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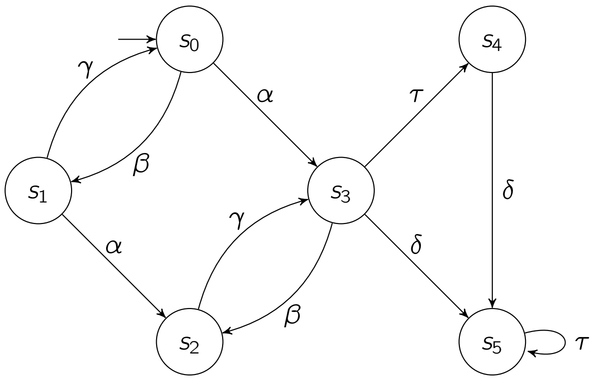
Exercise 1 (Independent actions): (2 points)

Consider the following transition system TS with action set $Act = \{\alpha, \beta, \gamma, \delta, \tau\}$ in which all states are equally labeled. Determine for each pair of unequal actions whether they are independent.



Hint for Exercise 2 and 3: Action $\alpha \in Act$ is dependent from $ample(s) \subseteq Act$ iff $\alpha \notin ample(s) \wedge \exists \beta \in ample(s)\colon \alpha$ and β are dependent.

Exercise 2 (Ample set conditions): (3+1+1 points)

Consider the transition system TS depicted below.
Further, assume the following ample sets.

(i) $ample(s_1) = \{\beta\}$

(ii) $ample(s_2) = \{\alpha\}$

(iii) $ample(s_5) = \{\alpha, \gamma\}$

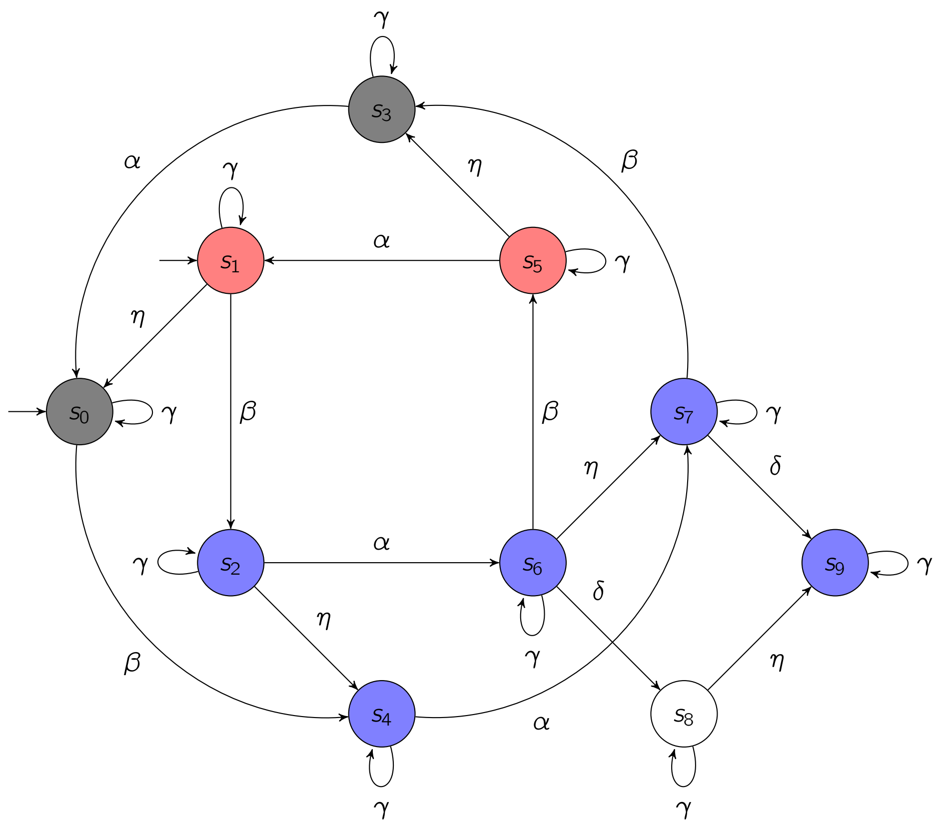
(iv) $ample(s_6) = \{\alpha, \beta, \delta\}$

and $ample(s_i) = Act(s_i)$ for all other states.

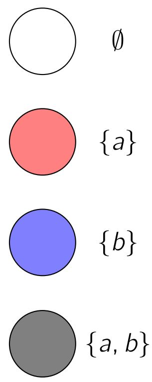
(a) For each of the given ample sets, indicate whether it satisfies conditions (A1) to (A3). Justify your answer.

(b) Check if condition (A4) is satisfied for the given ample sets. Justify your answer.

(c) In case some of the conditions (A1) to (A4) do not hold, provide a minimal extension of the ample sets to fix the issue. Justify your changes.



(a) TS



(b) Legend

Exercise 3 (Dependency condition): (3 points)

Let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP, L_i)$, $i \in \{1, \dots, n\}$ be finite action-deterministic transition systems such that $Act_i \cap Act_j \cap Act_k = \emptyset$ for $1 \leq i < j < k \leq n$. We consider the parallel composition with synchronization over common actions, i.e. the transition system

$$TS = TS_1 \parallel TS_2 \parallel \dots \parallel TS_n.$$

For each state $s = \langle s_1, \dots, s_n \rangle$ of TS, let $Act_i(s) = Act_i \cap Act(s)$ be the set of actions of TS_i that are enabled in s.

Show that the dependency Condition (A2) holds if for each state $s = \langle s_1, \dots, s_n \rangle$ of TS the following conditions (i) and (ii) hold:

(i) If $ample(s) \neq Act(s)$, then $ample(s) = Act_i(s)$ for some $i \in \{1, \dots n\}$.

(ii) If $ample(s) = Act_i(s) \neq Act(s)$ for some $i \in \{1, \dots n\}$, then

- (a) $ample(s) \cap (\bigcup_{\substack{1 \leq j \leq n \\ j \neq i}} Act_j) = \emptyset$, and
- (b) $Act_i(s) = Act(s_i)$