Exercise Sheet 6

General remarks:

- **Due date:** February 3rd.

- Since there is **no exercise class on January 27th**, you may either submit your solution via email to propro1617@i2.informatik.rwth-aachen.de, or put your solutions into the box labeled *Static Program Analysis* at the chair. In the latter case, please make sure to mark them as submissions to Probabilistic Programming. (before the exercise class).

- Solutions must be written in English.

- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

Exercise 1   (Random Walks)   33%

A monkey is standing one step from the edge of a cliff and takes repeated independent steps; forward, with probability p, or backward, with probability $1 - p$, as depicted below:
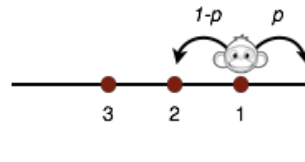
Show that if $p > 1 - p$, then the monkey will almost surely fall off the cliff, i.e. reach position 0.

Exercise 2   (Hardness of lower bound for program expectations)   34%

Racall that

$$\mathrm{E}_{P,\sigma}\left(f\right) \;=\; \sum_{k=0}^{\infty} \sum_{w \in \{L,\,R\}^{\leq k}} \wp\big(\mathrm{T}_k(\gamma_{P,\sigma},\, w),\, f\big)$$

gives the expected value of pGCL program P with respect to post–expectation f on initial state σ. The (outer–most) infinite sum intuitively ranges over the length of all terminating executions.

Given a positive rational number q prove that the problem of deciding whether

$$q < \mathrm{E}_{P,\sigma}\left(f\right)$$

belongs to Σ_1.

Exercise 3   (Hardness of approximating variances)   33%

Let P be a pGCL program (with observe), σ be an initial state and f be an expectation. Then the (conditional) *variance* of P with respect to f and σ is defined as

$$Var[P,\sigma](f) = \underline{\mathsf{cwp}}[P](f^2)(\sigma) - \underline{\mathsf{cwp}}[P](f)(\sigma) \cdot \underline{\mathsf{cwp}}[P](f)(\sigma).$$

Show that computing lower bounds on the variance, i.e. deciding whether $Var[P,\sigma](f) > q$ for some fixed rational number q, is Σ_2^0–hard.