General Remarks

• Please hand in your solutions in groups of 3. Either hand in your solutions at the beginning of the exercise class or put them into the box at the chair.

• If you have questions regarding the exercises and/or lecture, feel free to write us an email or visit us at our office.

Exercise 1 (LUB and GLB of Predicate Abstraction): (3 Points)

Given the set of predicates \( P = \{ p_1, \ldots, p_4 \} \) with

- \( p_1 := x > y \)
- \( p_2 := x > 3 \)
- \( p_3 := y \leq 2 \)
- \( p_4 := x = 2 \)

Provide the LUB \( \sqcup \) and GLB \( \sqcap \) of the following subsets of \( P \)

a) \( Q_1 = \{ p_1 \} \) and \( Q_2 = \{ p_3 \} \)

b) \( Q_1 = \{ p_1, p_2 \} \) and \( Q_2 = \{ p_2, p_4 \} \)

c) \( Q_1 = \{ p_1, \neg p_2 \} \) and \( Q_2 = \{ \neg p_3 \} \)

Exercise 2 (Predicate Abstraction and CEGAR): (7 Points)

Consider the following program fragment \( c \).

\[
\begin{align*}
[y := b]_1; \\
\text{if } [b > 0]_2 \text{ then } [y := y - 1]_3; \\
\text{if } ([y < 0]_4 \text{ then } [\text{skip}]_5 \text{ else } [\text{skip}]_6 \text{ end}; \\
\text{else } [\text{skip}]^7 \text{ end};
\end{align*}
\]

Show that label 5 is not reachable.

a) Give the abstract transition system of \( c \) for the set of predicates \( P = \emptyset \).

b) Provide a spurious counterexample for your initial abstraction from (a).

c) Compute the strongest postconditions \( P' \) for your counterexample from (b).

d) Execute one abstraction refinement step with the help of your counterexample from (b).

e) Is this refinement step sufficing to show that label 5 is not reachable?