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Concurrency Theory WS 2015/2016

— Series 9 —

Hand in until January 25th before the exercise class.

Exercise 1 (Branching Bisimulation) (2 + 1 + 1 Points)

A binary relation $\mathcal{R}$ over the set of states of an LTS is a *branching bisimulation* if and only if it is symmetric and, whenever $P \mathrel{\mathcal{R}} Q$ holds and $\alpha \in Act$ (including τ): if $P \xrightarrow{\alpha} P'$ then either $\alpha = \tau$ and $P' \mathrel{\mathcal{R}} Q$ or there is a $k \geq 0$ and a sequence of transitions

$$Q = Q_0 \xrightarrow{\tau} Q_1 \xrightarrow{\tau} \ldots \xrightarrow{\tau} Q_k \xrightarrow{\alpha} Q'$$

such that $P \mathrel{\mathcal{R}} Q_j$ holds for each $j \in \{0, \ldots, k\}$ and $P' \mathrel{\mathcal{R}} Q'$.

Two states P and Q are *branching bisimilar* if and only if there is a branching bisimulation $\mathcal{R}$ such that $P \mathrel{\mathcal{R}} Q$. The largest branching bisimulation is called *branching bisimilarity*.

1. Show that branching bisimilarity is contained in weak bisimilarity.
2. Prove or disprove: If P and Q are branching bisimilar then P and Q are weakly bisimilar.
3. Prove or disprove: If P and Q are weakly bisimilar then P and Q are branching bisimilar.

Exercise 2 (τ–Laws) (3 Points)

Let $\approx_{BB}$ denote the branching bisimilarty as introduced in the first exercise. Prove or disprove for $P, Q \in Prc$ and $\alpha \in Act$:

1. $\alpha.\tau.P \;\approx_{BB}\; \alpha.P$.
2. $P + \tau.P \;\approx_{BB}\; \tau.P$.
3. $\alpha.(P + \tau.Q) \;\approx_{BB}\; \alpha.(P + \tau.Q) + \alpha.Q$.

Exercise 3 (Characteristic HML Formula) (3 Points)

Consider the LTS provided below. Provide an HML formula F for each pair of states $s \neq s'$ such that $s \models F$ and $s' \not\models F$.