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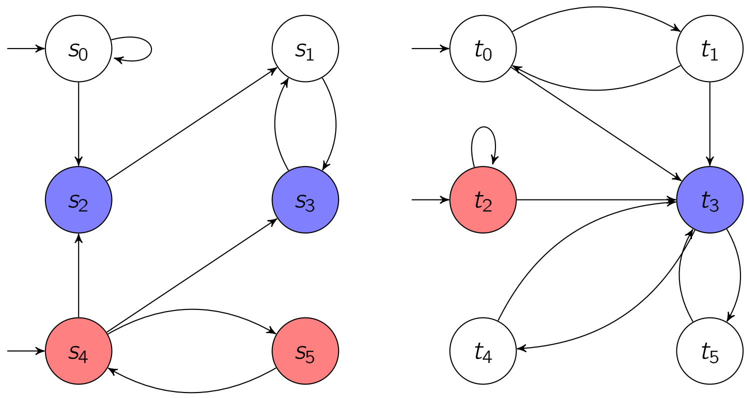
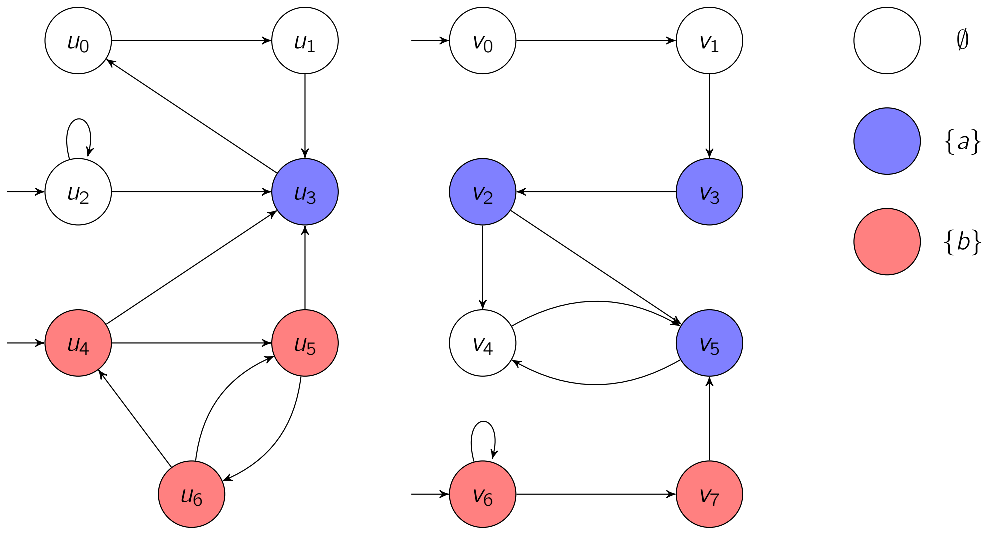
Exercise 1 (Equivalences): (1.5+1.5+1.5+1.5 points)

Consider the following transition systems.



(a) TS_1 (b) TS_2



(c) TS_3 (d) TS_4 (e) Legend

For each $i, j \in \{1 \dots 4\} \times \{1 \dots 4\}$, $i \neq j$ determine whether

(a) $TS_i \sim TS_j$

(b) $TS_i \approx TS_j$

(c) $TS_i \approx_{div} TS_j$

(d) $TS_i \triangleq TS_j$

holds. Justify your answers.

Exercise 2 (Observational equivalence): (2+2 points)

Observational equivalence $\approx_{obs}$ is a slight variant of stutter-bisimulation equivalence where state s_2 is allowed to perform a path fragment

$$\underbrace{s_2 u_1 \ldots u_m}_{stuttersteps} \underbrace{v_1 \ldots v_k s_2'}_{stuttersteps}$$

with arbitrary stutter steps at the beginning and at the end and $s_1' \approx_{obs} s_2'$ to simulate a transition $s_1 \to s_1'$ of an observational equivalent state s_1. I.e., it is not required that s_2 and states u_i are observationally equivalent, or that s_2' and v_i are observationally equivalent. For the special case where $s_1 \to s_1'$ is a stutter step the path fragment of length 0 (consisting of state $s_2 = s_2'$) can be used to simulate $s_1 \to s_1'$.
The formal definition of observational equivalence is as follows. Let TS_1 and TS_2 be two transition systems with state-spaces S_1 and S_2, respectively, and the same set AP of atomic propositions. A binary relation $\mathcal{R} \subseteq S_1 \times S_2$ is called an *observational bisimulation* for (TS_1, TS_2) iff the following conditions (A) and (B) are satisfied:

(A) Every initial state of TS_1 is related to an initial state of TS_2, and vice versa. That is,

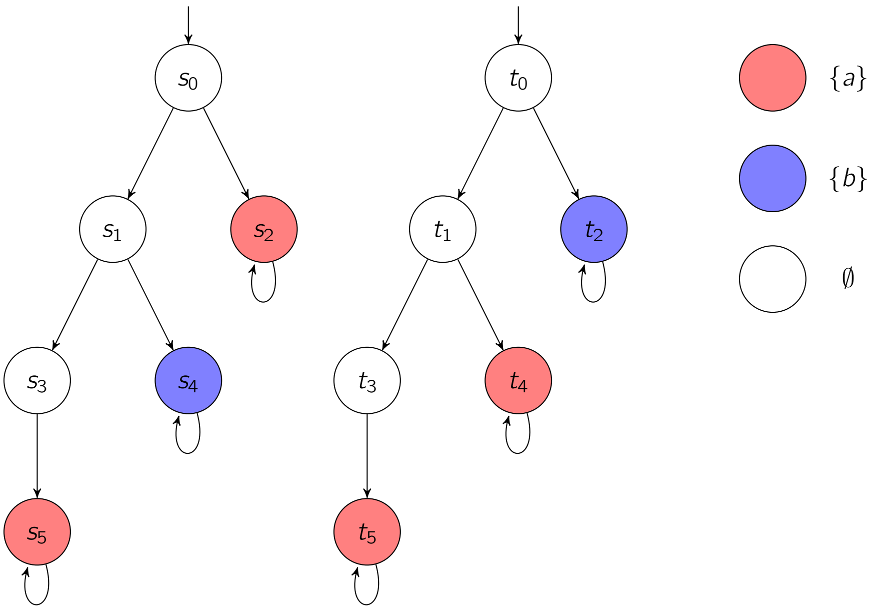
$$\forall s_1 \in I_1 \; \exists s_2 \in I_2.(s_1, s_2) \in \mathcal{R} \qquad \text{and} \qquad \forall s_2 \in I_2 \; \exists s_1 \in I_1.(s_1, s_2) \in \mathcal{R}$$

(B) For all $(s_1, s_2) \in \mathcal{R}$, the following conditions (I),(II) and (III) hold:
   (I) If $(s_1, s_2) \in \mathcal{R}$ then $L_1(s_1) = L_2(s_2)$.
   (II) If $(s_1, s_2) \in \mathcal{R}$ and $s_1' \in Post(s_1)$, then there exists a path fragment $u_0 u_1 \ldots u_n$ such that $n \geq 0$ and $u_0 = s_2$, $(s_1', u_n) \in \mathcal{R}$ and, for some $m \leq n$, $L_2(u_0) = L_2(u_1) = \ldots = L_2(u_m)$ and $L_2(u_{m+1}) = L_2(u_{m+2}) = \ldots = L_2(u_n)$.
   (III) If $(s_1, s_2) \in \mathcal{R}$ and $s_2' \in Post(s_2)$, then there exists a path fragment $u_0 u_1 \ldots u_n$ such that $n \geq 0$ and $u_0 = s_1$, $(u_n, s_2') \in \mathcal{R}$ and, for some $m \leq n$, $L_1(u_0) = L_1(u_1) = \ldots = L_1(u_m)$ and $L_1(u_{m+1}) = L_1(u_{m+2}) = \ldots = L_1(u_n)$.

TS_1 and TS_2 are called observational equivalent, denoted $TS_1 \approx_{obs} TS_2$, if there exists an observational bisimulation for (TS_1, TS_2).
Questions:
The goal of this exercise is to show that $\approx_{obs}$ is strictly coarser than stutter-bisimulation equivalence $\approx$.

(a) Show that $TS_1 \approx TS_2$ implies $TS_1 \approx_{obs} TS_2$.

(b) Consider the two transition systems TS_1 and TS_2 shown in the following figure. Does $TS_1 \approx TS_2$ or $TS_1 \approx_{obs} TS_2$ hold? Justify your answer.



(f) TS_1 (g) TS_2 (h) Legend