Notes:

- The exercise sheets should be solved in groups of 2 persons and handed in before the exercise class on Wednesday, 14:15.

Exercise 1 (Relation $\sim_n$): (2 + 1 points)

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system. The relations $\sim_n \subseteq S \times S$ are inductively defined by:

- $s_1 \sim_0 s_2$ iff $L(s_1) = L(s_2)$.
- $s_1 \sim_{n+1} s_2$ iff:
  - $L(s_1) = L(s_2)$,
  - for all $s_1' \in Post(s_1)$ there exists $s_2' \in Post(s_2)$ with $s_1' \sim_n s_2'$,
  - for all $s_2' \in Post(s_2)$ there exists $s_1' \in Post(s_1)$ with $s_1' \sim_n s_2'$.

Questions:

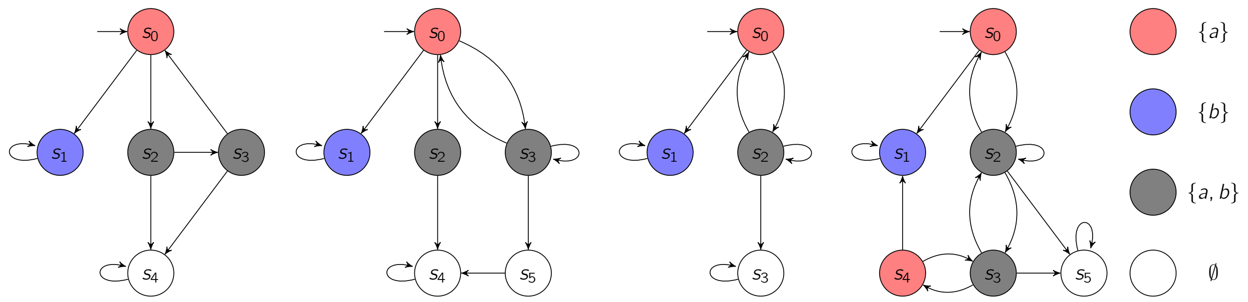
(a) Show that for *finite* TS it holds that $\sim_{TS} = \bigcap_{n\geq 0} \sim_n$, i.e.,

$$s_1 \sim_{TS} s_2 \text{ iff } s_1 \sim_n s_2 \text{ for all } n \geq 0$$

(b) Does this also hold for infinite transition systems (provide a proof or a counterexample)?

Exercise 2 (Equivalences): (2 + 2 points)

Consider the following transition systems.



(a) TS_1 (b) TS_2 (c) TS_3 (d) TS_4 (e) Legend

(a) Which transition systems are trace equivalent? Justify your answers by either providing the set of traces or a counterexample trace.

(b) Which transition systems are bisimulation equivalent? Justify your answers by either providing a bisimulation relation or a CTL$_{\setminus U}$ formula that distinguishes the considered transition systems. (Note: a CTL$_{\setminus U}$ formula contains neither an U-operator nor one of its derived operators such as $\Diamond$ and $\Box$.)

Exercise 3 (Distinguishing formulae): (1.5 + 1.5 points)

We show that both the $\bigcirc$ and U-operator are needed to create distinguishing formulae for transition systems. (Note: a $\text{CTL}_{\setminus\bigcirc}$ formula contains no $\bigcirc$-operator and similarly a $\text{CTL}_{\setminus \mathsf{U}}$ formula contains no U-operator.)

(a) Give two transition systems TS_1 and TS_2, s.t.

 (i) $\exists\text{CTL}_{\setminus\bigcirc}$ formula $\varphi : TS_1 \models \varphi$ but $TS_2 \not\models \varphi$.

 (ii) $\forall\text{CTL}_{\setminus \mathsf{U}}$ formula $\psi : TS_1 \models \psi \iff TS_2 \models \psi$.

(b) Give two transition systems TS_1 and TS_2, s.t.

 (i) $\exists\text{CTL}_{\setminus \mathsf{U}}$ formula $\varphi : TS_1 \models \varphi$ but $TS_2 \not\models \varphi$.

 (ii) $\forall\text{CTL}_{\setminus\bigcirc}$ formula $\psi : TS_1 \models \psi \iff TS_2 \models \psi$.