Modeling and Verification of Probabilistic Systems

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http://moves.rwth-aachen.de/teaching/ws-1516/movep15/

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Overview

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  - $\bigcirc^I \Phi$ asserts that a transition to a Φ-state can be made at time $t \in I$.
  - $\Phi \, \mathsf{U}^I \Psi$ asserts that a Ψ-state can be reached via Φ-states at time $t \in I$.

CTMCs — A transition system perspective

Continuous-time Markov chain

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Residence time

The average residence time in state s is $\frac{1}{r(s)}$.

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$$\Phi \; ::= \; \text{true} \;\Big|\; a \;\Big|\; \Phi_1 \wedge \Phi_2 \;\Big|\; \neg\Phi \;\Big|\; \mathbb{P}_J(\varphi)$$

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- CSL *path formulae* are formed according to the following grammar:

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  Abbreviate $\mathbb{P}_{[0,0.5]}(\varphi)$ by $\mathbb{P}_{\leqslant 0.5}(\varphi)$ and $\mathbb{P}_{]0,1]}(\varphi)$ by $\mathbb{P}_{>0}(\varphi)$.

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- $s \models \mathbb{P}_J(\varphi)$ if probability that paths starting in s fulfill φ lies in J.

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- . . . once there, remain there almost surely for the next 31 time units:

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The satisfaction relation $\models$ is defined for CSL state formulas by:

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\begin{array}{lll}
s \models a & \text{iff} & a \in L(s) \\
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This is as for PCTL, except that Pr is the probability measures on cylinder sets of timed paths in CTMC $\mathcal{C}$.

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- $X\Phi \;\equiv\; \bigcirc^I \Phi$ with $I = \mathbb{R}_{\geqslant 0}$.

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Proof:

Rather straightforward; left as an exercise.

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CSL model checking

CSL model checking problem

Input: a finite CTMC $\mathcal{C} = (S, \mathbf{P}, r, \iota_{\mathrm{init}}, AP, L)$, state $s \in S$, and CSL state formula Φ

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3. Check whether state s belongs to $Sat(\Phi)$.

Core model checking algorithm

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Propositional formulas

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$Sat(\cdot)$ is defined by structural induction as follows:

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Sat(\text{true}) &= S \\
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In order to determine whether $s \in Sat(\mathbb{P}_J(\varphi))$, the probability $Pr(s \models \varphi)$ for the event specified by φ needs to be established.

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Let us consider the computation of $Pr(s \models \varphi)$ for all possible φ.

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This is a slight generalisation of the Volterra integral equation system for timed reachability.

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Phrased using CSL state formulas

$$\underbrace{Pr(s \models \Phi \, \mathrm{U}^{\leqslant t} \, \Psi)}_{\text{timed reachability in } \mathcal{C}} \quad =$$

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The last step collapses all states in $S \setminus Sat(\exists(\Phi \, \mathrm{U} \, \Psi))$ into a single state, and does the same with all states in $Sat(\forall(\Phi \, \mathrm{U} \, \Psi))$.

Preservation of CSL-formulas

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Bisimulation and CSL-equivalence coincide

Let $\mathcal{C}$ be a finitely branching CTMC and s, t states in $\mathcal{C}$. Then:

$$s \sim_m t \quad \text{if and only if} \quad s \text{ and } t \text{ are CSL-equivalent.}$$

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If for CSL-formula Φ we have $s \models \Phi$ but $t \not\models \Phi$, then it follows $s \not\sim_m t$.

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Weak bisimulation and CSL-without-next-equivalence coincide

Let $\mathcal{C}$ be a finitely branching CTMC and s, t states in $\mathcal{C}$. Then:

$$s \approx_m t \quad \text{if and only if} \quad s \text{ and } t \text{ are CSL-without-next-equivalent.}$$

Here. CSL-without-next is the fragment of CSL where the next-operator $\bigcirc$ does not occur.

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For any finite CTMC $\mathcal{C}$ with state space S, $r \geqslant \max\{\, r(s) \mid s \in S \,\}$ and Φ a CSL-without-next-formula:

$$Sat^{\mathcal{C}}(\Phi) \;=\; Sat^{\mathcal{C}'}(\Phi) \quad \text{where } \mathcal{C}' = \mathit{unif}(r, \mathcal{C}).$$

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Uniformization and CSL

For any uniformized CTMC: CSL-equivalence coincides with CSL-without-next-equivalence.

Overview

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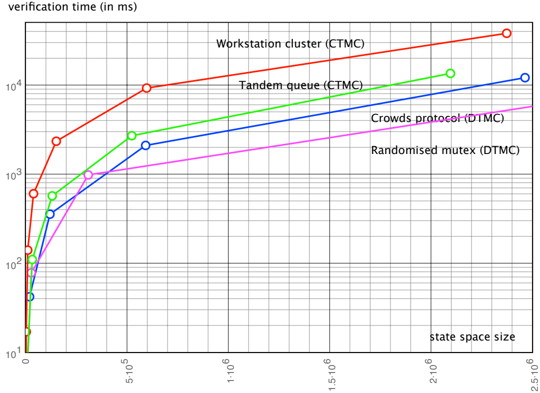
For finite CTMC $\mathcal{C}$ and CSL state-formula Φ, the CSL model-checking problem can be solved in time

$$\mathcal{O}\big(\, poly(size(\mathcal{C})) \cdot t_{\max} \cdot |\Phi| \,\big)$$

where $t_{\max} \;=\; \max\{\, t \mid \Psi_1 \,\mathsf{U}^{\leqslant t} \Psi_2 \text{ occurs in } \Phi \,\}$ with and $t_{\max} = 1$ if Φ does not contain a time-bounded until-operator.

Some practical verification times



- command-line tool MRMC ran on a Pentium 4, 2.66 GHz, 1 GB RAM laptop.
- CSL formulas are time-bounded until-formulas.

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- CSL model checking is performed by a recursive descent over Φ.
- The timed next operator amounts to a single vector-matrix multiplication.
- The time-bounded until-operator $\mathrm{U}^{\leqslant t}$ is solved by uniformization.
- The worst-case time complexity is polynomial in the size of the CTMC and linear in the size of the formula.