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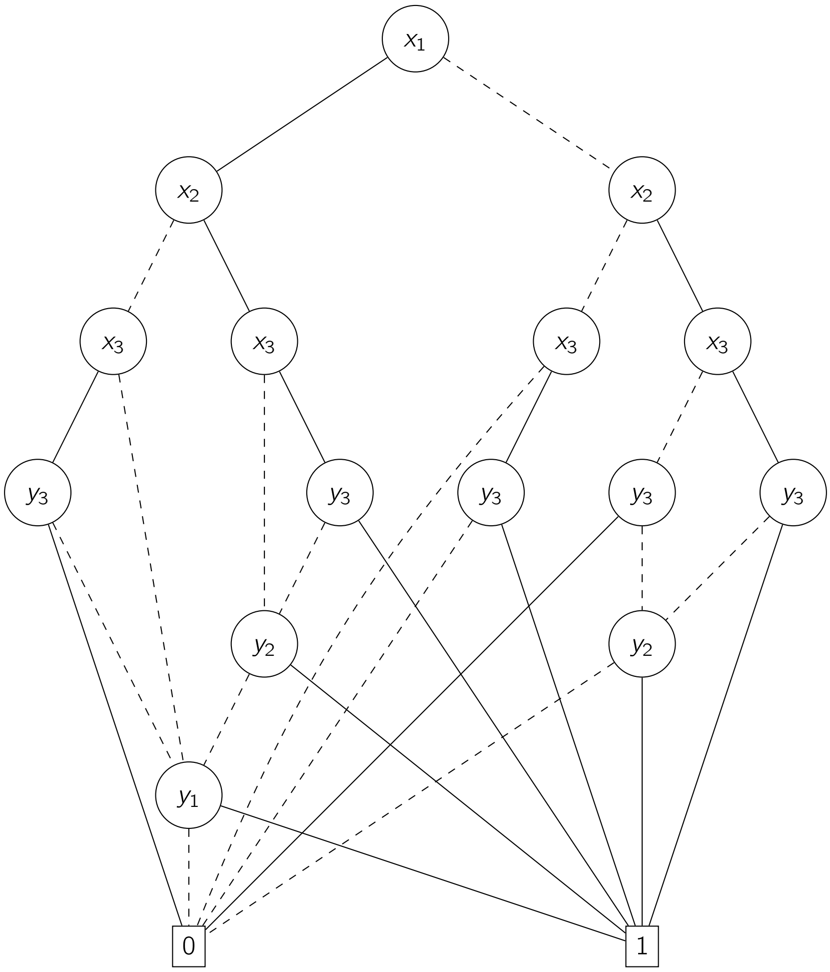
Exercise 1 (Boolean function representation): (1+1 points)

Consider the ROBDD depicted below.

a) We consider a new variable ordering given by $y_3 < x_3 < x_2 < y_2 < x_1 < y_1$. Give the resulting **R**OBDD.

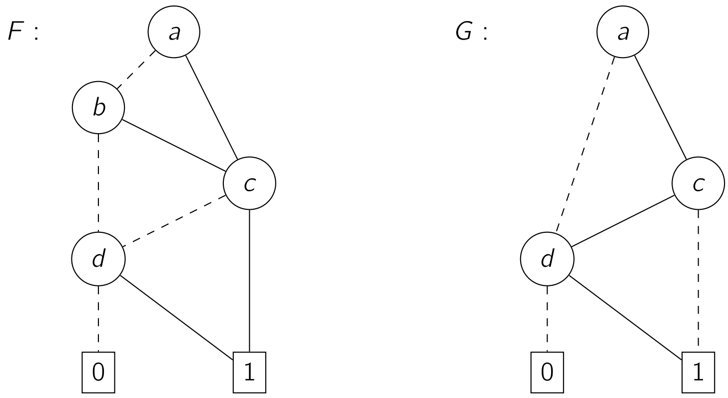
b) Determine the boolean function $f(x_1, x_2, x_3, y_1, y_2, y_3)$ that the ROBDD represents.



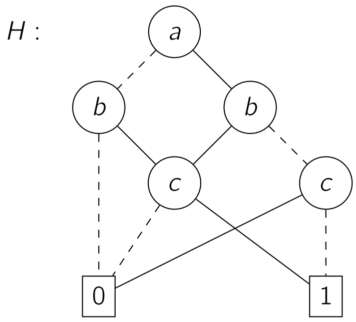
Exercise 2 (Operations on ROBDDs): (1+1+2 points)

Perform the following operations on ROBDDs. Your result should always be a **reduced** OBDD. Provide the intermediate steps of your computations. In particular, indicate which of the three rules for reducing OBDDs (slide 29) you apply. The variable ordering is given by $a < b < c < d$.

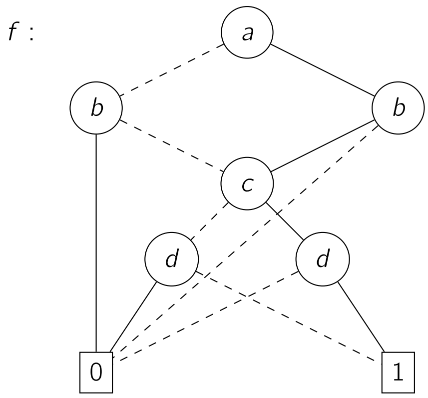
a) Compute $F \lor G$ for the following ROBDDs F and G.
Hint: Recall the similarities between ROBDDs and DFAs. A (possibly non-reduced) OBDD for $F \lor G$ can be constructed similarly to the well-known product construction for DFAs.



b) Compute $H|_{b=1}$ for the ROBDD H given below



c) Compute $\exists a.(\exists d.f(a,b,c,d))$ in the form of an ROBDD for the function f defined by the ROBDD below.
Hint: $\exists a.g(a,\dots) \equiv g(0,\dots) \vee g(1,\dots)$



Exercise 3 (ROBDD construction): (1+1 points)

Consider the family of functions (for $k > 0$) given by

$$f_k(x_0, \dots, x_{n-1}, a_0, \dots, a_{k-1}) = x_{|a|}$$

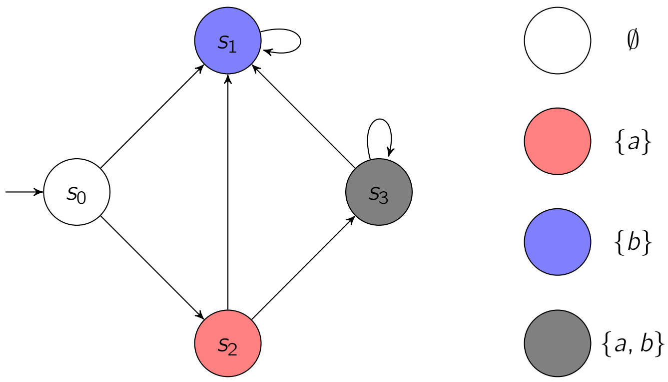
where $n = 2^k$, $x_i \in \{0,1\}$ for $0 \leq i < n$ and $a_j \in \{0,1\}$ for $0 \leq j < k$ and $|a| = \sum_{j=0}^{k-1} a_j 2^j$. In other words, the function returns the $|a|$-th bit of $(x_1 \dots x_n)$.

(a) Provide the ROBDDs that represent f_3 for the variable ordering $a_0 < \dots < a_{k-1} < x_0 < \dots < x_{n-1}$.

(b) Does this variable ordering yield an ROBDD with minimal size? If yes, justify your answer. If no, give a variable ordering and the corresponding ROBDD with strictly smaller size.

Exercise 4 (Transition system encoding): (1+1 points)

Consider the following transition system.



(a) TS (b) Legend

(a) Define the switching functions that represent the transition system for $\bar{x} = (x_0, x_1)$:
   - $\Delta(\bar{x}, \bar{x}')$ for the transition function
   - $f_a(\bar{x})$ the satisfaction sets for each atomic proposition $a \in Act$

   States are encoded according to the binary representation of their index, e.g., $enc(s_1) = (x_0, x_1) = (0, 1)$.

(b) Encode the switching functions from before using ROBDDs. Use variable ordering $x_0 < x_1$ for the satisfaction sets and interleave it ($x_0 < x_0' < x_1 < x_1'$) for the transition relation.