



# Concurrency Theory WS 2015/2016

## — Series 4 —

Hand in until November 30th before the exercise class.

### Exercise 1 (Dining Philosophers)

**(1+2+2+1 Points)**

The philosophical society employs two philosophers,  $\text{Phil}_1$  and  $\text{Phil}_2$ . Both spend their time either thinking or eating at a table with a large spaghetti bowl, one Spoon and one Fork. Each philosopher usually keeps thinking, but at any point in time, he may decide to eat. When philosopher  $\text{Phil}_1$  decides to eat, he picks up the fork, then picks up the spoon, then eats, then releases the fork and then releases the spoon. When philosopher  $\text{Phil}_2$  decides to eat, he picks up the spoon, then picks up the fork, then eats, then releases the spoon and then releases the fork.

- (a) Complete the following CCS process definition such that it describes the operation of the philosophical society! Use the set of actions names  $A = \{\text{eat}_1, \text{eat}_2, \text{pickUpFork}, \text{releaseFork}, \text{pickUpSpoon}, \text{releaseSpoon}\}$ !

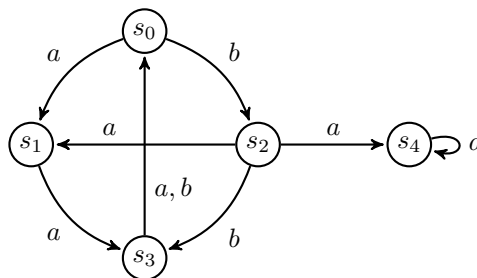
$$\begin{aligned} \text{Society} &= (\text{Phil}_1 \parallel \text{Phil}_2 \parallel \text{Spoon} \parallel \text{Fork}) \\ &\quad \setminus \{\text{pickUpFork}, \text{releaseFork}, \text{pickUpSpoon}, \text{releaseSpoon}\} \\ \text{Phil}_1 &= ? \\ \text{Phil}_2 &= ? \\ \text{Spoon} &= ? \\ \text{Fork} &= ? \end{aligned}$$

- (b) Draw the corresponding LTS and argue by observation of the LTS that the system exhibits a deadlock! You may use abstract names for the states.
- (c) Give an HML formula with one variable  $D$  which expresses the absence of a deadlock in any arbitrary LTS and argue by applying fixed-point iteration to the semantics of  $D$  with respect to the LTS from (c) that the system exhibits a deadlock!
- (d) What strong recommendation should the philosophical society give to philosopher  $\text{Phil}_2$  regarding his eating behavior such that the system no longer exhibits a deadlock? Justify your answer!

### Exercise 2 (Mutually Recursive Equation Systems)

**(2 Points)**

Consider the LTS



and the mutually recursive equation system

$$E = \begin{pmatrix} X_1 & \min & [a]X_1 \vee [b]X_2 \\ X_2 & \max & [b]X_2 \wedge [a]X_1 \end{pmatrix}.$$



Do the fixed-point iteration for  $\llbracket E \rrbracket$ !

### Exercise 3 (HML with One Variable)

**(2 Points)**

Prove that there exists no HML formula with one variable  $F$  such that for every LTS  $(S, Act, \rightarrow)$  with  $|S| \geq 2$  neither  $S$  nor  $\emptyset$  are a fixed-point of  $\llbracket F \rrbracket$ !