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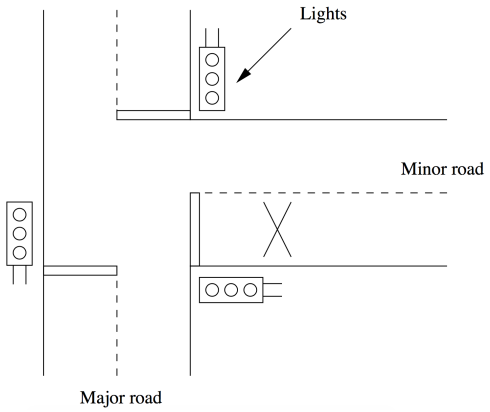
Exercise 1 (Modelling with timed automata): (1+2+2 points)

A control system must ensure the safe and correct functioning of a set of traffic lights (green, yellow, red) at a T-junction between a major and a minor road. The lights will be set to green on the major road and red on the minor road unless a vehicle is detected by a sensor in the road just before the lights on the minor road. In this case the lights will be switchable in the standard manner and allow traffic to leave the minor road. After a suitable interval the lights will revert to their default position to allow traffic to flow on the major road again. Once a vehicle is detected the sensor will be disabled until the minor-road lights are set to red again. A sketch of the T-junction is provided below.



Questions:

a) First we ignore all timing issues involved and concentrate on the qualitative aspects of the behavior of the traffic lights. Model the traffic lights for the major-road and minor-road as transition systems. For convenience, you may assume that the two major-road lights are fully synchronized and can be modeled as a single light. Complement your system model by adding a third transition system that regulates the arrival of cars in the minor road. Finally, give the resulting parallel composition of all three transition systems by syncronizing over the common actions.

b) Extend the transition systems of the traffic lights to timed automata that incorporate the following timing constraints. Deal with each timing constraint separately so as to reduce the complexity. Indicate for each timing constraint the necessary adaptations to your untimed model:

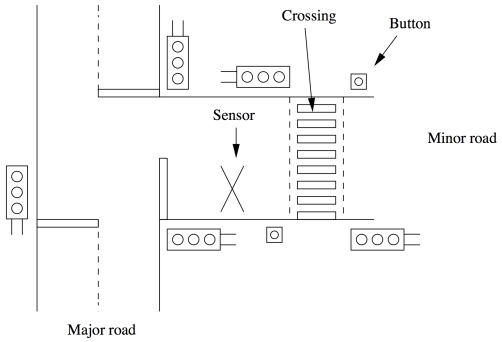
(i) a minor-road light stays on green for 30 seconds

(ii) all interim lights stay on for 5 seconds

(iii) there is a one second delay between switching from yellow to red.

(iv) the major-road lights must be on green for at least 30 seconds in each cycle

c) We extend the T-junction in the following way. Suppose there is a pedestrian crossing a short distance down the minor road but beyond the sensor. There is a button on each side of the road for pedestrians to indicate they wish to cross. The crossing should only allow people to cross when the 'minor lights' are set to red in order to minimize waiting times for traffic on the minor road. The new situation is sketched below.



Extend your timed model of the previous question in order to cope with this new situation.
Justify that the crossing indeed only allows pedestrians to cross when the 'minor lights' are set to red.

Exercise 2 (Timed automata semantics): (3+2 points)

a) Give the formal semantics of the following timed automaton TA by means of a transition system $TS(TA) = (S, Act, \rightarrow, I, AP, L)$. That is, formally define the components of the (infinite) transition system.

b) Compute the set of reachable states in $TS(TA)$.