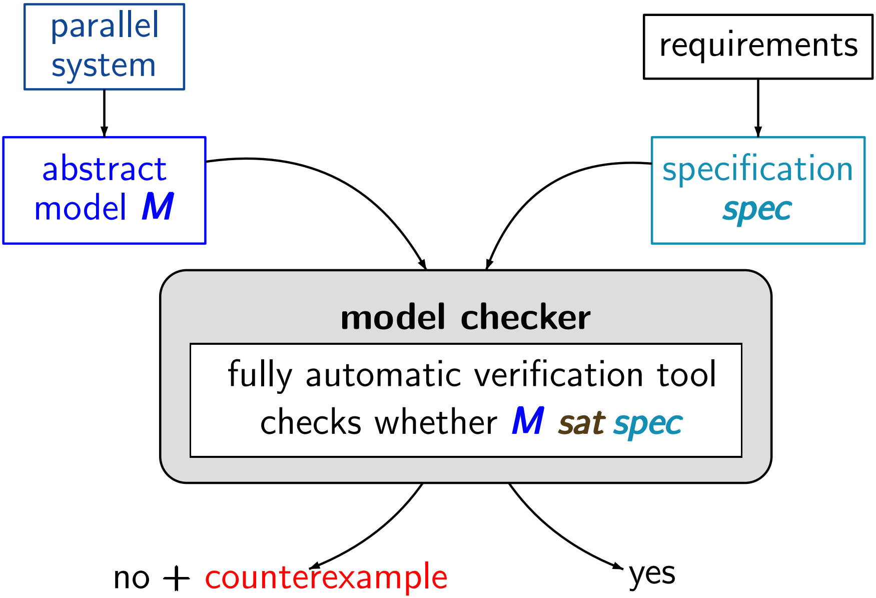
# Model checking





# Decidability of the model checking problem?

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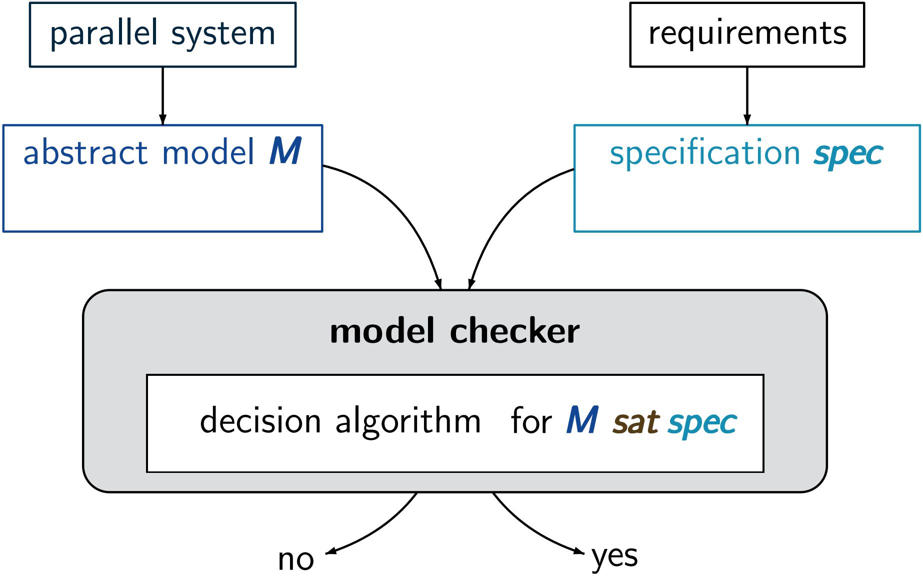
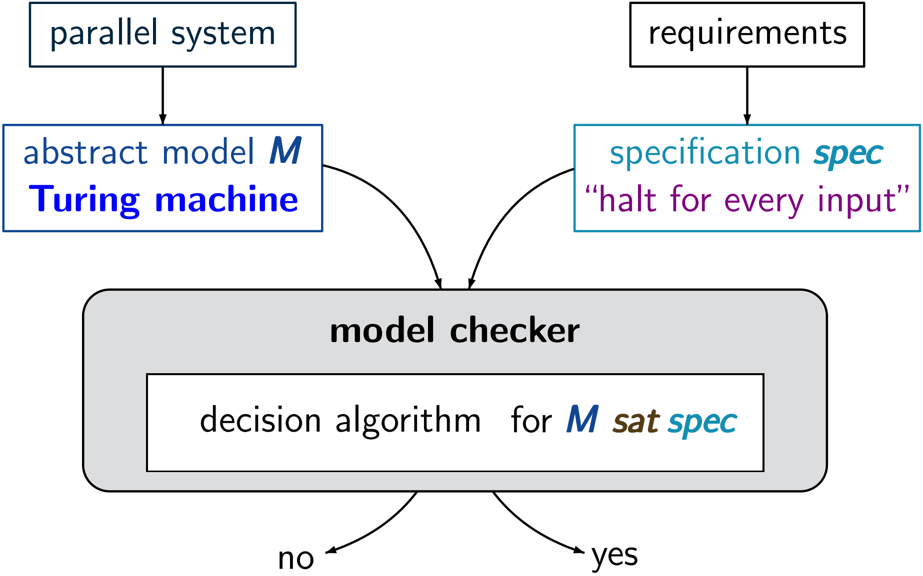
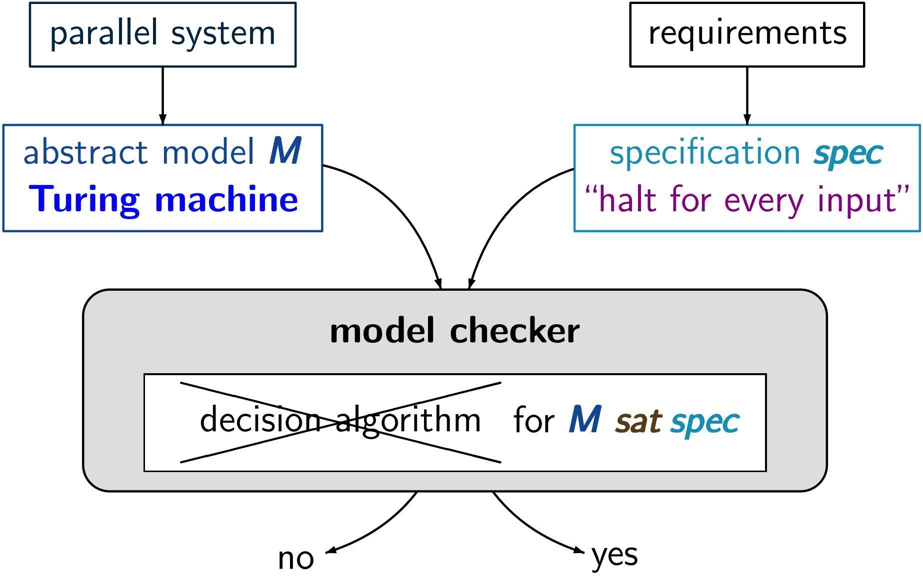


# Decidability of the model checking problem?





# General model checking problem is undecidable 



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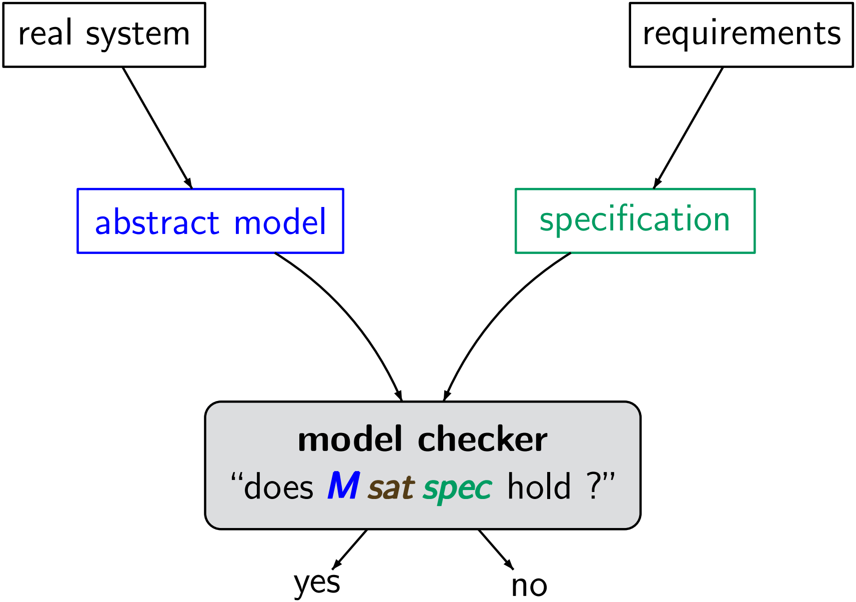

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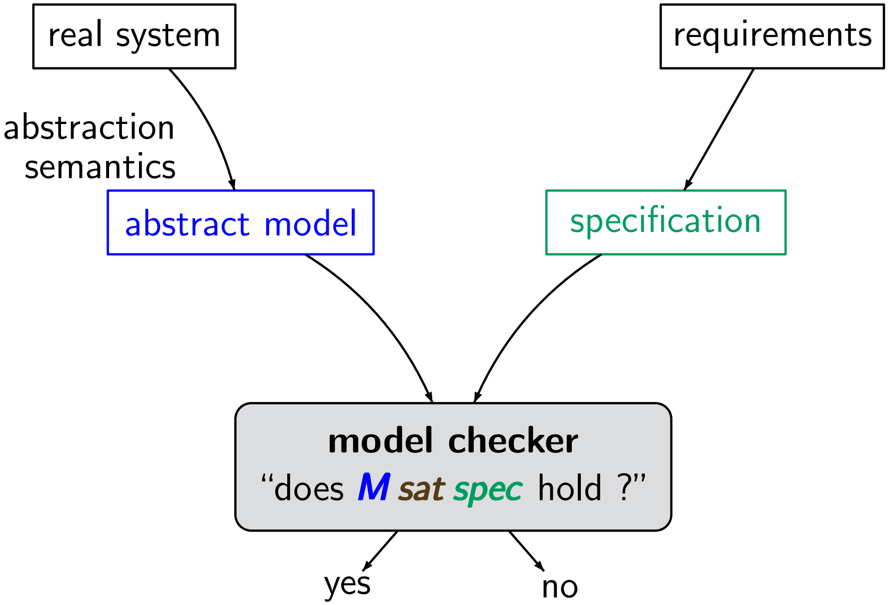

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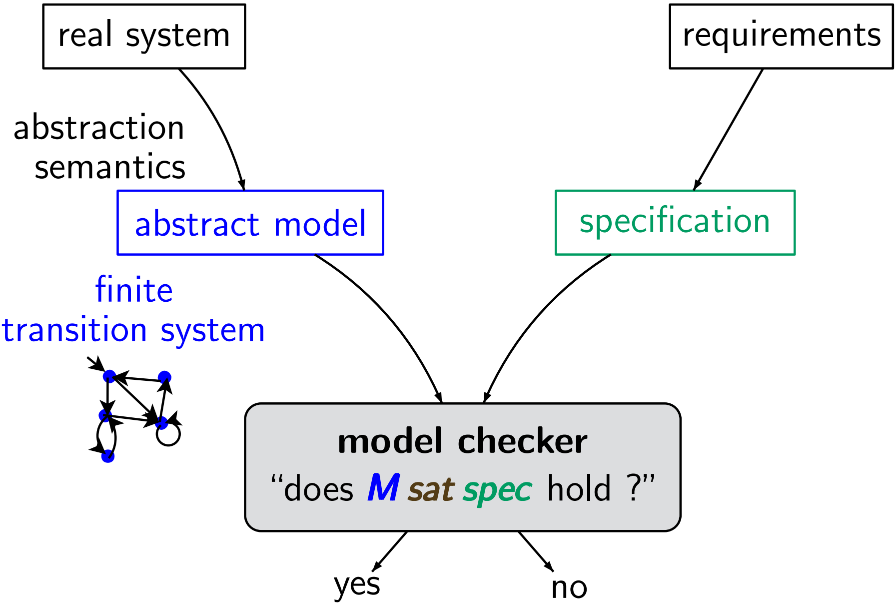

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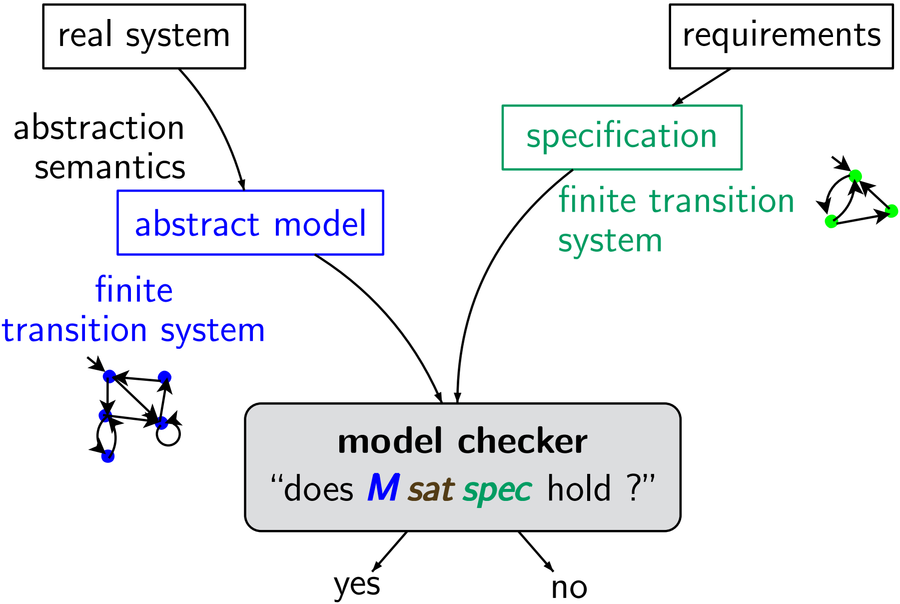

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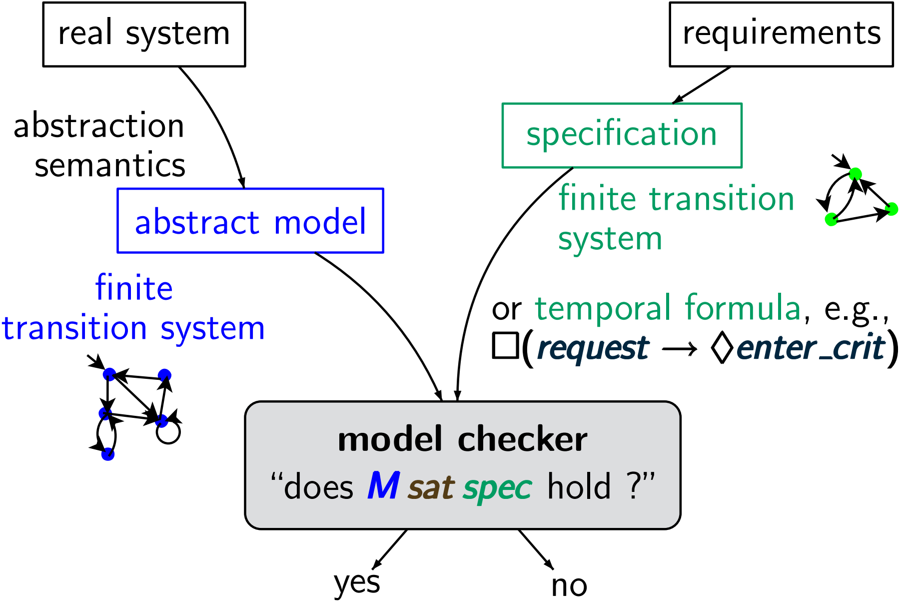

 Model checking vs other validation techniques 

The validation techniques (testing, simulation, deductive verification, model checking) are complementary to each other.

# Model checking vs other validation techniques 

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## model checking

- most efficient validation technique, fully automatic
- but mostly only applicable for finite models with "small" (or "sufficiently structured") state space
- industrial applications:
  - hardware systems
  - communication protocols
  - coordination protocols for distributed systems
  - ⋮

# Historical notes



| | | |
|---|---|---|
| 1976 | Keller | transition systems (TS)<br>to model parallel systems |
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| 1983 | Kanellakis/Smolka | model checking for homogeneous TS-based specifications |
| 1985 | Lichtenstein/Pnueli | model checking for **LTL** |
| 1986 | Vardi/Wolper | (model checking for **LTL**) |

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**state explosion problem**

> state space of industrial systems too large
> to be handled by naïve implementations of
> model checking algorithms

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model checking for infinite systems, quantitative analysis,
e.g., real-time systems, probabilistic systems

# Transition system (TS)



A transition system is a tuple

$$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$$

- $S$ is the state space, i.e., set of states,
- $Act$ is a set of actions,
- $\longrightarrow \subseteq S \times Act \times S$ is the transition relation,

> i.e., transitions have the form $s \xrightarrow{\alpha} s'$
> where $s, s' \in S$ and $\alpha \in Act$

- $S_0 \subseteq S$ the set of initial states,
- $AP$ a set of atomic propositions,
- $L : S \rightarrow 2^{AP}$ the labeling function

# "Behaviour" of transition systems



possible behaviours of a TS result from:

```
select nondeterministically an initial state s ∈ S_0
WHILE  s is non-terminal DO
      select nondeterministically a transition s --α--> s'
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*executions:* maximal "transition sequences"

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*reachable fragment:*

$Reach(\mathcal{T})$ = set of all states that are reachable from an initial state through some execution

# Linear-time vs branching-time

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transition system

$$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$$

# Linear-time vs branching-time



transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

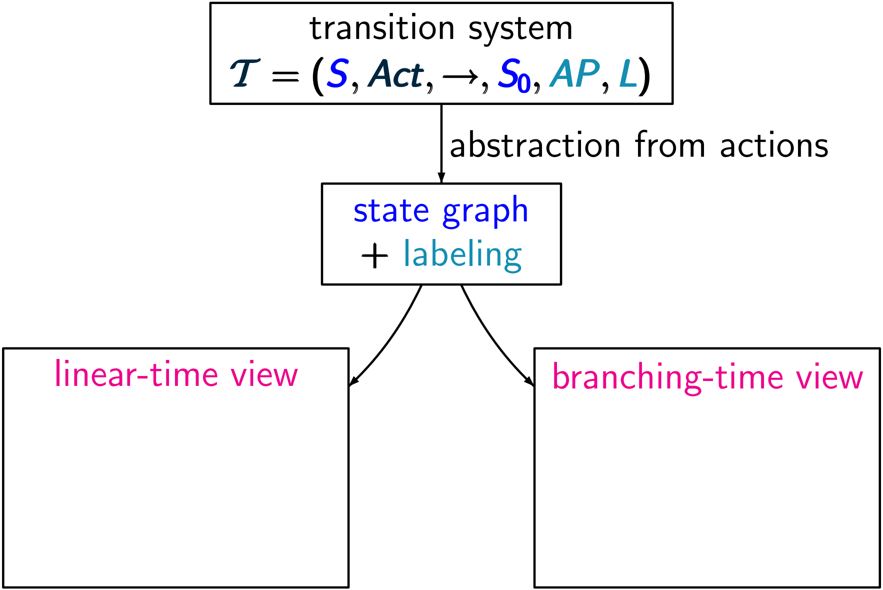
↓ abstraction from actions

state graph

+ labeling

# Linear-time vs branching-time





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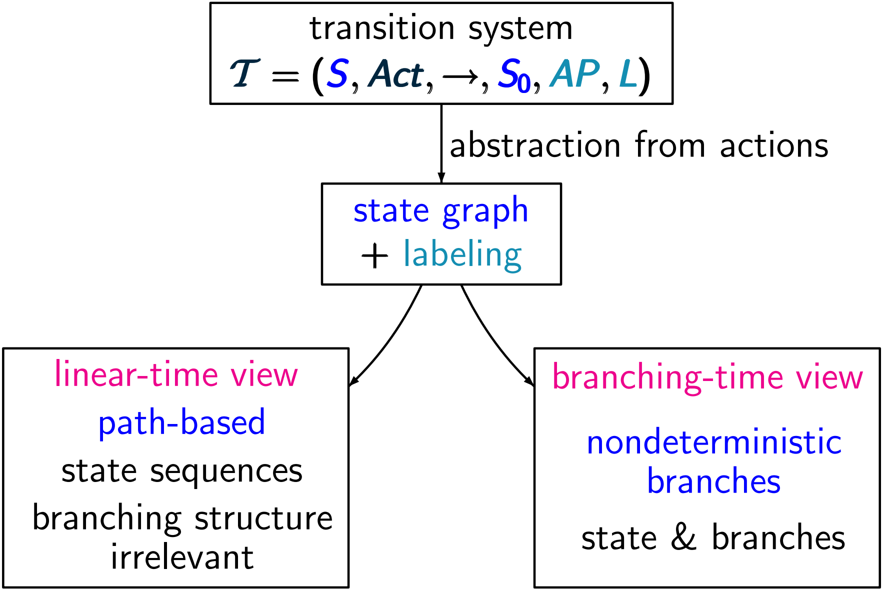

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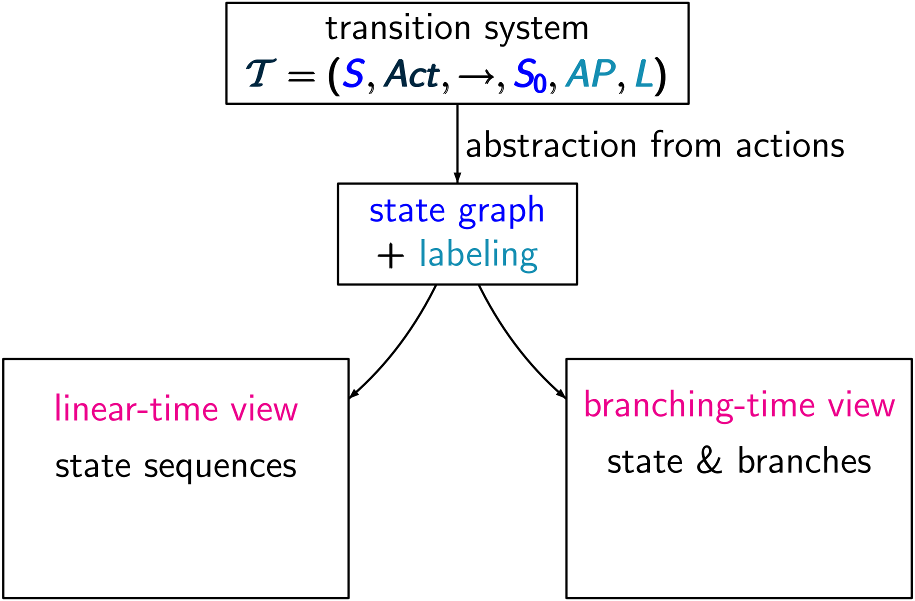

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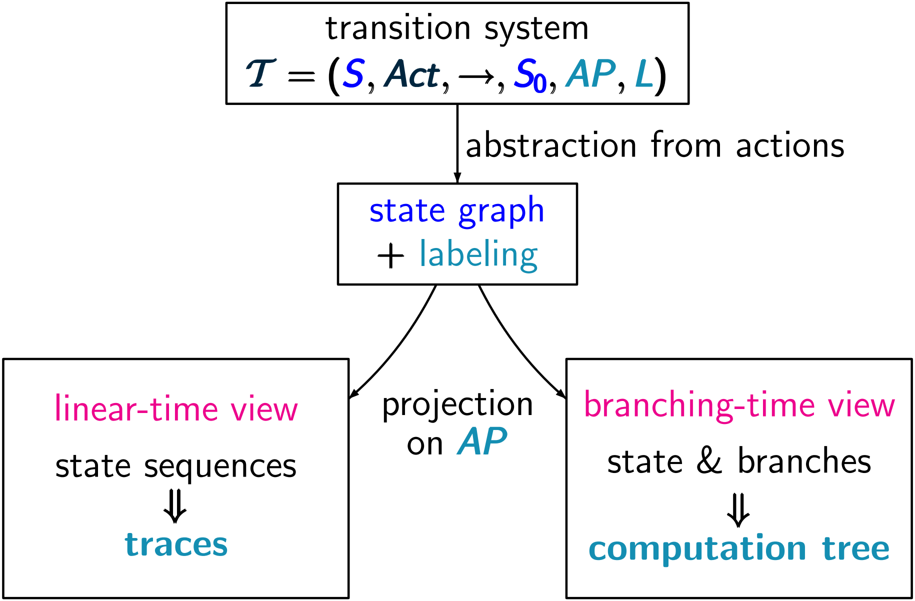

# Traces

LTB2.4-4

for TS with labeling function $L : S \to 2^{AP}$

*execution:* states + actions

$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ infinite or finite

↓

*paths:* sequences of states

$s_0\, s_1\, s_2 \ldots$ infinite or $s_0\, s_1 \ldots s_n$ finite

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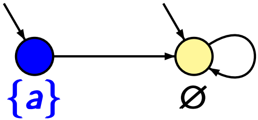
# Example: traces



Let $\mathcal{T}$ be a TS without terminal states.

$$Traces(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\pi) : \pi \in Paths(\mathcal{T})\} \subseteq (2^{AP})^\omega$$

$$Traces_{fin}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\widehat{\pi}) : \widehat{\pi} \in Paths_{fin}(\mathcal{T})\} \subseteq (2^{AP})^*$$



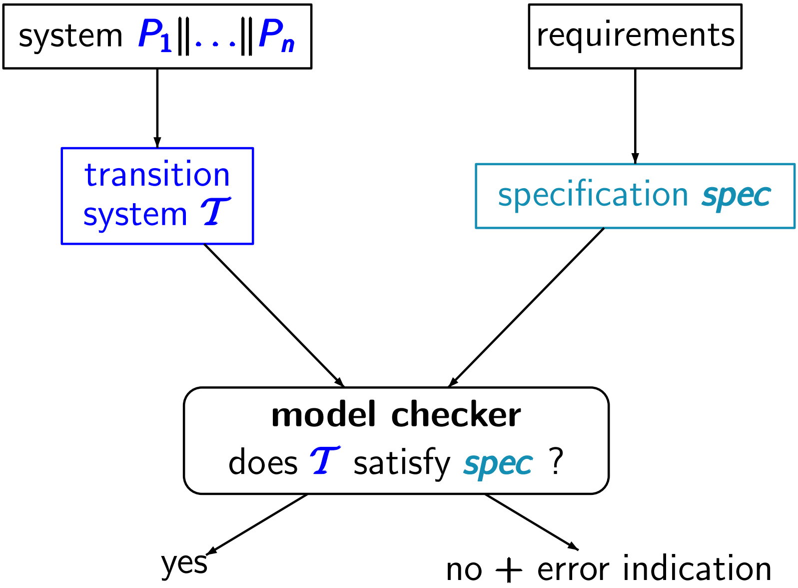

TS $\mathcal{T}$ with a single atomic proposition $a$

$$Traces(\mathcal{T}) = \{ \{a\}\varnothing^\omega, \varnothing^\omega \}$$

$$Traces_{fin}(\mathcal{T}) = \{\{a\}\varnothing^n : n \geq 0\} \cup \{\varnothing^m : m \geq 1\}$$

# Model checking





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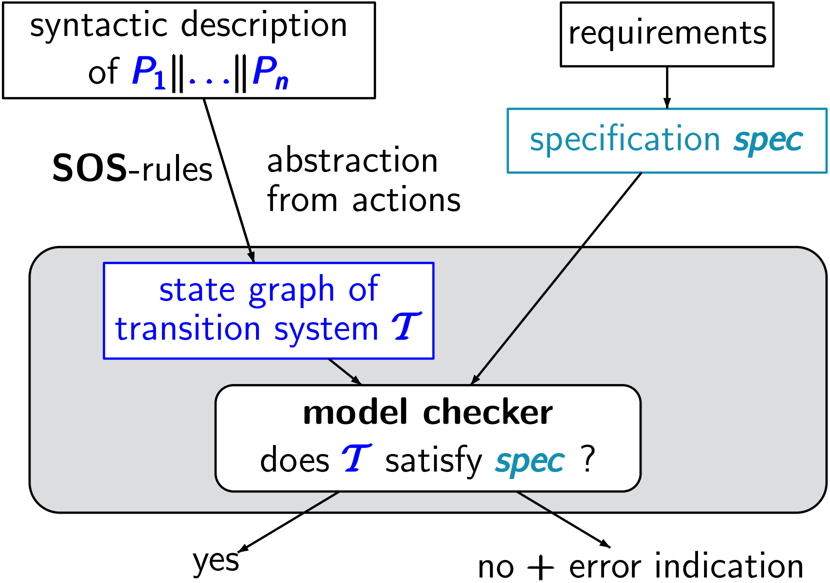

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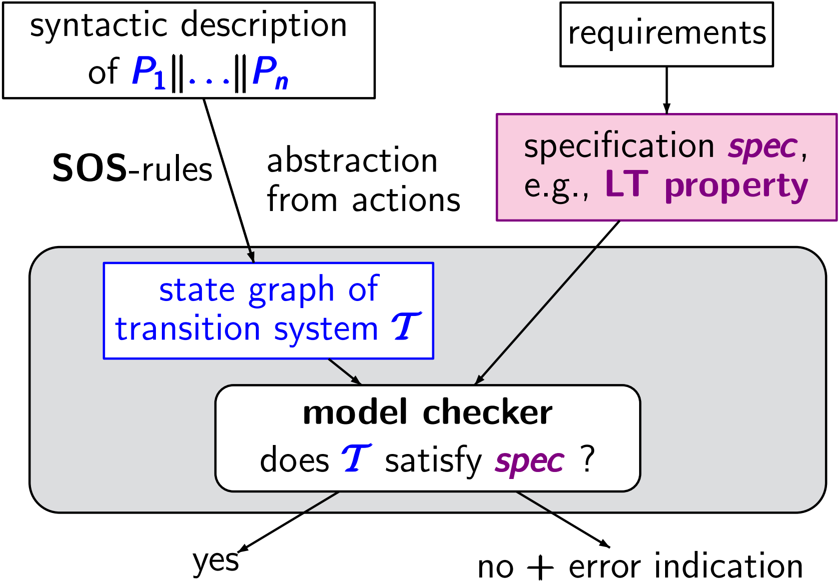

# Linear-time properties (LT properties)

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for TS over $AP$ without terminal states

An LT property over $AP$ is a language $E$ of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

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E.g., for mutual exclusion problems and
$AP = \{crit_1, crit_2, \ldots\}$

safety:

$MUTEX =$ set of all infinite words $A_0\,A_1\,A_2\ldots$ over $2^{AP}$ such that for all $i \in \mathbb{N}$:
$crit_1 \notin A_i$ or $crit_2 \notin A_i$

# Satisfaction relation for LT properties



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Satisfaction relation $\models$ for TS:

If $\mathcal{T}$ is a TS (without terminal states) over $AP$ and $E$ an LT property over $AP$ then

$$\mathcal{T} \models E \quad \text{iff} \quad Traces(\mathcal{T}) \subseteq E$$

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Satisfaction relation $\models$ for TS and states:

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---

If $s$ is a state in $\mathcal{T}$ then

$$s \models E \quad \text{iff} \quad Traces(s) \subseteq E$$

# LT properties and trace inclusion



An LT property over $AP$ is a language $E$ of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq (2^{AP})^\omega$.

If $\mathcal{T}$ is a TS over $AP$ then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over $AP$ then the following statements are equivalent:

(1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$

(2) for all LT-properties $E$ over $AP$: whenever $\mathcal{T}_2 \models E$ then $\mathcal{T}_1 \models E$

$(1) \Longrightarrow (2)$: $\checkmark$

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$(2) \Longrightarrow (1)$: consider $E = Traces(\mathcal{T}_2)$

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Transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set $AP$ of atomic propositions are called trace equivalent iff

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Trace equivalent TS satisfy the **same LT properties**

# LT properties and trace relations



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The following statements are equivalent:

(1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$

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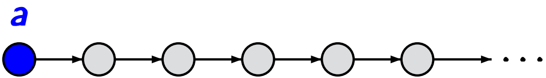
# Linear Temporal Logic (LTL)



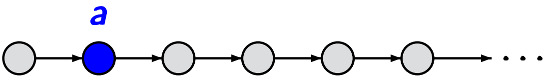
$$\varphi \; ::= \; \mathit{true} \; \big| \; a \; \big| \; \varphi_1 \wedge \varphi_2 \; \big| \; \neg\varphi \; \big| \; \bigcirc \varphi \; \big| \; \varphi_1 \, \mathsf{U} \, \varphi_2$$

where $a \in AP$ $\quad$ $\bigcirc \;\widehat{=}\;$ next $\quad$ $\mathsf{U} \;\widehat{=}\;$ until

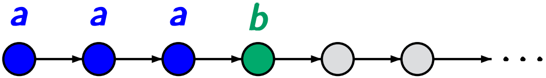
atomic proposition $a \in AP$




next operator $\bigcirc a$




until operator $a \, \mathsf{U} \, b$



# Derived operators in LTL



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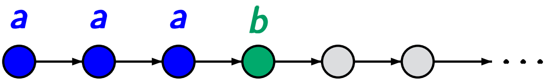
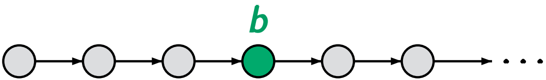



eventually
$\Diamond b$



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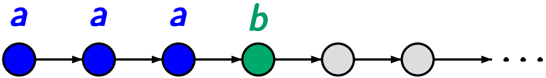
| | | | |
|---|---|---|---|
| $\Diamond\varphi$ | $\overset{\text{def}}{=}$ | $\mathit{true}\,\mathsf{U}\,\varphi$ | eventually |
| $\Box\varphi$ | $\overset{\text{def}}{=}$ | $\neg\Diamond\neg\varphi$ | always |

until operator
$a \,\mathsf{U}\, b$

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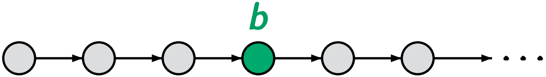


always
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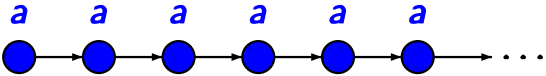
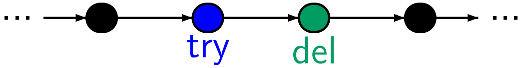


# Next $\bigcirc$, until U and eventually $\Diamond$



$\Box$ (try_to_send $\rightarrow$ $\bigcirc$ delivered)



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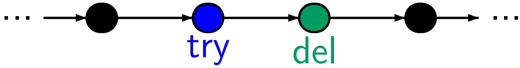
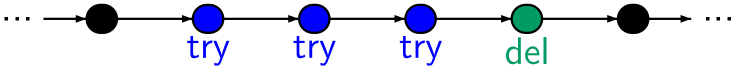
$$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$$

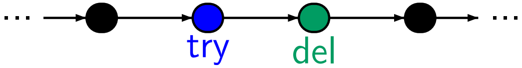



$$\Box\,(\text{try\_to\_send} \rightarrow \text{try\_to\_send}\ \mathbf{U}\ \text{delivered})$$



# Next $\bigcirc$, until $\mathsf{U}$ and eventually $\Diamond$

$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$

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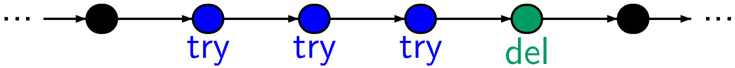
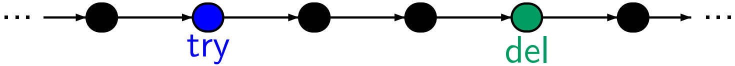



$\Box\,(\text{try\_to\_send} \rightarrow \Diamond\ \text{delivered})$



# Infinitely often and eventually forever



$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

| | | | |
|---|---|---|---|
| eventually | $\Diamond\varphi$ | $\overset{\text{def}}{=}$ | $\mathit{true}\,\mathsf{U}\,\varphi$ |
| always | $\Box\varphi$ | $\overset{\text{def}}{=}$ | $\neg\Diamond\neg\varphi$ |
| infinitely often | $\Box\Diamond\varphi$ | | |
| eventually forever | $\Diamond\Box\varphi$ | | |

| | | |
|---|---|---|
| e.g., | unconditional fairness | $\Box\Diamond \mathit{crit}_i$ |
| | strong fairness | $\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$ |
| | weak fairness | $\Diamond\Box \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$ |

# LTL-semantics

LTLSF3.1-6A

# LTL-semantics



interpretation of **LTL** formulas over traces, i.e.,
infinite words over $2^{AP}$

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interpretation of **LTL** formulas over traces, i.e., infinite words over $2^{AP}$

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$

# Semantics of LTL over infinite words



for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

| | | |
|---|---|---|
| $\sigma \models \mathit{true}$ | | |
| $\sigma \models a$ | iff | $A_0 \models a$ ,i.e., $a \in A_0$ |
| $\sigma \models \varphi_1 \wedge \varphi_2$ | iff | $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$ |
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$\sigma \models \varphi_1 \, \mathsf{U} \, \varphi_2$ iff there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j \, A_{j+1} \, A_{j+2} \, \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i \, A_{i+1} \, A_{i+2} \, \ldots \models \varphi_1$ for $0 \leq i < j$

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*remind:* LT property of an LTL formula:

$$Words(\varphi) = \left\{ \sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi \right\}$$

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satisfaction relation for LT properties

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finite trace inclusion and equivalence:

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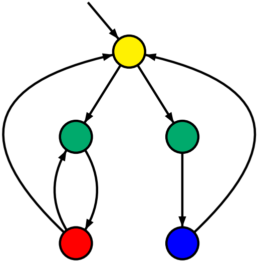
* **minimization ???**

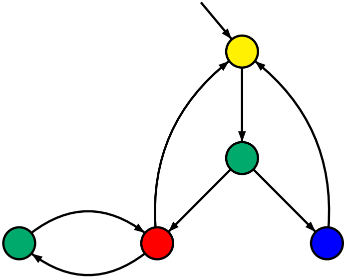
# Minimization w.r.t. trace equivalence?



$\mathcal{T}_1$:



$\mathcal{T}_2$:



- $\mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$

  but $\mathcal{T}_1$ and $\mathcal{T}_2$ are not isomorphic
- $\mathcal{T}_1$, $\mathcal{T}_2$ have **5** states and **7** transitions each
- there is no smaller TS that is trace-equivalent to $\mathcal{T}_i$

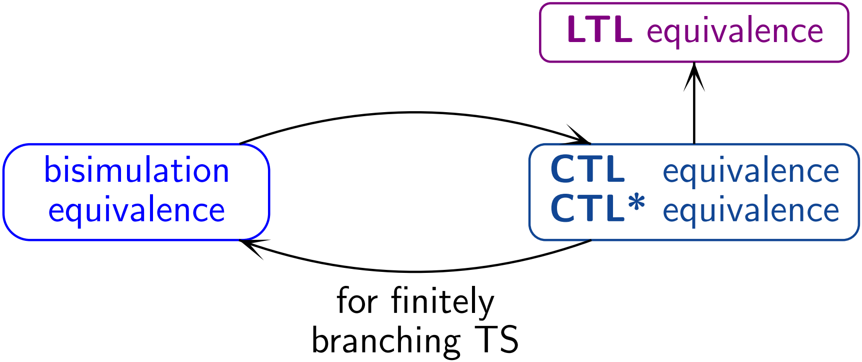
# Classification of implementation relations

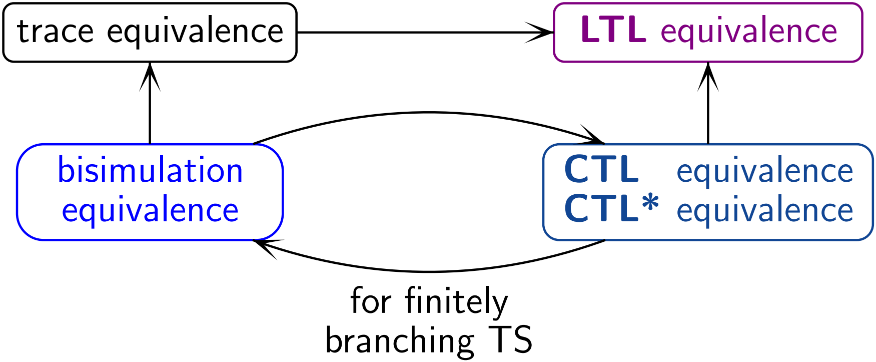


- linear vs. branching time
  - linear time: trace relations
  - branching time: (bi)simulation relations
- (nonsymmetric) preorders vs. equivalences:
  - preorders: trace inclusion, simulation
  - equivalences: trace equivalence, bisimulation
- strong vs. weak relations
  - strong: reasoning about all transitions
  - weak: abstraction from stutter steps

# Summary: equivalences

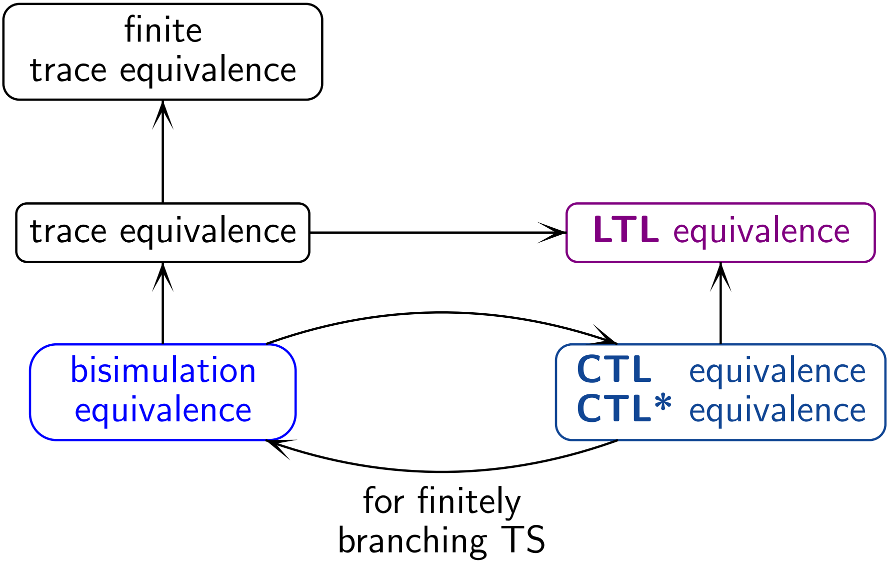





trace equivalence
LTL equivalence
bisimulation equivalence
CTL equivalence
CTL* equivalence
for finitely branching TS

# Summary: equivalences





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