Concurrency Theory

HML and strong bisimilarity

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Overview

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Summary so far

- Weak and strong bisimilarity are based on mutually mimicking of processes.
- They possess the required properties of behavioural equivalences.[1]
- In particular, $\sim$ and $\approx^c$ are deadlock sensitive.
- Hennessy-Milner logic is a logic for expressing properties of processes.

Aim of this lecture

1. Study the connection between strong bisimilar processes and HML.

[1] For weak bisimilarity the notion of observation congruence was needed.

Overview

Verifying correctness of reactive systems

Equivalence checking approach

$$Impl \equiv Spec$$

- $\equiv$ is an abstract equivalence, e.g. $\sim$ or $\approx^c$
- *Spec* is often expressed in the same language as *Impl*, e.g., CCS
- *Spec* provides the full specification of the intended behaviour.

Model checking approach

$$Impl \models Property$$

- $\models$ is the satisfaction relation
- *Property* is a particular feature, often expressed via a logic, e.g., HML
- *Property* is a partial specification of the intended behaviour

Overview

HML syntax

Syntax of HML formulae [Hennessy & Milner, 1985]

$$F, G \ ::= \ \text{true} \mid \text{false} \mid F \wedge G \mid F \vee G \mid \langle a \rangle\, F \mid [a]\, F$$

where a is an action.

Intuitive interpretation

- true, all processes satisfy this property
- false, no process satisfies this property
- $\wedge$, $\vee$ logical conjunction and disjunction
- $\langle a \rangle\, F$, there is at least one a-successor that satisfies F
- $[a]\, F$, all a-successors have to satisfy F.

Note that negation is not an elementary operation in HML.

HML semantics

HML semantics

Let $P \in Prc$, $F.G$ HML-formulae, and a an action. Then:

$$\begin{array}{rl} P \models \text{true} & \text{for each } P \in Prc \\ P \models \text{false} & \text{for no } P \in Prc \\ P \models F \wedge G & \text{iff } P \models F \text{ and } P \models G \\ P \models F \vee G & \text{iff } P \models F \text{ or } P \models G \\ P \models \langle a \rangle\, F & \text{iff } P \xrightarrow{a} P' \text{ for some } P' \in Prc \text{ with } P' \models F \\ P \models [a]\, F & \text{iff } P' \models F \text{ for all } P' \in Prc \text{ with } P \xrightarrow{a} P'. \end{array}$$

We write $P \not\models F$ whenever P does not satisfy F, i.e., not $(P \models F)$.

Examples

Negation

Complement of an HML-formula

$$\begin{array}{rcl@{\qquad}rcl} \text{true}^c & = & \text{false} & (F \vee G)^c & = & F^c \wedge G^c \\ \text{false}^c & = & \text{true} & (\langle a \rangle\, F)^c & = & [a]\, F^c \\ (F \wedge G)^c & = & F^c \vee G^c & ([a]\, F)^c & = & \langle a \rangle\, F^c \end{array}$$

Theorem

For any $P \in Prc$ and HML-formula F:

1. $P \models F$ implies $P \not\models F^c$
2. $P \models F^c$ implies $P \not\models F^c$.

Proof.

By structural induction on F. Rather straightforward. □

Overview

Strong bisimulation

Strong bisimulation [Park, 1981, Milner, 1989]

A binary relation $\mathcal{R} \subseteq Prc \times Prc$ is a strong bisimulation whenever for every $(P, Q) \in \mathcal{R}$, and $\alpha \in Act$:

1. if $P \xrightarrow{\alpha} P'$ then there exists $Q' \in Prc$ s.t. $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \mathcal{R}$
2. if $Q \xrightarrow{\alpha} Q'$ then there exists $P' \in Prc$ s.t. $P \xrightarrow{\alpha} P'$ and $(P', Q') \in \mathcal{R}$.

Strong bisimilarity

The processes P and Q are strongly bisimilar, denoted $P \sim Q$, iff there is a strong bisimulation $\mathcal{R}$ with $(P, Q) \in \mathcal{R}$. Thus,

$$\sim \;=\; \bigcup \{ \mathcal{R} \mid \mathcal{R} \text{ is a strong bisimulation} \}.$$

Relation $\sim$ is called strong bisimilarity.

Image-finite transition system

Image-finite process

A process P is image-finite iff the set $\{ P' \in Prc \mid P \xrightarrow{\alpha} P' \}$ is finite for every action α (possibly $\alpha = \tau$). A labeled transition system is image-finite if so is each of its states.

Examples

The process $A_{rep} \;=\; a.\text{nil} \,||\, A_{rep}$ is not image-finite. By induction on n, one can prove that for each $n \in \mathbb{N}$:

$$A_{rep} \;\xrightarrow{a}\; \underbrace{a.\text{nil} \,||\, \cdots \,||\, a.\text{nil}}_{n \text{ times}} \,||\, \text{nil} \,||\, A_{rep}$$

Also the process $A^{\omega} \;=\; \sum_{i \geqslant 0} a^i$ with $a^0 = \text{nil}$ and $a^{i+1} = a.a^i$ is not image-finite.

Relationship HML and trace equivalence

Recall from Lecture 4:

HML and trace equivalence

If $P, Q \in Prc$ satisfy the same HML-formulae, i.e., for every HML-formula F it holds $P \models F$ iff $Q \models F$, then $Tr(P) = Tr(Q)$. The converse does not hold.

Relationship HML and strong bisimilarity

Hennessy-Milner theorem

Let $(Prc, Act, \{ \xrightarrow{a} \mid a \in Act \})$ be an image-finite LTS and $P, Q \in Prc$. Then:

$$P \sim Q$$

if and only if

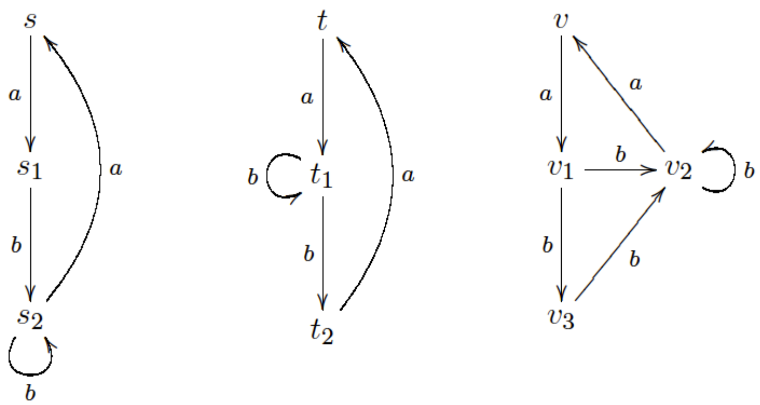
$$\text{for every HML-formula } F : (P \models F \text{ iff } Q \models F)\,.$$

Proof.

On the board. □

Showing $P \not\sim Q$ thus amounts to finding a single HML-formula F with $P \models F$ and $Q \not\models F$.

Example



It follows $s \not\sim t$ and $s \not\sim v$ and $t \not\sim v$. Distinguishing HML-formulas for:

- s and t is: $F = \langle a \rangle \langle b \rangle [b]\,\text{false}$ as $t \models F$ and $s \not\models F$
- s and v is: $F = \langle a \rangle \langle b \rangle [a]\,\text{false}$ as $v \models F$ and $s \not\models F$
- t and v is: $F = \langle a \rangle \langle b \rangle (\langle a \rangle\,\text{true} \wedge \langle b \rangle\,\text{true})$ as $v \models F$ and $t \not\models F$.

Counterexample for non image-finite processes

Overview

Characteristic properties

The Hennessy-Milner theorem asserts that for image-finite processes, strong bisimilarity and HML-equivalence coincide.

As a next step, we show that for finite transition systems, the equivalence classes under $\sim$ can be characterised with a single formula in HML extended with recursion.

For finite process P, this HML-formula X_P is called P's characteristic property.

The need for recursion

The need for recursion

There is no recursion-free HML-formula F_p that can characterize the process P defined by $X = a.X$ up to strong bisimilarity.

Proof.

By contraposition. Let HML-formula F_p with $[\![F_p]\!] = \{ Q \mid P \sim Q \}$. In particular, $P \models F_p$ and $Q \models F_p$ implies $P \sim Q$ for each Q. We will show that this cannot hold for any formula F_p. Let $P_0, P_1, P_2, \ldots$ be defined by $P_0 = \text{nil}$ and $P_{i+1} = a.P_i$. P_i can execute i a-actions in a row and then terminates. Nothing else. Obviously $P \not\sim P_i$ for every i. Claim: for every HML-formula F it holds $P \models F$ iff $P_k \models F$ where k is the modal depth[2] of F. This can be proven by induction on the structure of F. Thus, $P \sim P_k$. Contradiction. □

[2] The maximal number of nested occurrences of modal operators in F.

HML with recursion

Syntax of recursive HML formulae [Hennessy & Milner, 1985]

Let $\mathcal{X} = \{ X_1, \ldots, X_n \}$ be a set of variables. The syntax of HML over $\mathcal{X}$ is defined by the grammar:

$$F, G \;::=\; X_i \mid \text{true} \mid \text{false} \mid F \wedge G \mid F \vee G \mid \langle a \rangle F \mid [a] F$$

where $0 < i \leqslant n$ and a is an action. A mutually recursive equation system has the form

$$(X_i = F_{X_i} \mid 0 < i \leqslant n)$$

where F_{X_i} is a HML-formula over $\mathcal{X}$ for every $0 < i \leqslant n$.

We skip the details of the semantics; see Lecture 5 for the details.

Characteristic formula

Consider the finite LTS $(\{ P_1, \ldots, P_n \}, Act, \longrightarrow)$ and let $\mathcal{X} = \{ X_{P_1}, \ldots, X_{P_n}, \ldots \}$ contain (at least) n variables.

Intuitively, X_P is the syntactic symbol for the characteristic formula of process P.

A characteristic formula for P has to describe which actions P can perform, which actions it cannot perform and what happens after performing an action.

Example

A coffee machine (again) on the black board.

Characteristic property

Characteristic property [Ingolfsdottir et. al, 1987]

For finite process $P \in Prc$, let recursive HML-formula X_P be defined by:

$$X_P \overset{max}{=} \bigwedge_{a, P'. P \xrightarrow{a} P'} \langle a \rangle X_{P'} \;\wedge\; \bigwedge_{a} [a] \left(\bigvee_{a, P'. P \xrightarrow{a} P'} X_{P'} \right)$$

Then: $Q \models X_P$ iff $P \sim Q$ for every $Q \in Prc$.
The formula X_P is called the characteristic property of P.

Proof.

Outside the scope of this lecture. □

Overview

Summary

1. Strong bisimilarity and HML-equivalence coincide for image-finite processes.
2. This result does not hold for processes that are not image-finite.
3. Any two strong bisimilar processes satisfy the same HML formulas.
4. For finite processes a recursive HML-formula does exist that precisely characterises the strong bisimilar processes.