# Concurrency Theory

## Interleaving Semantics of Petri Nets

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http://moves.rwth-aachen.de/teaching/ws-1516/ct

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# Overview

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# Carl Adam Petri (1926-2010)

The original work[1] does not contain a single (graphical) Petri net!

[1] Petri's PhD dissertation, 1962.

# Semantics: executions and traces

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States: current configurations of the machine

- One or more initial states
- Possibly some distinguished final states

Transitions: moves between configurations

| | | | |
|---|---|---|---|
| Lambda calculus | $(\lambda x.xx)(\lambda y.y)$ | $\longrightarrow$ | $(\lambda y.y)(\lambda z.z)$ |
| Turing machine | $0010q_1011$ | $\longrightarrow$ | $001q_201011$ |
| Finite automaton | $q_1$ | $\xrightarrow{a}$ | $q_2$ |
| Pushdown automaton | $(q_1, XYYZ)$ | $\xrightarrow{a}$ | $(q_2, XYXYYZ)$ |

Executions: alternating sequences of states and transitions

# Petri's question

C.A. Petri points out a discrepancy between how Theoretical Physics and Theoretical Computer Science described systems in 1962:

Theoretical Physics describes systems as a collection of interacting particles (subsystems), without a notion of global clock or simultaneity

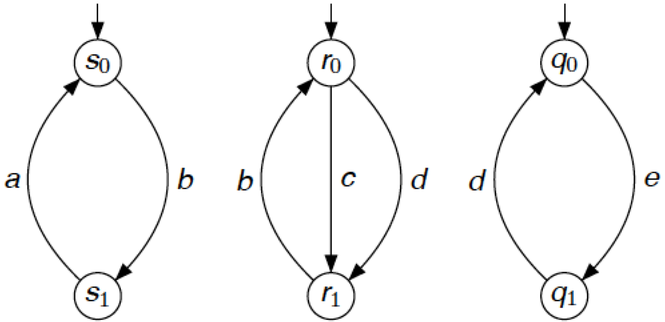
Theoretical Computer Science describes systems as sequential virtual machines going through a temporally ordered sequence of global states

Petri's question:

Which kind of abstract machine should be used to describe the physical implementation of a Turing machine?

# Petri net

A graphical representation of interacting finite automata:



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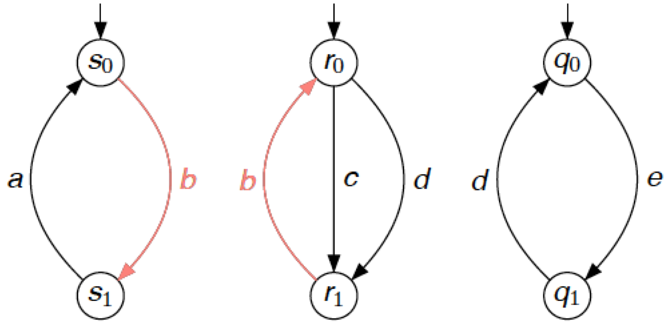

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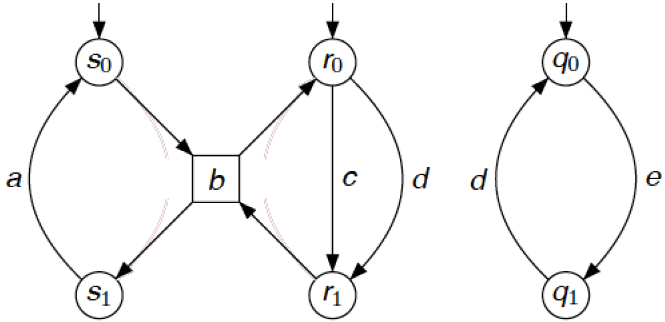

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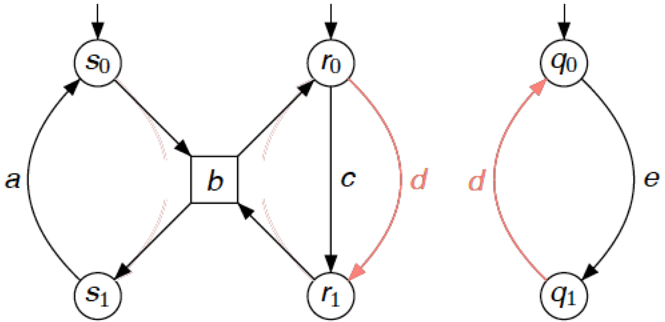

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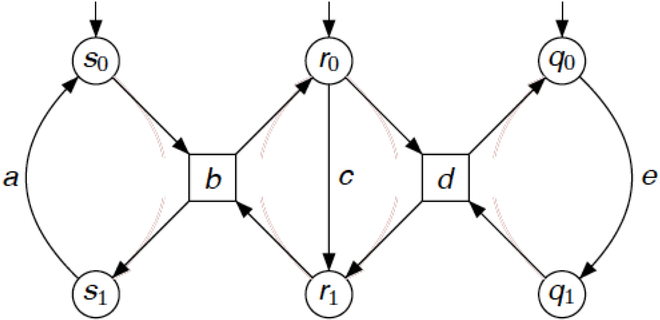

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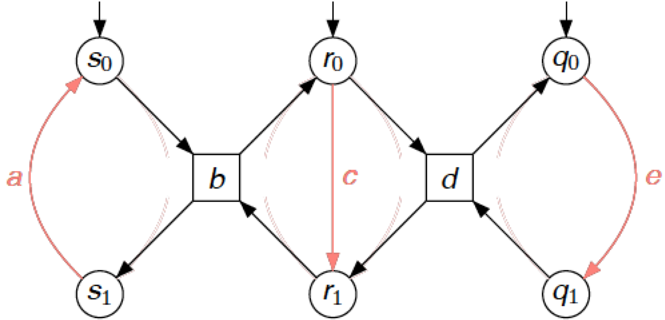

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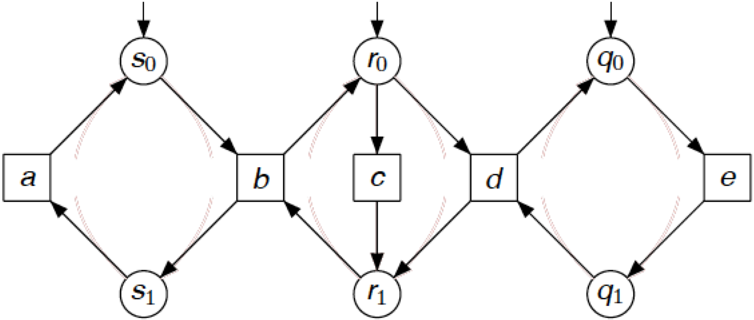

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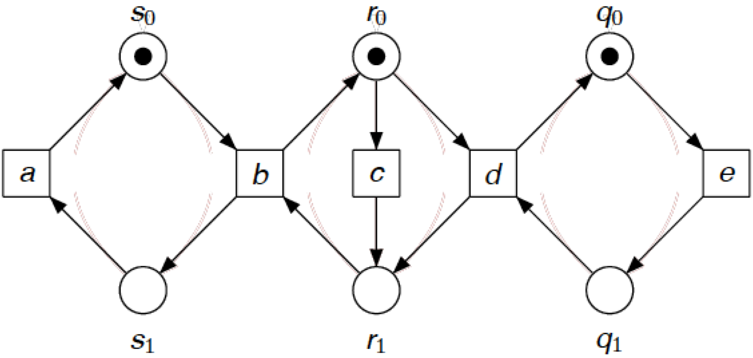

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A Petri net is a structure with two kinds of elements: places and transitions. They are connected by arcs.

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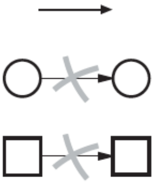

# Nets

## Net

A Petri net $N$ is a triple $(P, T, F)$ where:

- $P$ is the finite set of places
- $T$ is the finite set of transitions with $P \cap T = \varnothing$
- $F \subseteq (P \times T) \cup (T \times P)$ are the arcs[2]

Places and transitions are generically called nodes.

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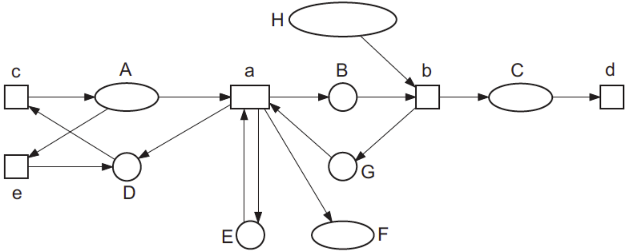


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# The pre- and post-sets

## Pre- and post-sets

Let node $x \in P \cup T$.
The pre-set of $x$ is defined by: ${}^\bullet x = \{ y \mid (y, x) \in F \}$.
The post-set of $x$ is defined by: $x^\bullet = \{ y \mid (x, y) \in F \}$.

Two nodes $x, y \in N$ form a loop if $x \in {}^\bullet y$ and $y \in {}^\bullet x$.

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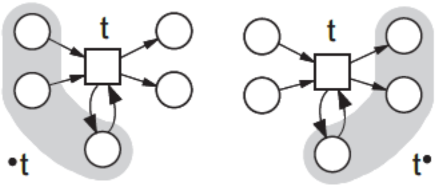

# Markings

## Marking

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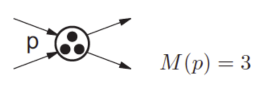

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$M(p) = 3$

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## Intuition

Note: a marking is a multiset. It defines a distribution of tokens across places. Tokens are depicted as black dots.

# Transition firing

## Enabling and firing of a transition

Let $(P, T, F, M)$ be an elementary net. Marking $M$ enables a transition $t$ if $M(p) \geqslant 1$ for each place $p \in {}^{\bullet}t$.

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Transition $t$ can fire in marking $M$ if $t$ is enabled at $M$. Its firing leads to marking $M'$, denoted by step $M \xrightarrow{t} M'$, defined for place $p \in P$ by:

$$M'(p) \;=\; M(p) - F(p, t) + F(t, p).$$

where we represent $F$ by its characteristic function.

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Transition $t$ is enabled whenever every $p \in {}^\bullet t$ holds at least one token. On $t$'s firing, one token is removed from each place in ${}^\bullet t$, and one token is put in each place in $t^\bullet$:

$$M'(p) \;=\; \begin{cases} M(p) - 1 & \text{if } p \in {}^\bullet t \text{ and } p \notin t^\bullet \\ M(p) + 1 & \text{if } p \in t^\bullet \text{ and } p \notin {}^\bullet t \\ M(p) & \text{otherwise} \end{cases}$$

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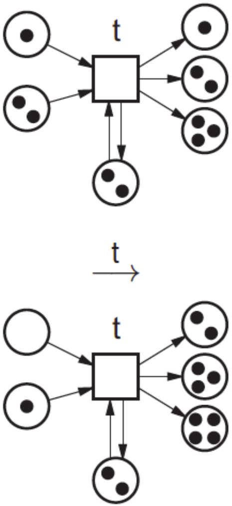


# Overview

# The interleaving semantics of Petri nets

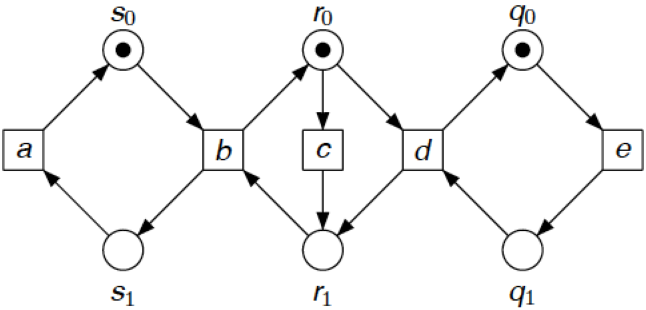
## An execution semantics

States: markings (distributions of tokens over the net)

Transitions: $M \xrightarrow{t} M'$

Sequential runs: $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \ldots$

# The interleaving semantics of Petri nets



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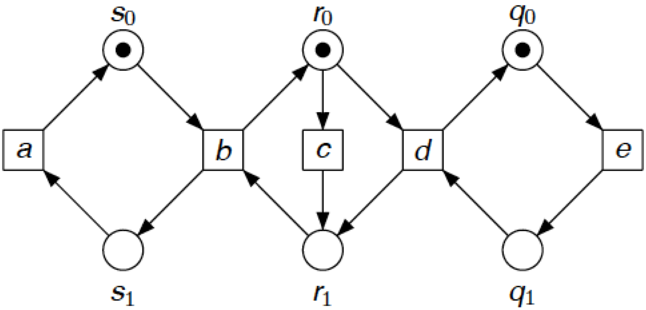


$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As the marking for $s_0$ is the complement of $s_1$, the marking for $s_0$ is omitted.

The same applies to the places $r_0$ and $q_0$.

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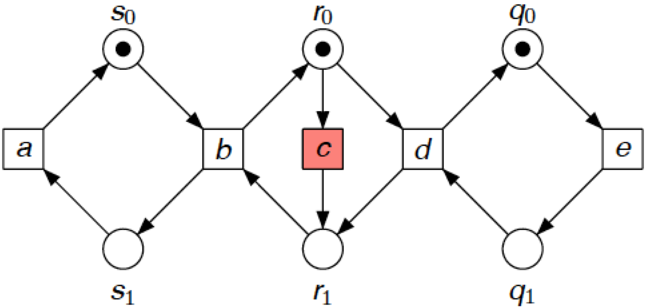


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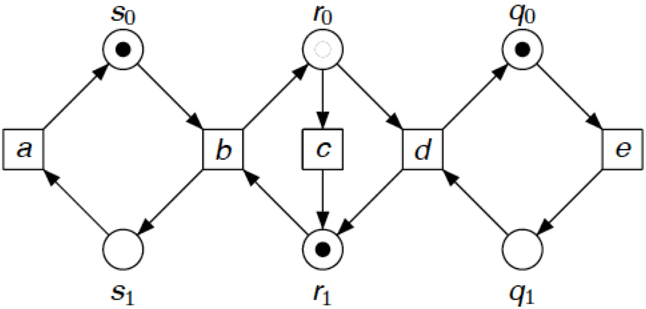


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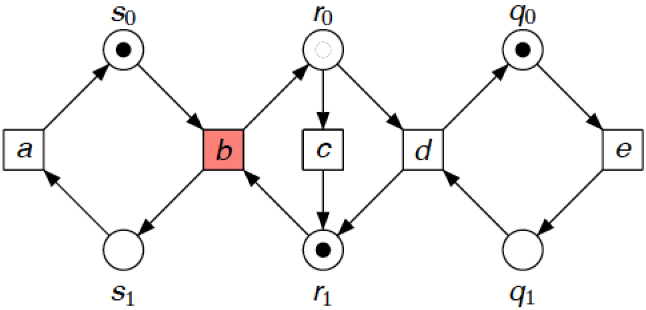


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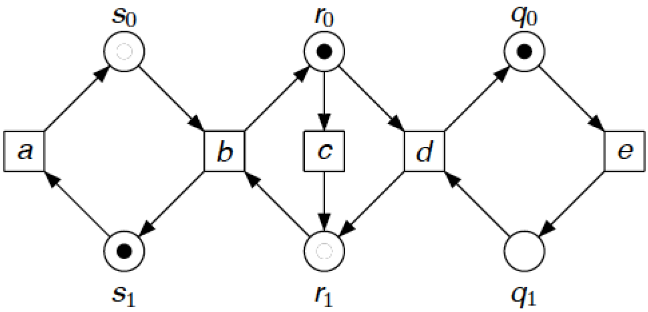


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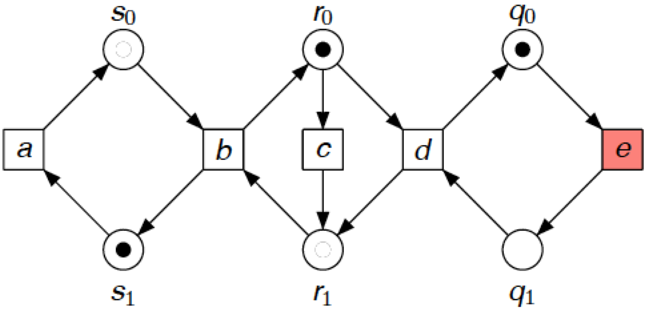


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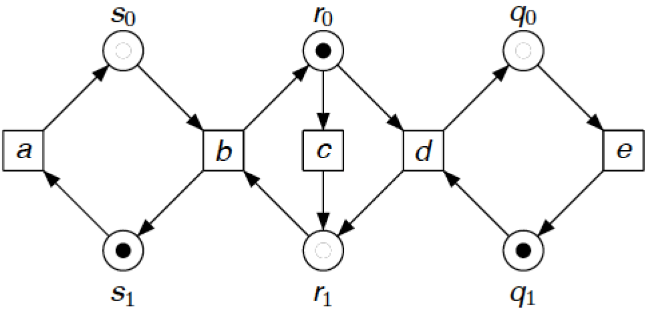


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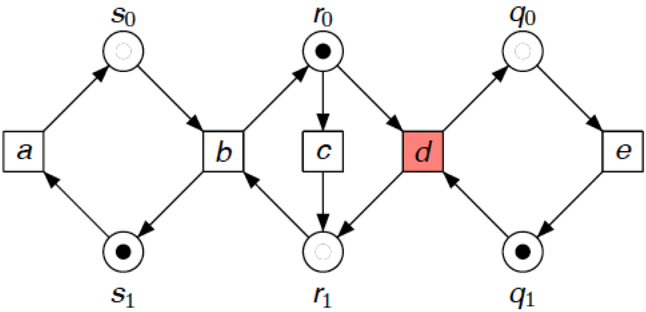


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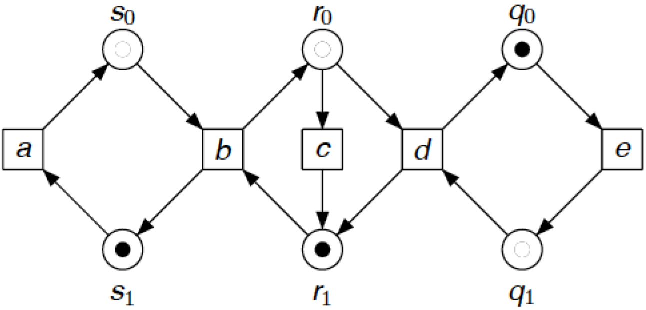


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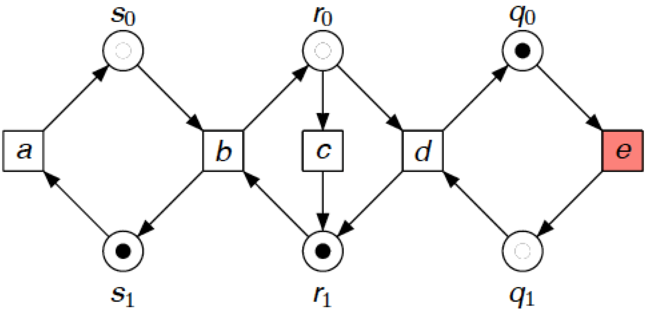


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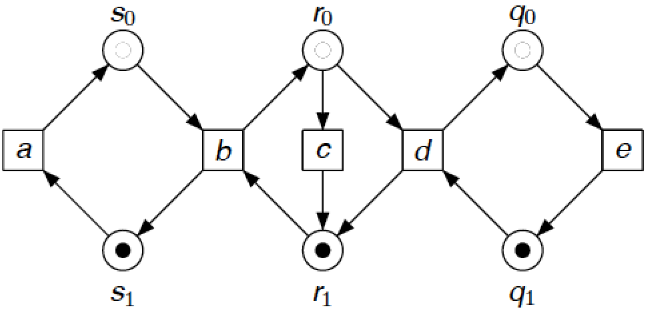


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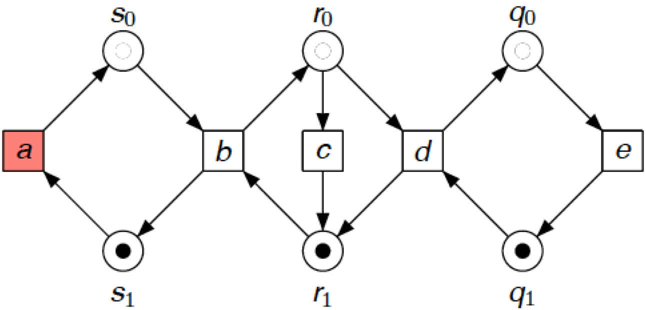


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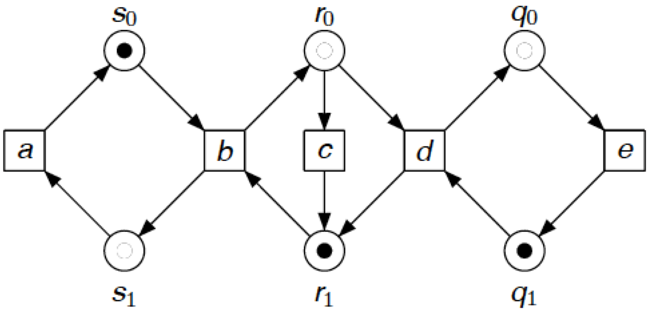


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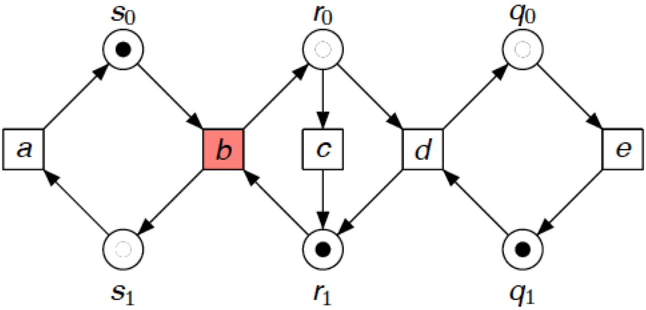


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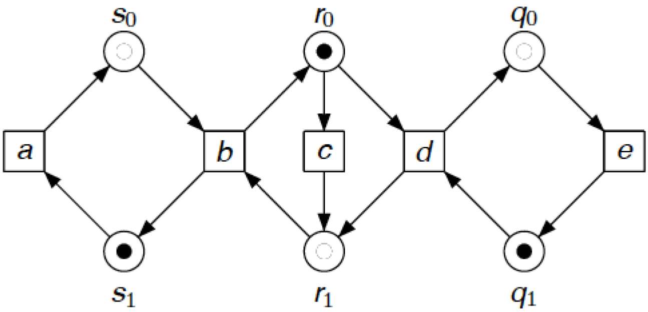


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# Reachable markings

## Step sequence

- A sequence of transitions $\sigma = t_1\, t_2 \ldots t_n$ is a step sequence if there exist markings $M_1$ through $M_n$ such that:

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \cdots \xrightarrow{t_{n-1}} M_{n-1} \xrightarrow{t_n} M_n$$

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- Marking $M_n$ is reached by the occurrence of $\sigma$, denoted $M_0 \xrightarrow{\sigma} M_n$.

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## Step sequence

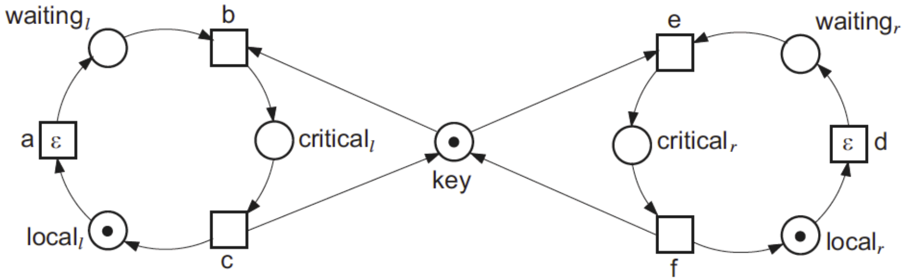
- A sequence of transitions $\sigma = t_1\, t_2 \ldots t_n$ is a step sequence if there exist markings $M_1$ through $M_n$ such that:

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \ldots \xrightarrow{t_{n-1}} M_{n-1} \xrightarrow{t_n} M_n$$

- Marking $M_n$ is reached by the occurrence of $\sigma$, denoted $M_0 \xrightarrow{\sigma} M_n$.
- $M$ is a reachable marking if there exists a step sequence $\sigma$ with $M_0 \xrightarrow{\sigma} M$.

# Mutual exclusion

Two processes cycling through the states local, waiting and critical.



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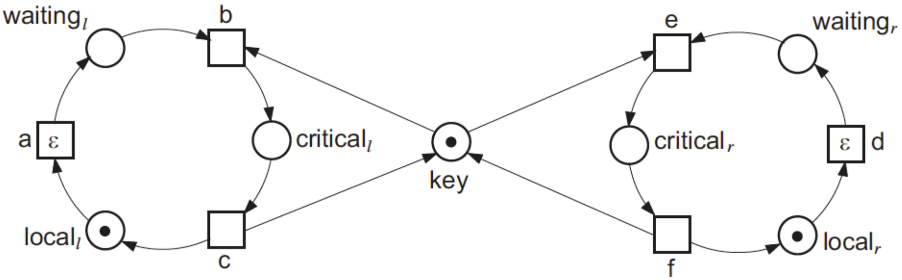
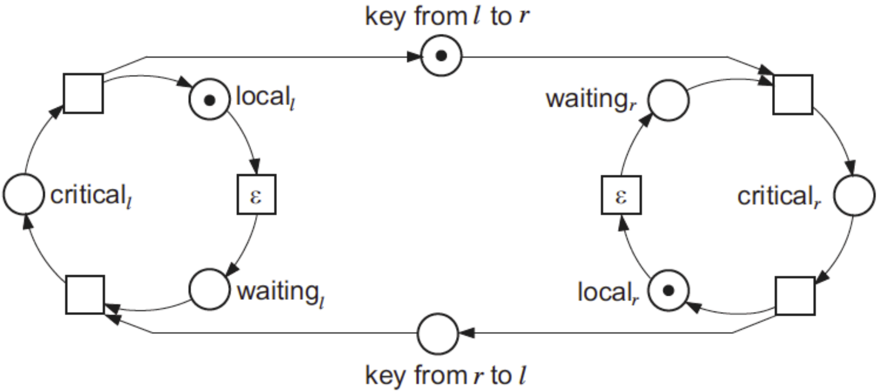



Between transitions b and e a conflict can arise infinitely often. No strategy has been modeled to solve this conflict.

# Mutual exclusion

A strategy where processes are acquired access in an alternating fashion:



# One-bounded elementary nets

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$$M(p) \leqslant 1.$$

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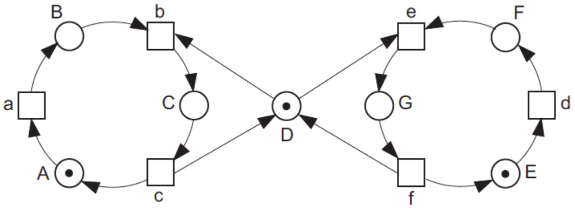
Markings of 1-bounded elementary nets can be described as a string of marked places, e.g., $ADE$. Two steps begin with this marking: $ADE \xrightarrow{a} BDE$ and $ADE \xrightarrow{d} ADF$.



# Overview

# Sequential runs

## Sequential run

Let $N$ be an elementary net. A sequential run of $N$ is a sequence

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \cdots$$

of steps of $N$ starting with the initial marking $M_0$. A run can be finite or infinite. A finite run $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} M_n$ is complete if $M_n$ does not enable any transition, otherwise incomplete.

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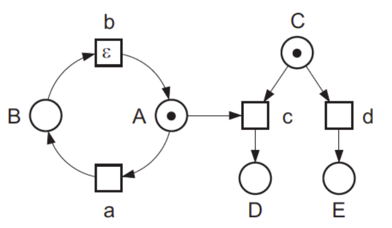


A sample complete run is:

$$AC \xrightarrow{a} BC \xrightarrow{b} AC \xrightarrow{c} D$$

A sample incomplete run is:

$$AC \xrightarrow{d} AE \xrightarrow{a} BE$$

# Marking graph

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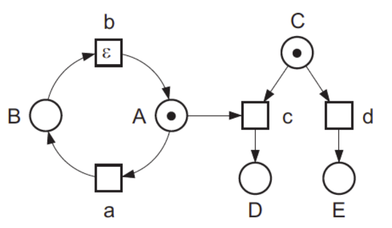


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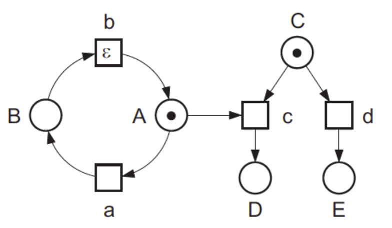


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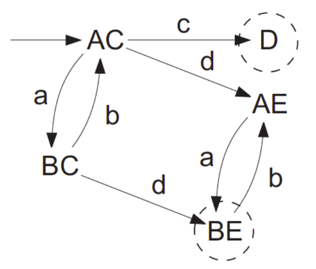


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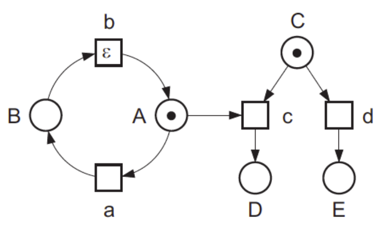


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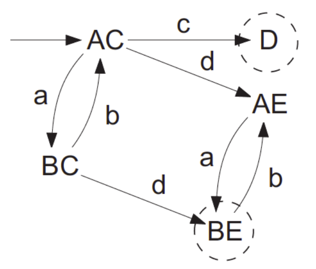


Its marking graph

**Observation:** 1-bounded nets induce finite marking graphs

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- The marking graph is the interleaving semantics of a net.