# Concurrency Theory

## Winter Semester 2015/16

## Lecture 7: Modelling and Analysing Mutual Exclusion Algorithms


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# Outline of Lecture 7

# Recap: Mutually Recursive Equational Systems

## Syntax of Mutually Recursive Equational Systems

**Definition (Syntax of mutually recursive equational systems)**

Let $\mathcal{X} = \{X_1, \ldots, X_n\}$ be a set of variables. The set $HMF_{\mathcal{X}}$ of Hennessy-Milner formulae over $\mathcal{X}$ is defined by the following syntax:

$$
\begin{array}{rlll}
F ::= & X_i & & \text{(variable)} \\
\mid & \mathrm{tt} & & \text{(true)} \\
\mid & \mathrm{ff} & & \text{(false)} \\
\mid & F_1 \wedge F_2 & & \text{(conjunction)} \\
\mid & F_1 \vee F_2 & & \text{(disjunction)} \\
\mid & \langle \alpha \rangle F & & \text{(diamond)} \\
\mid & [\alpha] F & & \text{(box)}
\end{array}
$$

where $1 \leq i \leq n$ and $\alpha \in Act$. A mutually recursive equational system has the form

$$(X_i = F_{X_i} \mid 1 \leq i \leq n)$$

where $F_{X_i} \in HMF_{\mathcal{X}}$ for every $1 \leq i \leq n$.

## Semantics of Recursive Equational Systems I

As before: semantics of formula depends on states satisfying the variables

**Definition (Semantics of mutually recursive equational systems)**

Let $(S, Act, \longrightarrow)$ be an LTS and $E = (X_i = F_{X_i} \mid 1 \leq i \leq n)$ a mutually recursive equational system. The semantics of $E$, $[\![E]\!] : (2^S)^n \to (2^S)^n$, is defined by

$$[\![E]\!](T_1, \ldots, T_n) := ([\![F_{X_1}]\!](T_1, \ldots, T_n), \ldots, [\![F_{X_n}]\!](T_1, \ldots, T_n))$$

where

$$\begin{aligned}
[\![X_i]\!](T_1, \ldots, T_n) &:= T_i \\
[\![\mathrm{tt}]\!](T_1, \ldots, T_n) &:= S \\
[\![\mathrm{ff}]\!](T_1, \ldots, T_n) &:= \emptyset \\
[\![F_1 \wedge F_2]\!](T_1, \ldots, T_n) &:= [\![F_1]\!](T_1, \ldots, T_n) \cap [\![F_2]\!](T_1, \ldots, T_n) \\
[\![F_1 \vee F_2]\!](T_1, \ldots, T_n) &:= [\![F_1]\!](T_1, \ldots, T_n) \cup [\![F_2]\!](T_1, \ldots, T_n) \\
[\![\langle \alpha \rangle F]\!](T_1, \ldots, T_n) &:= \langle \cdot \alpha \cdot \rangle([\![F]\!](T_1, \ldots, T_n)) \\
[\![[\alpha] F]\!](T_1, \ldots, T_n) &:= [\cdot \alpha \cdot]([\![F]\!](T_1, \ldots, T_n))
\end{aligned}$$

# Recap: Mutually Recursive Equational Systems

## Semantics of Recursive Equational Systems II

**Lemma**

*Let $(S, Act, \longrightarrow)$ be a finite LTS and $E = (X_i = F_{X_i} \mid 1 \leq i \leq n)$ a mutually recursive equational system. Let $(D, \sqsubseteq)$ be given by $D := (2^S)^n$ and*

$$(T_1, \ldots, T_n) \sqsubseteq (T_1', \ldots, T_n')$$

*iff $T_i \subseteq T_i'$ for every $1 \leq i \leq n$.*

1. $(D, \sqsubseteq)$ *is a complete lattice with*
$$\bigsqcup\{(T_1^i, \ldots, T_n^i) \mid i \in I\} = \left(\bigcup\{T_1^i \mid i \in I\}, \ldots, \bigcup\{T_n^i \mid i \in I\}\right)$$
$$\bigsqcap\{(T_1^i, \ldots, T_n^i) \mid i \in I\} = \left(\bigcap\{T_1^i \mid i \in I\}, \ldots, \bigcap\{T_n^i \mid i \in I\}\right)$$
2. $[\![E]\!]$ *is monotonic w.r.t.* $(D, \sqsubseteq)$
3. $\mathrm{fix}([\![E]\!]) = [\![E]\!]^m(\emptyset, \ldots, \emptyset)$ *for some* $m \in \mathbb{N}$
4. $\mathrm{FIX}([\![E]\!]) = [\![E]\!]^M(S, \ldots, S)$ *for some* $M \in \mathbb{N}$

**Proof.**

omitted □

# Outline of Lecture 7

## A Mutually Recursive Specification

### Example 7.1

$$\begin{tikzcd} s \arrow[r, bend left, "a"] & s_1 \arrow[l, bend left, "b"] & s_2 \arrow[l, "a"] \arrow[r, "a"] & s_3 \arrow[loop right, "b"] \end{tikzcd}$$

Let $S := \{s, s_1, s_2, s_3\}$ and $E$ given by

$$X \stackrel{max}{=} \langle a \rangle Y \wedge [a] Y \wedge [b]\mathrm{ff}$$
$$Y \stackrel{max}{=} \langle b \rangle X \wedge [b] X \wedge [a]\mathrm{ff}$$

Computation of $\mathrm{FIX}(\llbracket E \rrbracket)$: on the board

## Outline of Lecture 7

Recap: Mutually Recursive Equational Systems

An Example

Mixing Least and Greatest Fixed Points

Modelling Mutual Exclusion Algorithms

Evaluating the CCS Model

Model Checking Mutual Exclusion

Alternative Verification Approaches

## Mixing Least and Greatest Fixed Points I

- **So far:** least/greatest fixed point of overall system
- **But:** too restrictive

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*"It is possible for the system to reach a state which has a livelock (i.e., an infinite sequence of internal steps)."*

## Mixing Least and Greatest Fixed Points I

- **So far:** least/greatest fixed point of overall system
- **But:** too restrictive

### Example 7.2

*"It is possible for the system to reach a state which has a livelock (i.e., an infinite sequence of internal steps)."*

can be specified by

$$Pos(Livelock)$$

where

$$Pos(F) \stackrel{min}{=} F \vee \langle Act \rangle Pos(F) \qquad \text{(cf. Example 4.6)}$$
$$Livelock \stackrel{max}{=} \langle \tau \rangle Livelock$$

(thus, $Livelock \equiv Forever(\tau)$ [cf. Example 6.3])

## Mixing Least and Greatest Fixed Points II

**Caveat:** arbitrary mixing can entail non-monotonic behaviour

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Fixed-point iteration:

$$(\perp, \top) = (\emptyset, S)$$

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**Solution:** nesting of specifications by partitioning equations into a sequence of blocks such that all equations in one block

- are of same type (either *min* or *max*) and
- use only variables defined in the same or subsequent blocks

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- are of same type (either *min* or *max*) and
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$\Longrightarrow$ bottom-up, block-wise evaluation by fixed-point iteration

## Mixing Least and Greatest Fixed Points III

### Example 7.4 (cf. Example 7.2)

$$PosLL \overset{min}{=} Livelock \lor \langle Act \rangle PosLL$$
$$Livelock \overset{max}{=} \langle \tau \rangle Livelock$$

$$\begin{tikzcd} s \arrow[r, "a"] & p \arrow[r, "\tau"] \arrow[loop below, "\tau"] & q \arrow[r, "\tau"] & r \end{tikzcd}$$

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### Example 7.4 (cf. Example 7.2)

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$$s \xrightarrow{a} p \xrightarrow{\tau} q \xrightarrow{\tau} r \qquad p \circlearrowleft \tau$$

1. Fixed-point iteration for $Livelock : T \mapsto \langle \cdot \tau \cdot \rangle (T)$:

$$S = \{s, p, q, r\}$$

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$$S = \{s, p, q, r\} \mapsto \{p, q\} \mapsto \{p\} \mapsto \{p\}$$

2. Fixed-point iteration for $PosLL : T \mapsto \{p\} \cup \langle \cdot Act \cdot \rangle (T)$:

$$\emptyset$$

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# Mixing Least and Greatest Fixed Points

## The Modal $\mu$-Calculus

- Logic that supports free mixing of least and greatest fixed points:
  - D. Kozen: *Results on the Propositional $\mu$-Calculus*, Theoretical Computer Science 27, 1983, 333–354

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  (in NP $\cap$ co-NP; linear for HML with one variable)
- Generally undecidable for infinite LTSs and HML with one variable (CCS, Petri nets, ...)

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- Overview paper:
  - O. Burkart, D. Caucal, F. Moller, B. Steffen: *Verification on Infinite Structures*, Chapter 9 of *Handbook of Process Algebra*, Elsevier, 2001, 545–623

## Outline of Lecture 7

## Peterson's Mutual Exclusion Algorithm

- **Goal:** ensuring exclusive access to non-shared resources
- Here: two competing processes $P_1, P_2$ and shared variables
  - $b_1, b_2$ (Boolean, initially false)
  - $k$ (in $\{1, 2\}$, arbitrary initial value)
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**Algorithm 7.5 (Peterson's algorithm for $P_i$)**

while true do
    *"non-critical section"*;
    $b_i :=$ true;
    $k := j$;
    while $b_j \wedge k = j$ do skip;
    *"critical section"*;
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## Representing Shared Variables in CCS

- Not directly expressible in CCS (communication by message passing)
- Idea: consider variables as processes that communicate with environment by processing read/write requests

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### Example 7.6 (Shared variables in Peterson's algorithm)

- Encoding of $b_1$ with two (process) states $B_{1t}$ (value tt) and $B_{1f}$ (ff)
- Read access along ports $b1rt$ (in state $B_{1t}$) and $b1rf$ (in state $B_{1f}$)
- Write access along ports $b1wt$ and $b1wf$ (in both states)

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- Possible behaviours: $B_{1f} = \overline{b1rf}.B_{1f} + b1wf.B_{1f} + b1wt.B_{1t}$
  $B_{1t} = \overline{b1rt}.B_{1t} + b1wf.B_{1f} + b1wt.B_{1t}$

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- Possible behaviours:
$$\begin{aligned} B_{1f} &= \overline{b1rf}.B_{1f} + b1wf.B_{1f} + b1wt.B_{1t} \\ B_{1t} &= \overline{b1rt}.B_{1t} + b1wf.B_{1f} + b1wt.B_{1t} \end{aligned}$$
- Similarly for $b_2$ and $k$:
$$\begin{aligned} B_{2f} &= \overline{b2rf}.B_{2f} + b2wf.B_{2f} + b2wt.B_{2t} \\ B_{2t} &= \overline{b2rt}.B_{2t} + b2wf.B_{2f} + b2wt.B_{2t} \end{aligned}$$
$$\begin{aligned} K_1 &= \overline{kr1}.K_1 + kw1.K_1 + kw2.K_2 \\ K_2 &= \overline{kr2}.K_2 + kw1.K_1 + kw2.K_2 \end{aligned}$$

## Modelling the Processes in CCS

**Assumption:** $P_i$ cannot fail or terminate within critical section

**Peterson's algorithm**

while true do
  *"non-critical section"*;
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### CCS representation

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\begin{aligned}
P_1 &= \overline{b1wt}.\overline{kw2}.P_{11}\\
P_{11} &= b2rf.P_{12} + \\
&\phantom{=}\ b2rt.(kr1.P_{12} + kr2.P_{11})\\
P_{12} &= enter_1.exit_1.\overline{b1wf}.P_1\\
P_2 &= \overline{b2wt}.\overline{kw1}.P_{21}\\
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P_{22} &= enter_2.exit_2.\overline{b2wf}.P_2
\end{aligned}
$$

$$Peterson = (P_1 \parallel P_2 \parallel B_{1f} \parallel B_{2f} \parallel K_1) \setminus L$$

$$\text{for } L = \{b1rf, b1rt, b1wf, b1wt, b2rf, b2rt, b2wf, b2wt, kr1, kr2, kw1, kw2\}$$

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\text{for } L &= \{b1rf, b1rt, b1wf, b1wt, \\
&\quad b2rf, b2rt, b2wf, b2wt, \\
&\quad kr1, kr2, kw1, kw2\}
\end{aligned}
$$



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| Peterson's algorithm | CCS representation |
|---|---|
| while true do<br>*"non-critical section"*;<br>$b_i :=$ true;<br>$k := j$;<br>while $b_j \land k = j$ do skip;<br>*"critical section"*;<br>$b_i :=$ false;<br>end | $P_1 = \overline{b1wt}.\overline{kw2}.P_{11}$<br>$P_{11} = b2rf.P_{12} + b2rt.(kr1.P_{12} + kr2.P_{11})$<br>$P_{12} = enter_1.exit_1.\overline{b1wf}.P_1$<br>$P_2 = \overline{b2wt}.\overline{kw1}.P_{21}$<br>$P_{21} = b1rf.P_{22} + b1rt.(kr1.P_{21} + kr2.P_{22})$<br>$P_{22} = enter_2.exit_2.\overline{b2wf}.P_2$<br>$Peterson = (P_1 \parallel P_2 \parallel B_{1f} \parallel B_{2f} \parallel K_1) \setminus L$<br>for $L = \{b1rf, b1rt, b1wf, b1wt, b2rf, b2rt, b2wf, b2wt, kr1, kr2, kw1, kw2\}$ |

## Modelling the Processes in CCS

**Assumption:** $P_i$ cannot fail or terminate within critical section

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**CCS representation**

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&\quad b2rt.(kr1.P_{12} + kr2.P_{11}) \\
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for $L = \{b1rf, b1rt, b1wf, b1wt, b2rf, b2rt, b2wf, b2wt, kr1, kr2, kw1, kw2\}$

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### Peterson's algorithm

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## Outline of Lecture 7

Recap: Mutually Recursive Equational Systems

An Example

Mixing Least and Greatest Fixed Points

Modelling Mutual Exclusion Algorithms

Evaluating the CCS Model

Model Checking Mutual Exclusion

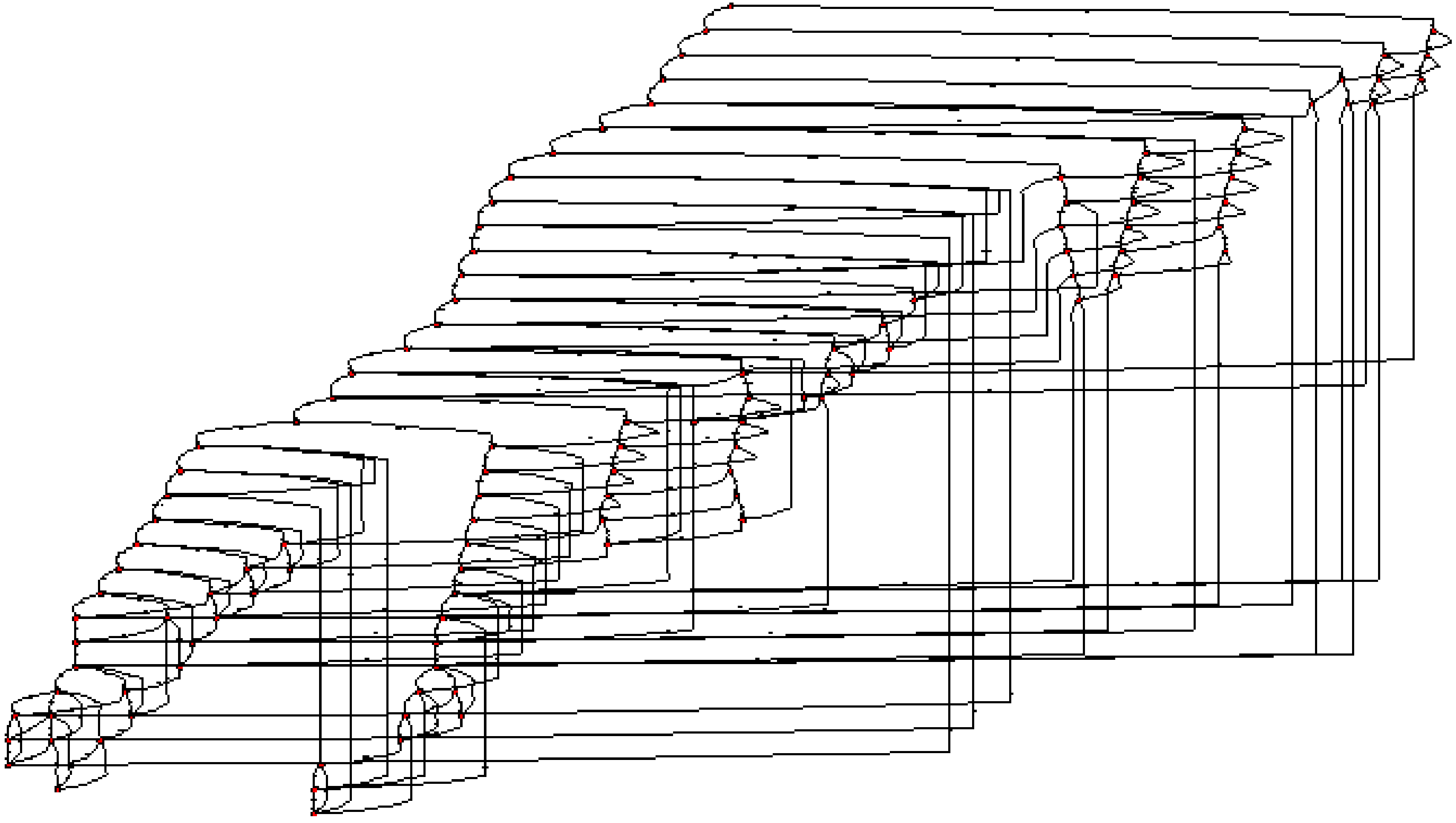
Alternative Verification Approaches

## Obtaining the LTS I

### Alternatives:

- By hand (really painful)
- By tools:
  - CAAL (Concurrency Workbench, Aalborg Edition): `http://caal.cs.aau.dk`
    - smart editor
    - visualisation of generated LTS
    - equivalence checking w.r.t. several bisimulation, simulation and trace equivalences
    - generation of distinguishing formulae for nonequivalent processes
    - model checking of recursive HML formulae
    - (bi)simulation and model checking games.
    - see exercises
  - TAPAs (Tool for the Analysis of Process Algebras): `http://rap.dsi.unifi.it/tapas/`
    - CCS specification of Peterson's algorithm available as example
    - yields LTS with 115 states (see next slide)
  - CWB (Edinburgh Concurrency Workbench): `http://homepages.inf.ed.ac.uk/perdita/cwb/`
    - somewhat outdated

## Obtaining the LTS II



# Outline of Lecture 7

# Model Checking Mutual Exclusion

## The Mutual Exclusion Property

- **Done:** formal description of Peterson's algorithm
- **To do:** analysing its behaviour (manually or with tool support)
- **Question:** what does "ensuring mutual exclusion" formally mean?

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- **Question:** what does "ensuring mutual exclusion" formally mean?

**Mutual exclusion**

At no point in the execution of the algorithm, processes $P_1$ and $P_2$ will both be in their critical section at the same time.

Alternatively:
It is always the case that either $P_1$ or $P_2$ or both are not in their critical section.

## Specifying Mutual Exclusion in HML

**Mutual exclusion**

It is always the case that either $P_1$ or $P_2$ or both are not in their critical section.

# Model Checking Mutual Exclusion

## Specifying Mutual Exclusion in HML

**Mutual exclusion**

It is always the case that either $P_1$ or $P_2$ or both are not in their critical section.

**Observations:**

- Mutual exclusion is an invariance property ("always")
- $P_i$ is in its critical section iff action $exit_i$ is enabled

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**Mutual exclusion in HML**

$$\begin{aligned} MutEx &:= Inv(F) \\ Inv(F) &\stackrel{max}{=} F \wedge [Act]Inv(F) \qquad \text{(cf. Theorem 6.2)} \\ F &:= [exit_1]\mathrm{ff} \vee [exit_2]\mathrm{ff} \end{aligned}$$

## Model Checking Mutual Exclusion

- Using TAPAs Tool
- Supports property specifications in $\mu$-calculus:

```
property MutEx:
max x. (([exit1] false | [exit2] false) & ([*] x))
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# Model Checking Mutual Exclusion

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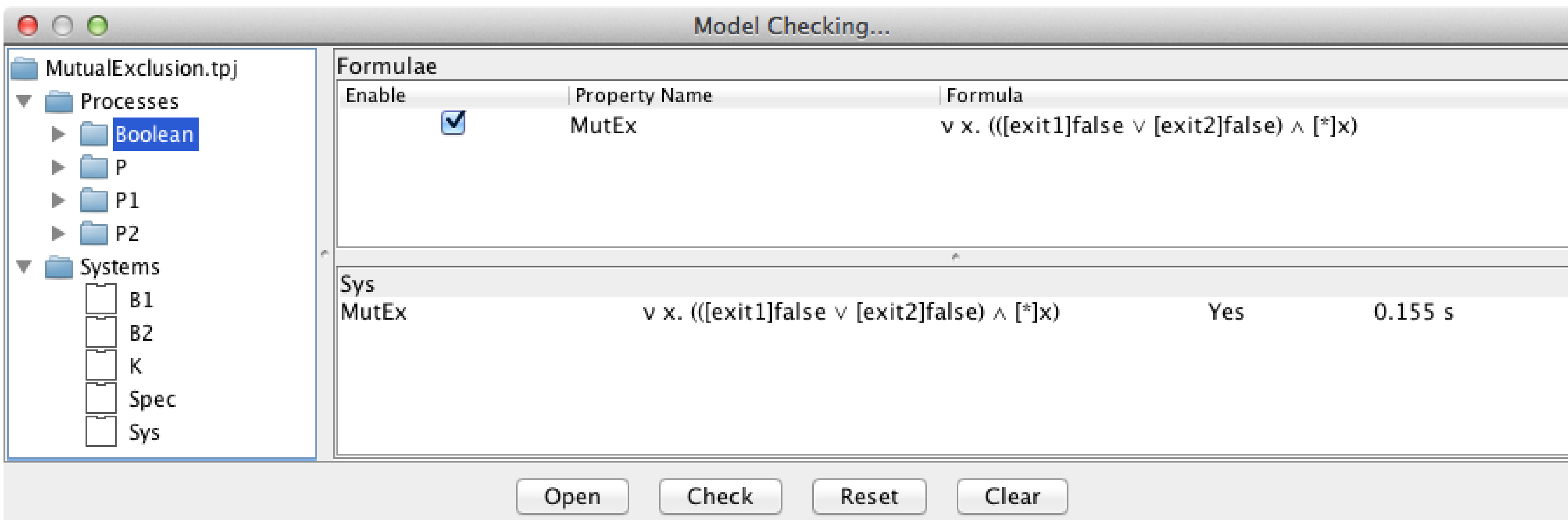

## Outline of Lecture 7

# Alternative Verification Approaches

## Verification by Bisimulation Checking

- Alternative to logic-based approaches
- **Idea:** establish equivalence between (concrete) “implementation” and (abstract) “specification”

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## Verification by Bisimulation Checking

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### Example 7.7 (Two-place buffers (cf. Example 2.5))

1. Sequential specification:

$$\begin{aligned} B_0 &= in.B_1 \\ B_1 &= \overline{out}.B_0 + in.B_2 \\ B_2 &= \overline{out}.B_1 \end{aligned}$$

2. Parallel implementation:

$$\begin{aligned} B_\| &= (B[f] \parallel B[g]) \setminus com \\ B &= in.\overline{out}.B \end{aligned}$$

   where $f := [out \mapsto com]$ and $g := [in \mapsto com]$

Later: (1) and (2) are “weakly bisimilar” (i.e., bisimilar up to $\tau$-transitions)

## Specifying Mutual Exclusion in CCS

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$$\mathit{MutExSpec} = \mathit{enter}_1.\mathit{exit}_1.\mathit{MutExSpec} + \mathit{enter}_2.\mathit{exit}_2.\mathit{MutExSpec}$$

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### Mutual exlusion in CCS

$$MutExSpec = enter_1.exit_1.MutExSpec + enter_2.exit_2.MutExSpec$$

Again: *Peterson* and *MutExSpec* are "weakly bisimilar"