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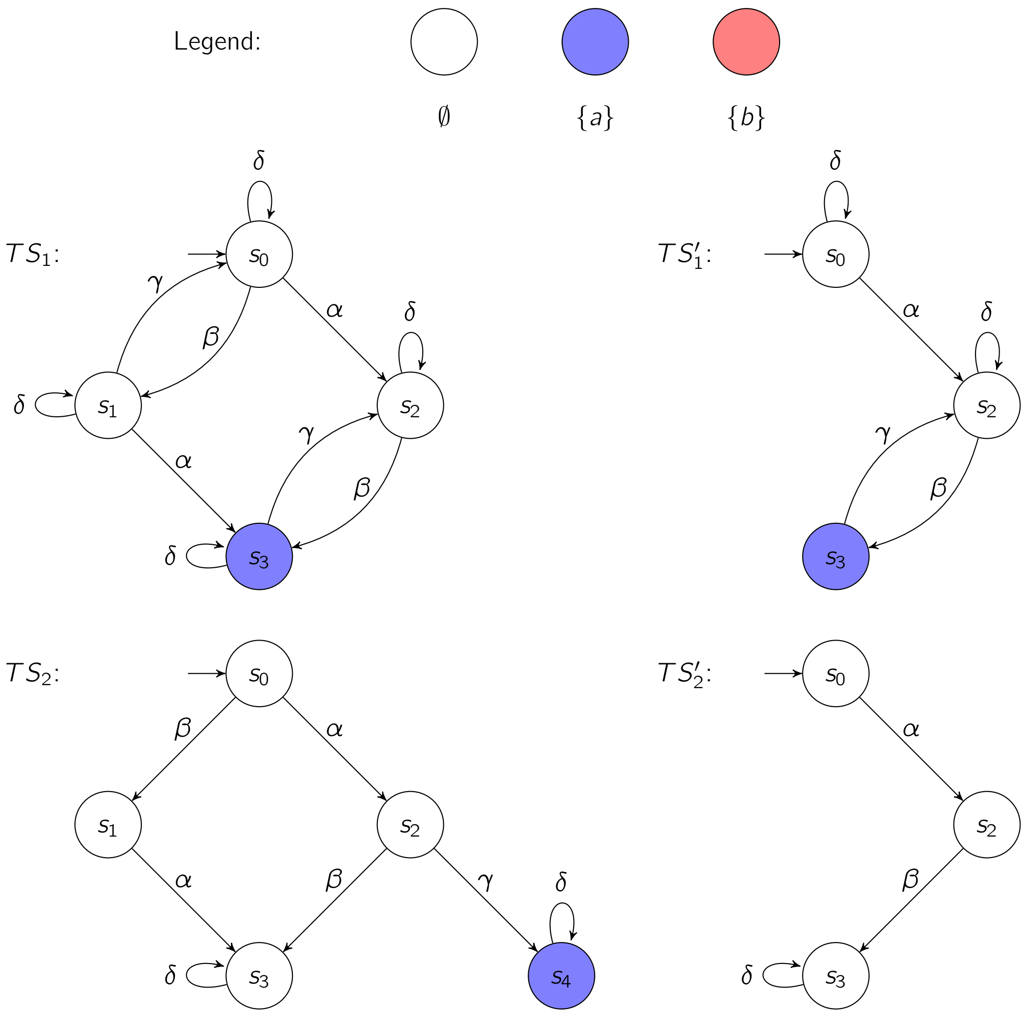
Exercise 1 (Ample set conditions): (4 points)

Consider the following four pairs (TS_i, TS_i') of transition systems where TS_i' results from reducing TS_i using the appropriate ample sets. For each pair (TS_i, TS_i') ...

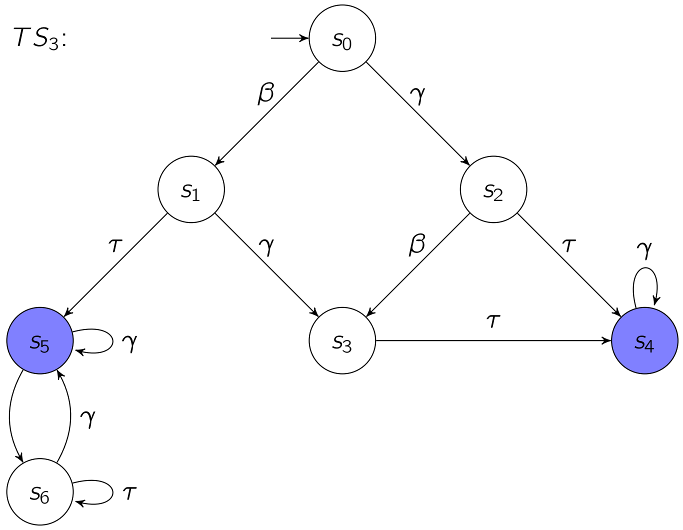
(i) Indicate **all** of the ample set conditions $(A1)$–$(A5)$ (including the branching condition $(A5)$ from lecture 11) that are violated. Justify your answers.

(ii) Indicate whether the two transition systems are divergence sensitive stutter bisimular. If not, provide a distinguishing CTL$\setminus_{\bigcirc}$ formula.

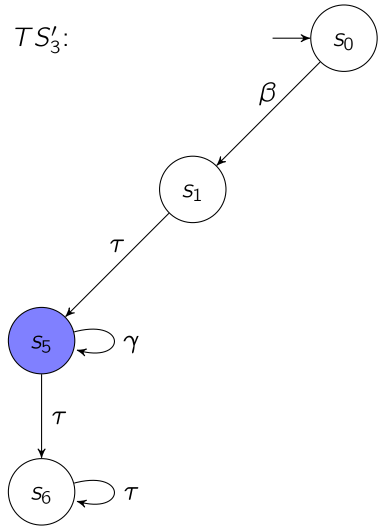
(iii) Indicate whether the two transition systems are stutter trace equivalent. If not, provide a distinguishing trace.



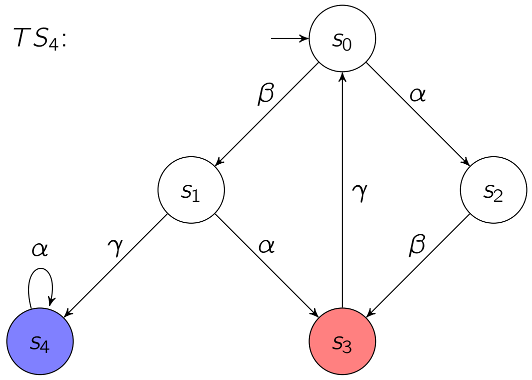
TS_3:



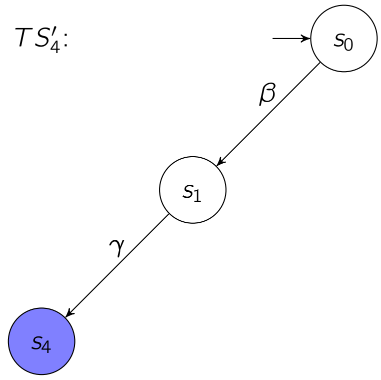
TS_3':



TS_4:

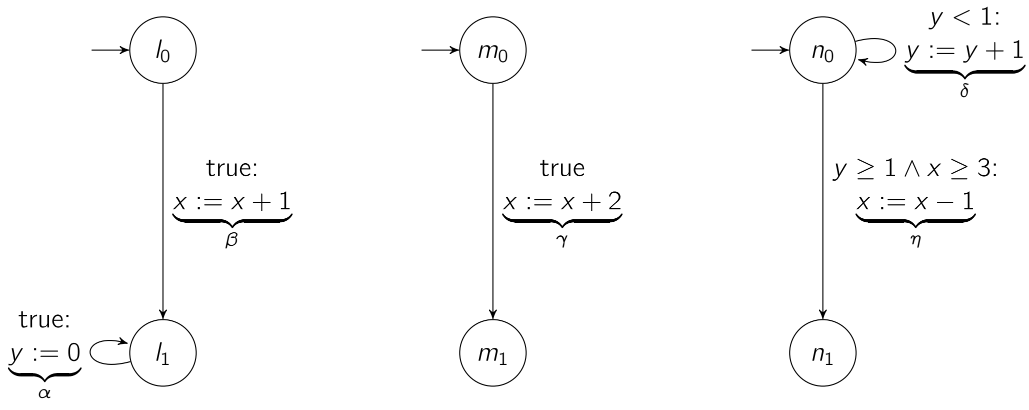


TS_4':



Exercise 2 (POR algorithm): (3 points)

Consider the following three program graphs PG_1, PG_2 and PG_3 over the shared variables x and y.



(a) PG_1 (b) PG_2 (c) PG_3

Prove or refute that the invariant $\varphi = \Box\neg n_1$ holds on $TS(PG_1 \parallel PG_2 \parallel PG_3)$ (where only n_1 is considered as an atomic proposition) with the initial condition $x = 0 \wedge y = 0$. For this, use the POR-based algorithm presented in the lecture (slide 153); in particular use the presented method to derive ample sets (slide 219). Whenever you are required to pick a process (i.e., program graph) by any of the algorithms, try PG_2 first, then PG_1 and only then PG_3. Choosing the order of explored actions (in the ample set) is up to you. Write down all steps that you performed.

Exercise 3 (Stutter permutation equivalence): (3 points)

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be an action-deterministic transition system and let $\mathcal{I}_{st}$ be the set of all pairs $(\alpha, \beta) \in Act \times Act$ of independent actions α and β where α or β (or both) is a stutter action. Let *stutter permutation equivalence* $\cong_{perm}$ be the finest equivalence on Act^* such that

$$\bar{\gamma}\alpha\beta\bar{\delta} \cong_{perm} \bar{\gamma}\beta\alpha\bar{\delta} \qquad \text{if} \quad \bar{\gamma}, \bar{\delta} \in Act^* \quad \text{and} \quad (\alpha, \beta) \in \mathcal{I}_{st}$$

The extension of $\cong_{perm}$ to an equivalence for infinite action sequences is defined as follows. If $\tilde{\alpha} = \alpha_1\alpha_2\alpha_3\ldots$ and $\tilde{\beta} = \beta_1\beta_2\beta_3\ldots$ are actions sequences in Act^ω, then $\tilde{\alpha} \sqsubseteq_{perm} \tilde{\beta}$ if for all finite prefixes $\alpha_1\ldots\alpha_n$ of $\tilde{\alpha}$ there exists a finite prefix $\beta_1\ldots\beta_m$ of $\tilde{\beta}$ with $m \geq n$ and a finite word $\bar{\gamma} \in Act^*$ such that

$$\alpha_1\ldots\alpha_n\bar{\gamma} \cong_{perm} \beta_1\ldots\beta_m$$

We then define the binary relation $\cong^\omega_{perm}$ on Act^ω by

$$\tilde{\alpha} \cong^\omega_{perm} \tilde{\beta} \qquad \text{iff} \qquad \tilde{\alpha} \sqsubseteq_{perm} \tilde{\beta} \quad \text{and} \quad \tilde{\beta} \sqsubseteq_{perm} \tilde{\alpha}$$

Show that $\cong^\omega_{perm}$ is an equivalence.