

**Exercise 1 (Weak Until):**

**(4 points)**

Prove the following theorem: For any finite DTMC it holds that

$$\mathbb{P}_{=1}(\diamond a) \equiv \forall((\exists \diamond a) W a)$$

where  $W$  is the weak until operator defined by  $\Phi W \Psi = (\Phi U \Psi) \vee \square \Phi$ .

**Exercise 2 (PCTL Equivalences):**

**(2 points)**

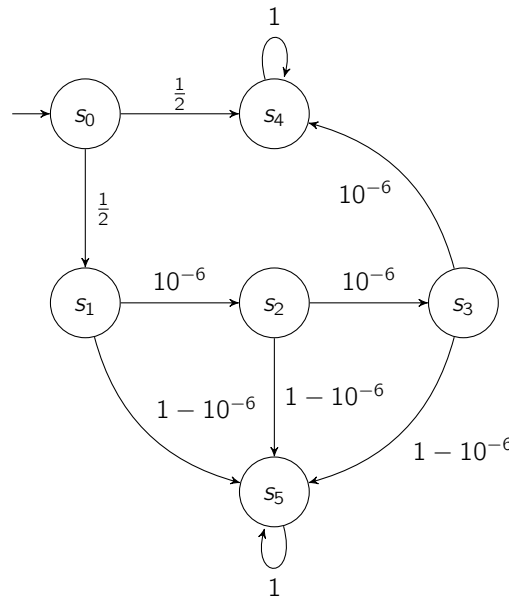
Prove or disprove the following *PCTL* equivalences.

- a)  $\mathbb{P}_{>0.5}(\bigcirc \mathbb{P}_{>0.5}(\diamond a)) \equiv \mathbb{P}_{>0.5}(\diamond \mathbb{P}_{>0.5}(\bigcirc a))$
- b)  $\mathbb{P}_{=1}(\bigcirc \mathbb{P}_{=1}(\diamond a)) \equiv \mathbb{P}_{=1}(\diamond \mathbb{P}_{=1}(\bigcirc a))$

**Exercise 3 (PCTL Model Checking):**

**(4 points)**

- a) Consider the following DTMC  $D$ .



- (i) Without any computation, decide whether  $D \models \underbrace{\mathbb{P}_{\leq 0.5}(\diamond s_4)}_{\Phi}$ .

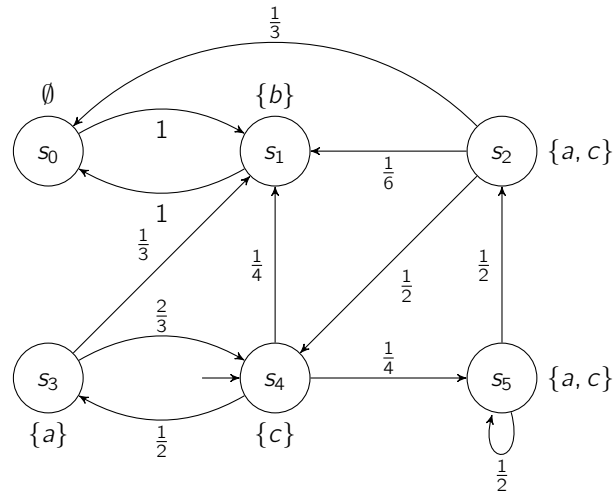
(ii) Write a PRISM model that precisely captures the above DTMC.

(iii) Use PRISM to verify or refute your result for  $\Phi$ .

To document your work, please include your model and the (relevant) output of PRISM. Additionally, explain any remarkable observations.

*Hint: PRISM and further documentation can be obtained on [www.prismmodelchecker.org](http://www.prismmodelchecker.org).*

- b) Consider the DTMC  $D'$ .



Determine the set  $Sat(\Phi)$  where  $\Phi = \mathbb{P}_{< \frac{3}{4}}(\bigcirc \mathbb{P}_{\geq \frac{1}{3}}(a \text{ U}^{\leq 3} (b \vee \neg c)))$  using the algorithm from the lecture. Please give intermediate results and explain your reasoning. A full computation, however, is not needed.