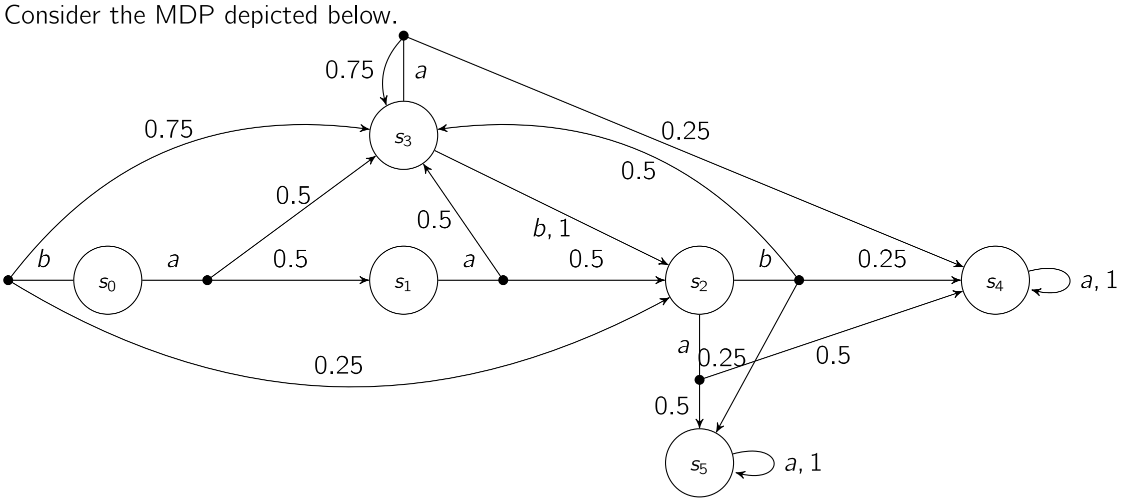
Exercise 1 (Value Iteration): (5 points)

Consider the MDP depicted below.



Consider the probabilities for the following property: $\mathrm{Pr}^{\mathcal{M}}_{max}(s \models \Diamond s_4)$ for all $s \in S$.

a) Execute policy iteration, start with taking a in all states. If you need an ordering on the states, use the ordering of the state numbers.

b) Give the LP formulation to characterize the property above.

c) Execute the preprocessing to eliminate $S^{=0}, S^{=1}$ as discussed in the lecture. Then, execute two iterations of value iteration.

Exercise 2 (Minimal Counterexamples): (5 points)

For an MDP $\mathcal{M} = (S, \mathrm{Act}, \mathbf{P}, \mathbf{s}_{init}, \mathrm{AP}, \mathbf{L})$, a *subsystem* is an MDP $\mathcal{M}' = (S', \mathrm{Act}', \mathbf{P}', \mathbf{s}'_{init}, \mathrm{AP}', \mathbf{L}')$ with $S' \subset S$. M' shall have the exact same behavior as M for those states, that are included in S'.

a) Give a formal definition for a subsystem of an MDP. What changes need to be made to the original Definition of MDPs?

b) A *mixed integer linear program* is a linear program where certain variables are allowed to be integer. The formal definition reads as follows: Let $A \in \mathbb{Q}^{m \times n}$, $B \in \mathbb{Q}^{m \times k}$, $b \in \mathbb{Q}^m$, $c \in \mathbb{Q}^n$, and $d \in \mathbb{Q}^k$. A *mixed integer linear program* (MILP) consists of computing $\min c^T x + d^T y$ such that $Ax + By \leq b$ and $x \in \mathbb{R}^n,\ y \in \mathbb{Z}^k$.

Consider an MDP $\mathcal{M} = (S, \mathrm{Act}, \mathbf{P}, \mathbf{s}_{init}, \mathrm{AP}, \mathbf{L})$ with a single initial state and a reachability property $\varphi = \mathbb{P}_{\leq \lambda}(\Diamond \texttt{target})$ that is violated for s_{init}.

Give an MILP formulation that computes a subsystem $\mathcal{M}'$ of an MDP $\mathcal{M}$ which is minimal in terms of the number of states such that s_{init} is included in S' and φ is also violated for s_{init} inside M'. (*Hint:* Use integer variables to count the states of the subsystem.)