General Remarks

- Please hand in your solutions in groups of 3. Either hand in your solutions at the beginning of the exercise class or put them into the box at the chair.
- If you have questions regarding the exercises and/or lecture, feel free to write us an email or visit us at our office.

Exercise 1 (Extending Interval Analysis): (2 Points)

The WHILE-language presented in the lecture does not feature a division operator /. In this exercise we aim to incorporate this operator in the language and adapt the interval analysis accordingly. For simplicity, we assume that a division by zero yields an arbitrary value.

a) Extend the val_δ function for interval analysis to account for division (be as precise as possible).

b) Show that the transfer function of interval analysis (including your extension for division) is monotonic.

Exercise 2 (Limiting Fixed Point Iterations): (1 Points)

Let $(D, \sqsubseteq)$ be a complete lattice with greatest element $\top$ and least element $\bot$. Moreover, let $F : D \to D$ be a monotone function. For a fixed $k \in \mathbb{N}$, we define $\nabla_k : D \times D \to D$ as

$$d_1 \nabla_k d_2 = \begin{cases} d_1 \sqcup d_2 & \text{if } (d_1 \sqcup d_2) \sqsubseteq F^k(\bot) \\ \top & \text{otherwise} \end{cases}$$

Prove or disprove: For every $k \in \mathbb{N}$, ∇_k is a widening operator with respect to $(D, \sqsubseteq)$.

Exercise 3 (Widening): (6 Points)

Consider the domain $D = (\mathbb{N} \times \{0, 1\}) \cup \{\infty\}$.

a) Define a relation $\sqsubseteq \subseteq D \times D$ such that $(D, \sqsubseteq)$ is a complete lattice that contains both infinite ascending and infinite descending chains. Justify your answer.

b) Define a relation $\preceq \subseteq D \times D$ such that $(D, \preceq)$ is a complete lattice that contains both infinitely many pairwise disjoint infinite ascending and infinitely many pairwise disjoint infinite descending chains. Justify your answer.

c) Define widening operators for both $(D, \sqsubseteq)$ and $(D, \preceq)$. Justify your answer.

Exercise 4 (Widening as Upper Bounds): (1 Points)

Prove or disprove: For every complete lattice $(D, \sqsubseteq)$ and every widening $\nabla : D \times D \to D$, there exists a non-empty set $S \subseteq D$ such that $(S, \preceq)$, where $s \preceq s'$ iff $s \nabla s' = s'$, is a complete lattice.