General Remarks

- Please hand in your solutions in groups of 3. Either hand in your solutions at the beginning of the exercise class or put them into the box at the chair.
- If you have questions regarding the exercises and/or lecture, feel free to write us an email or visit us at our office.

Exercise 1 (Worklist Algorithm):

Perform a live variable analysis on the following program using the worklist algorithm.

```plaintext
skip;
while ¬(x ≥ 3 * y) do x := x + y end
y := x;
```

Exercise 2 (Complexity of LVA Fixpoint Iteration):

In the lecture we saw that fixpoint iteration requires at most $m \cdot n$ steps, where $m$ is the height of the PO while $n$ is the number of program points (i.e. labels). But how fast is the iterative algorithm for a concrete analysis (here: live variables)?

a) Show that LVA has the following property:
   Let $c \in Cmd$, $x \in Var_c$ and $l \in L_c$. If $x$ is live on the exit of $l$, then there exists an acyclic path from $B^l$ to a use of $x$ that does not re-define $x$.

b) Show that (standard) fixpoint iteration as seen in the lecture requires at most $|L_c|$ steps for convergence in case of LVA.

Exercise 3 (Definedness Analysis):

a) Formally describe how the constant propagation analysis can be turned into a “definedness” analysis that determines for every block which of the variables have a defined value (i.e. were initialised with an expression containing constants and initialised variables only). Assume that initially all variables have an undefined value.

b) Prove or disprove: The MOP solution for the resulting analysis is decidable.

Exercise 4 (Distributivity of Transfer Functions):

Consider a partial order of the form $(D, \subseteq)$, where $D = 2^M$ and $M$ a finite set. A transfer function $\varphi_l : D \to D$ is called distributive iff for any $d_1, d_2 \in D$:

$$\varphi_l(d_1 \sqcup d_2) = \varphi_l(d_1) \cup \varphi_l(d_2)$$

Show that any transfer function of the form $\varphi_l(d) = (d \setminus \text{kill}(B^l)) \cup \text{gen}(B^l)$ is distributive in this setting.