ like (l_x, l_r)
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- **Problem:** procedure calls $(l_c; l_n)$ and procedure returns $(l_x; l_r)$ treated like goto's
  $\Longrightarrow$ nesting of calls and returns ignored
  $\Longrightarrow$ too many paths considered
  $\Longrightarrow$ analysis information possibly imprecise (but still correct)

Naive Formulation II

Example 18.4 (Fibonacci numbers)

$\texttt{proc}\ [\texttt{Fib(val x, y, res z)}]^{1}\ \texttt{is}$
$\quad \texttt{if}\ [\texttt{x < 2}]^{2}\ \texttt{then}$
$\quad\quad [\texttt{z := y + 1}]^{3}$
$\quad \texttt{else}$
$\quad\quad [\texttt{call Fib(x-1, y, z)}]^{4}_{5};$
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\end{array}$$

- “Valid” path: $[9, 1, 2, 3, 8, 10]$

Naive Formulation II

Example 18.4 (Fibonacci numbers)

$\texttt{proc [Fib(val x, y, res z)]}^1\ \texttt{is}$
$\quad\texttt{if [x < 2]}^2\ \texttt{then}$
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$\quad\texttt{end}$
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$\texttt{[call Fib(5, 0, v)]}^9_{10}$

- "Valid" path: $[9, 1, 2, 3, 8, 10]$
- "Invalid" path: $[9, 1, 2, 4, 1, 2, 3, 8, 10]$

Naive Formulation III

Example 18.5 (Impreciseness of constant propagation analysis)

$[\texttt{proc}\ [\texttt{P(val x, res y)}]^1\ \texttt{is}$

$\quad [\texttt{y := x}]^2$

$[\texttt{end}]^3;$

$\texttt{if}\ [\texttt{y = 0}]^4\ \texttt{then}$

$\quad [\texttt{call P(1, y)}]^5_6;$

$\quad [\texttt{y := y - 1}]^7$

$\texttt{else}$

$\quad [\texttt{call P(2, y)}]^8_9;$

$\quad [\texttt{y := y - 2}]^{10}$

$\texttt{end};$

$[\texttt{skip}]^{11}$

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\texttt{else} \\
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\quad [\texttt{y := y - 2}]^{10} \\
\texttt{end;} \\
[\texttt{skip}]^{11}
\end{array}
$$

Two “valid” and two “invalid” paths:

- Valid: $[4, 5, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $\texttt{y} = 0$ at label 11

Naive Formulation III

Example 18.5 (Impreciseness of constant propagation analysis)

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proc [P(val x, res y)]^1 is
  [y := x]^2
[end]^3;
if [y = 0]^4 then
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  [y := y - 1]^7
else
  [call P(2, y)]_9^8;
  [y := y - 2]^10
end;
[skip]^11
```

Two “valid” and two “invalid” paths:

- Valid: $[4, 5, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Valid: $[4, 8, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = 0$ at label 11

Naive Formulation III

Example 18.5 (Impreciseness of constant propagation analysis)

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proc [P(val x, res y)]^1 is
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[end]^3;
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else
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  [y := y - 2]^10
end;
[skip]^11
```

Two “valid” and two “invalid” paths:

- Valid: $[4, 5, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Valid: $[4, 8, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Invalid: $[4, 5, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = -1$ at label 11

Naive Formulation III

Example 18.5 (Impreciseness of constant propagation analysis)

$\texttt{proc}\ [\texttt{P(val x, res y)}]^1\ \texttt{is}$
$\quad[\texttt{y := x}]^2$
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$\texttt{if}\ [\texttt{y = 0}]^4\ \texttt{then}$
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Two “valid” and two “invalid” paths:

- Valid: $[4, 5, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Valid: $[4, 8, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Invalid: $[4, 5, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = -1$ at label 11
- Invalid: $[4, 8, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $y = 1$ at label 11

Naive Formulation III

Example 18.5 (Impreciseness of constant propagation analysis)

```
proc [P(val x, res y)]^1 is
  [y := x]^2
[end]^3;
if [y = 0]^4 then
  [call P(1, y)]_6^5;
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else
  [call P(2, y)]_9^8;
  [y := y - 2]^10
end;
[skip]^11
```

Two "valid" and two "invalid" paths:

- Valid: $[4, 5, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Valid: $[4, 8, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = 0$ at label 11
- Invalid: $[4, 5, 1, 2, 3, 9, 10, 11]$
  $\Longrightarrow$ $y = -1$ at label 11
- Invalid: $[4, 8, 1, 2, 3, 6, 7, 11]$
  $\Longrightarrow$ $y = 1$ at label 11

$\Longrightarrow$ actually always $y = 0$ at 11, but naive method yields $y = \top$

Outline of Lecture 18

Interprocedural Dataflow Analysis

Intraprocedural vs. Interprocedural Analysis

The MVP Solution

Valid Paths I

- Consider only paths with correct nesting of procedure calls and returns
- Will yield MVP solution (Meet over all Valid Paths)

Definition 18.6 (Valid path fragments)

Given a dataflow system $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ and $l_1, l_2 \in Lab$, the set of valid paths from l_1 to l_2 is generated by the nonterminal symbol $P[l_1, l_2]$ according to the following context-free grammar:

$$
\begin{array}{rll}
P[l_1, l_2] & \to l_1 & \text{whenever } l_1 = l_2 \\
P[l_1, l_3] & \to l_1, P[l_2, l_3] & \text{whenever } (l_1, l_2) \in F \\
P[l_c, l] & \to l_c, P[l_n, l_x], P[l_r, l] & \text{whenever } (l_c, l_n, l_x, l_r) \in \text{iflow}
\end{array}
$$

Valid Paths II

Example 18.7 (Fibonacci numbers; cf. Example 18.4)

```
proc [Fib(val x, y, res z)]^1 is
  if [x < 2]^2 then
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[end]^8;
[call Fib(5, 0, v)]^9_10
```

Reminder:

$P[l_1, l_2] \to l_1$ for $l_1 = l_2$

$P[l_1, l_3] \to l_1, P[l_2, l_3]$ for $(l_1, l_2) \in F$

$P[l_c, l] \to l_c, P[l_n, l_x], P[l_r, l]$

for $(l_c, l_n, l_x, l_r) \in$ iflow

Valid Paths II

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\begin{array}{l}
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Reminder:

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P[l_1, l_2] & \to l_1 \text{ for } l_1 = l_2 \\
P[l_1, l_3] & \to l_1, P[l_2, l_3] \text{ for } (l_1, l_2) \in F \\
P[l_c, l] & \to l_c, P[l_n, l_x], P[l_r, l] \\
 & \quad \text{for } (l_c, l_n, l_x, l_r) \in \text{iflow}
\end{array}
$$

Valid paths from 9 to 10:

$$
\begin{array}{rl}
P[9, 10] & \to 9, P[1, 8], P[10, 10] \\
P[1, 8] & \to 1, P[2, 8] \\
P[2, 8] & \to 2, P[3, 8] \\
P[2, 8] & \to 2, P[4, 8] \\
P[3, 8] & \to 3, P[8, 8] \\
P[4, 8] & \to 4, P[1, 8], P[5, 8] \\
P[5, 8] & \to 5, P[6, 8] \\
P[6, 8] & \to 6, P[1, 8], P[7, 8] \\
P[7, 8] & \to 7, P[8, 8] \\
P[8, 8] & \to 8 \\
P[10, 10] & \to 10
\end{array}
$$

Valid Paths II

Example 18.7 (Fibonacci numbers; cf. Example 18.4)

```
proc [Fib(val x, y, res z)]^1 is
  if [x < 2]^2 then
    [z := y + 1]^3
  else
    [call Fib(x-1, y, z)]^4_5;
    [call Fib(x-2, z, z)]^6_7
  end
[end]^8;
[call Fib(5, 0, v)]^9_10
```

Reminder:

$P[l_1, l_2] \to l_1$ for $l_1 = l_2$

$P[l_1, l_3] \to l_1, P[l_2, l_3]$ for $(l_1, l_2) \in F$

$P[l_c, l] \to l_c, P[l_n, l_x], P[l_r, l]$ for $(l_c, l_n, l_x, l_r) \in iflow$

Valid paths from 9 to 10:

$P[9, 10] \to 9, P[1, 8], P[10, 10]$

$P[1, 8] \to 1, P[2, 8]$

$P[2, 8] \to 2, P[3, 8]$

$P[2, 8] \to 2, P[4, 8]$

$P[3, 8] \to 3, P[8, 8]$

$P[4, 8] \to 4, P[1, 8], P[5, 8]$

$P[5, 8] \to 5, P[6, 8]$

$P[6, 8] \to 6, P[1, 8], P[7, 8]$

$P[7, 8] \to 7, P[8, 8]$

$P[8, 8] \to 8$

$P[10, 10] \to 10$

Thus $[9, 1, 2, 3, 8, 10] \in L(P[9, 10])$,
$[9, 1, 2, 4, 1, 2, 3, 8, 10] \notin L(P[9, 10])$

The MVP Solution I

Definition 18.8 (Complete valid paths)

Let $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system. For every $l \in Lab$, the set of valid paths up to l is given by

$$VPath(l) := \{[l_1, \ldots, l_{k-1}] \mid k \geq 1, l_1 \in E, l_k = l, [l_1, \ldots, l_k] \text{ valid path from } l_1 \text{ to } l_k\}.$$

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For $\pi = [l_1, \ldots, l_{k-1}] \in VPath(l)$, we define the transfer function $\varphi_\pi : D \to D$ by

$$\varphi_\pi := \varphi_{l_{k-1}} \circ \ldots \circ \varphi_{l_1} \circ \mathrm{id}_D$$

(so that $\varphi_{[]} = \mathrm{id}_D$).

The MVP Solution II

Definition 18.9 (MVP solution)

Let $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system where $Lab = \{l_1, \ldots, l_n\}$.
The MVP solution for S is determined by

$$\mathrm{mvp}(S) := (\mathrm{mvp}(l_1), \ldots, \mathrm{mvp}(l_n)) \in D^n$$

where, for every $l \in Lab$,

$$\mathrm{mvp}(l) := \bigsqcup\{\varphi_\pi(\iota) \mid \pi \in VPath(l)\}.$$

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Corollary 18.10

1. $\mathrm{mvp}(S) \sqsubseteq \mathrm{mop}(S)$
2. *The MVP solution is undecidable.*

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Corollary 18.10

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Proof.

1. since $VPath(l) \subseteq Path(l)$ for every $l \in Lab$
2. as $\mathrm{mvp}(S) = \mathrm{mop}(S)$ in intraprocedural case and MOP solution undecidable (Thm. 7.1) □