Exercise 1 (Symbolic bisimulation): (0.5+0.5+1 points)

Let $\bar{x}, \bar{x}'$ and $\bar{b}$ be three vectors of Boolean variables of size $n > 0$ and let $a_i$ denote the $i$-th variable in a vector $\bar{a}$. Assume that the transition relation of a transition system $TS$ with state space $S = Eval(\bar{x})$ is given by means of a switching function $\Delta : Eval(\bar{x}, \bar{x}') \rightarrow \{0, 1\}$ (as seen in the lecture). Let $Q = \{B_0, ..., B_{2^n-1}\}$ be a partition of the state space ($S = \bigcup_{i=0}^{2^n-1} B_i$ and $B_i = \emptyset$ is possible) represented as a switching function $f_Q : Eval(\bar{x}, \bar{b}) \rightarrow \{0, 1\}$ defined by

$$f_Q(s, B_i) = 1 \iff s \in B_i$$

(a) Using the available switching functions and the usual operations on them, define another switching function $f_{\text{sig}} : Eval(\bar{x}, \bar{b}) \rightarrow \{0, 1\}$ given by

$$f_{\text{sig}}(s, B_i) = 1 \iff \exists s' \in \text{Post}(s) : s' \in B_i$$

(b) Let the characteristic functions $\chi_{Sat}(c)$ and $\chi_{Sat}(d)$ for the only two atomic propositions $c, d \in AP$ in $TS$ be given. Define a switching function $f_Q^* : Eval(\bar{x}, \bar{b}) \rightarrow \{0, 1\}$ that represents the partition $Q^* = \{|s| L(s) = A| A \in 2^{AP}\}$, i.e., the coarsest partition that respects the labeling.

You may use the usual operations on switching functions as well as the “special” switching functions $i|_E : Eval(\bar{b}) \rightarrow \{0, 1\}$ that evaluate to 1 if and only if the input is the binary encoding of the number $i$.

(c) Conceptually explain how the functions $f_Q(s, B_i)$ and $f_{\text{sig}}(s, B_i)$ can be used to compute the coarsest bisimulation equivalence on $TS$.

Exercise 2 (Bounded paths): (0.5+0.5 points)

For LTL formula $\varphi$ we write $TS \models_k \exists \varphi$ iff there exists a path $\pi \in \text{Paths}(TS)$ for which $\pi \models_k \varphi$. Notations like $TS \models_k \forall \varphi$ and $TS \models \exists \varphi$ are analogous.

Consider the transition system $TS$ depicted below.

Determine all $k \geq 0$ for which the following holds.

a) $TS \models_k \forall a$

b) $TS \models_k \exists \Box a$

Justify your answers!

Exercise 3 (Bounded model checking): (1+3+1 points)

Perform bounded model checking on the following transition system $TS$.

(a) Let a state $s$ be represented as $s = (s[0], s[1])$. Give the SAT representation of the transition relation $T(s, s')$, the initial state(s) $I(s)$ and the atomic proposition $a(s)$. 
(b) Given the property $\rho := \Box(\neg a \lor \Diamond \neg a)$ and the bound $k := 3$ generate the SAT encoding for the bounded model checking problem $[TS, \neg \rho]_3$ by especially specifying:
- The unfolding of the transition relation: $[TS]_3$
- The loop condition: $L_3$
- The translation for paths without loops: $[\neg \rho]_3$
- The translation for paths with loops: $\ell[\neg \rho]_3$ (for variable $\ell$)

(c) Try to find a satisfying assignment for the SAT encoding and give the resulting counterexample if one can be found.

**Exercise 4 (Bounded semantics):**

(1+1 points)  

Proof the following statements for finite transition systems $TS$ without terminal states and LTL formulas of the form

$$\varphi ::= a \mid \neg a \mid \Box a \mid \Diamond a \mid \Diamond a$$

where $a$ is an atomic proposition.

a) For any $\pi \in Paths(TS)$ and $k \geq 0$ it holds that $\pi \models_k \varphi \implies \pi \models \varphi$.

b) If $TS \models \exists \varphi$ then there exists $k \geq 0$ with $TS \models_k \exists \varphi$. 
