General Remarks

- Please hand in your solutions in groups of 3.
- If you cannot access the L2P due to registration issues please contact us such that we can add you manually.
- We will publish solutions to the exercises in the L2P. All other material, such as slides and exercise sheets, are distributed on our webpage.
- If you have questions regarding the exercises and/or lecture, feel free to write us an email or visit us at our office.

Exercise 1 (Soundness): (1 Points)

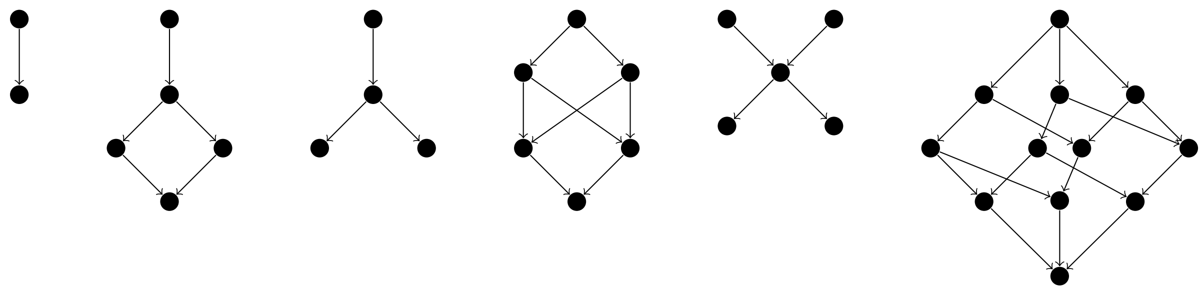
Consider an alternative transfer function $\phi(V) := (V \cup gen_{LV}(B^{l'})) \setminus kill_{LV}(B^{l'})$ for the live variable analysis. Prove or disprove its soundness (i.e., the analysis generates exactly the same results as the live variable analysis presented in lecture).

Exercise 2 (Lattice Graphs): (2 Points)

One can depict a finite partial order as a graph, where the elements are represented by the nodes and the order relation is given as (the transitive closure of) directed edges. An edge between two nodes indicates, that the corresponding elements are comparable where the direction of the edge is determined by the order and there is no element that lies "in between".

a) Depict the lattice of LVA with $Var_c := \{x, y, z\}$ where $\top$ is the upper node.

b) Consider the following graphs and decide for each if it represents a lattice. Justify you answer.



Exercise 3 (Chains): (2 Points)

a)
- Modify the domain $(\mathbb{Z}_{\geq 0}, \geq)$ such that it forms a complete lattice satisfying ACC.
- Consider the set $S := \{A \in 2^{\mathbb{N}} \mid A \text{ is downward closed w.r.t. } \leq\}$[1]. Does $(S, \subseteq)$ satisfy ACC? Does there exist a $k \in \mathbb{N}$, such that the height of S is bounded by k?

Argue why your answers are correct.

b) Show that a lattice satisfies ACC if all its chains are finite.

[1] A partially ordered set is downward closed if for every element of the set all smaller elements are also in it.

Exercise 4 (Exploiting the Framework): (5 Points)

a) Develop a (non-trivial) *sign analysis* for program variables based on an abstraction that maps all negative numbers to the symbol $-$, zero to the symbol 0 and all positive numbers to $+$. E.g. the set $\{0, 1, 2, \ldots\}$ is abstracted to $\{0, +\}$.

b) Show that the domain defined in your solution to a) satisfies ACC.

c) Show that the transfer functions defined in your solution to a) are indeed monotonic.

d) Perform your analysis on the following program:

```
x := 0;
y := -1;
z := x * y;
if (z = 0)
  x := x + 1;
else
  x := y - x;
z := x * y;
```