Static Program Analysis

Lecture 9: Dataflow Analysis VIII (Conditional Interval Analysis & Java Virtual Machine)

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http://moves.rwth-aachen.de/teaching/ws-1415/spa/

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Outline

Taking Conditional Branches into Account II

- **Solution:** introduce transfer functions for branches
- **First approach:** attach (negated) conditions as labels to control flow edges
  - advantage: no language modification required
  - disadvantage: entails extension of DFA framework
  - will not further be considered here
- **Second approach:** encode conditions as assertions (statements)
  - advantage: DFA framework can be reused
  - disadvantage: entails extension of WHILE language
  - the way we will follow

Extending the Syntax of WHILE Programs

Definition (Labelled WHILE programs with assertions)

The syntax of labelled WHILE programs with assertions is defined by the following context-free grammar:

$$
\begin{array}{rcl}
a & ::= & z \mid x \mid a_1+a_2 \mid a_1-a_2 \mid a_1*a_2 \in AExp \\
b & ::= & t \mid a_1=a_2 \mid a_1>a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in BExp \\
c & ::= & [\texttt{skip}]^l \mid [x \texttt{ := } a]^l \mid c_1 ; c_2 \mid \\
 & & \texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \mid \texttt{while } [b]^l \texttt{ do } c \mid [\texttt{assert } b]^l \in Cmd
\end{array}
$$

To be done:

- Definition of transfer functions for `assert` blocks (depending on analysis problem)
- Idea: assertions as filters that let only “valid” information pass

Outline

Interval Analysis with Assertions I

So far:

- The domain $(Int, \subseteq)$ of intervals over $\mathbb{Z}$ is defined by
  $$Int := \{[z_1, z_2] \mid z_1 \in \mathbb{Z} \cup \{-\infty\}, z_2 \in \mathbb{Z} \cup \{+\infty\}, z_1 \leq z_2\} \cup \{\emptyset\}$$
  where
  - $-\infty \leq z$, $z \leq +\infty$, and $-\infty \leq +\infty$ (for all $z \in \mathbb{Z}$)
  - $\emptyset \subseteq J$ (for all $J \in Int$)
  - $[y_1, y_2] \subseteq [z_1, z_2]$ iff $y_1 \geq z_1$ and $y_2 \leq z_2$
- Transfer functions $\{\varphi_l \mid l \in Lab\}$ are defined by
  $$\varphi_l(\delta) := \begin{cases} \delta & \text{if } B^l = \texttt{skip} \text{ or } B^l \in BExp \\ \delta[x \mapsto val_\delta(a)] & \text{if } B^l = (x \texttt{ := } a) \end{cases}$$
  where
  $$\begin{aligned} val_\delta(x) &:= \delta(x) \\ val_\delta(z) &:= [z, z] \end{aligned} \qquad \begin{aligned} val_\delta(a_1\texttt{+}a_2) &:= val_\delta(a_1) \oplus val_\delta(a_2) \\ val_\delta(a_1\texttt{-}a_2) &:= val_\delta(a_1) \ominus val_\delta(a_2) \\ val_\delta(a_1\texttt{*}a_2) &:= val_\delta(a_1) \odot val_\delta(a_2) \end{aligned}$$
  with
  $$\emptyset \oplus J := J \oplus \emptyset := \emptyset \ominus J := \ldots := \emptyset$$
  $$\begin{aligned} [y_1, y_2] \oplus [z_1, z_2] &:= [y_1 + z_1, y_2 + z_2] \\ [y_1, y_2] \ominus [z_1, z_2] &:= [y_1 - z_2, y_2 - z_1] \\ [y_1, y_2] \odot [z_1, z_2] &:= [\textstyle\bigsqcap\{y_1z_1, y_1z_2, y_2z_1, y_2z_2\}, \bigsqcup\{y_1z_1, y_1z_2, y_2z_1, y_2z_2\}] \end{aligned}$$

Interval Analysis with Assertions II

Additionally for $B^l = (\texttt{assert}\ b)$, $\delta : Var_c \to Int$ and $x \in Var_c$:

$$\varphi_l(\delta)(x) := \begin{cases} \emptyset & \text{if } Z = \emptyset \\ \left[\bigsqcap_{\mathbb{Z}\cup\{-\infty\}} Z, \bigsqcup_{\mathbb{Z}\cup\{+\infty\}} Z\right] & \text{otherwise} \end{cases}$$

where

- $Z := \{\sigma(x) \mid \sigma \in \Sigma_\delta, val_\sigma(b) = \mathrm{true}\}$
- $\Sigma_\delta := \{\sigma : Var_c \to \mathbb{Z} \mid \forall y \in Var_c : \sigma(y) \in \delta(y)\}$
  (and thus $\Sigma_\delta = \emptyset$ iff $\delta(y) = \emptyset$ for some $y \in Var_c$)
- $val_\sigma : BExp \to \mathbb{B}$ as before

Interval Analysis with Assertions III

Example 9.1

$$Var_c = \{\mathtt{x}, \mathtt{y}\},\ \delta = (\underbrace{[-\infty, 2]}_{\mathtt{x}}, \underbrace{[0, +\infty]}_{\mathtt{y}})$$

$$\begin{aligned} \Longrightarrow\ & \varphi_{\mathtt{assert\ x>0}}(\delta) = ([1, 2], [0, +\infty]) \\ & \varphi_{\mathtt{assert\ x=y}}(\delta) = ([0, 2], [0, 2]) \\ & \varphi_{\mathtt{assert\ x>y}}(\delta) = ([1, 2], [0, 1]) \\ & \varphi_{\mathtt{assert\ x<y}}(\delta) = ([-\infty, 2], [0, +\infty]) \end{aligned}$$

Interval Analysis with Assertions III

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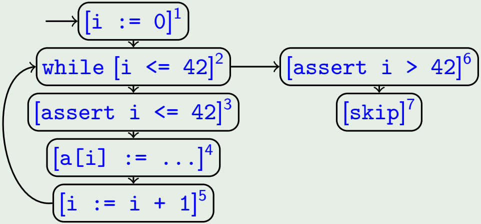
& \varphi_{\texttt{assert x<y}}(\delta) = ([-\infty, 2], [0, +\infty])
\end{aligned}$$

Remarks:

- Again for $B^l = (\texttt{assert}\ b)$ and $\delta : Var_c \to Int$, $\varphi_l(\delta) \sqsubseteq \delta$ and hence $\Sigma_{\varphi_l(\delta)} \subseteq \Sigma_\delta$ ("filter")
- Again if $\mathrm{AI}_l(x) = \emptyset$ for some $l \in Lab_c$ and $x \in Var_c$, then l is unreachable (and $\mathrm{AI}_l(y) = \emptyset$ for all $y \in Var_c$)

Example 9.2 (Interval analysis for array index; cf. Example 7.6)



Interval Analysis with Assertions IV

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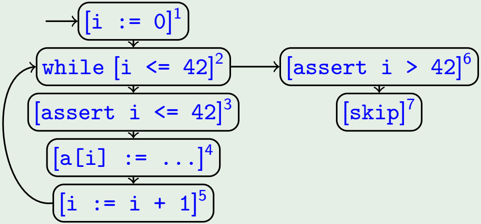

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\varphi_1(J) &= [0, 0] \\
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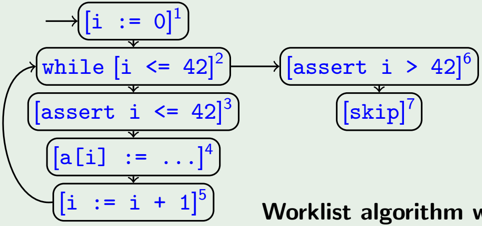

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Worklist algorithm with widening:

W	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$12, 23, 34, 45, 52, 26, 67$	$[-\infty, +\infty]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

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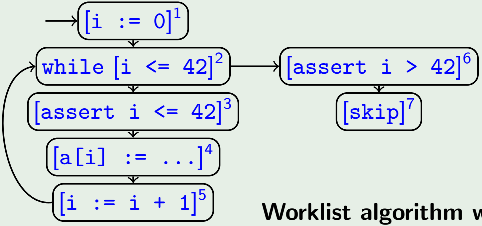

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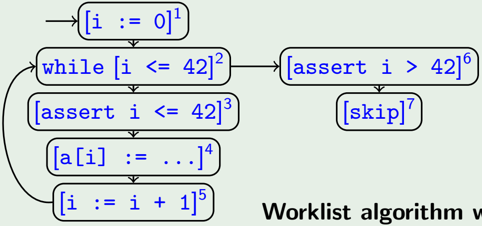

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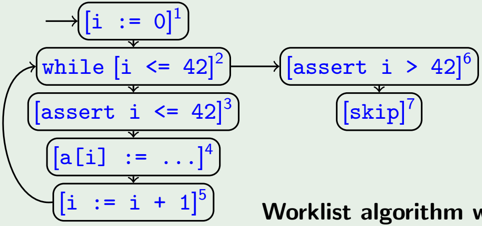

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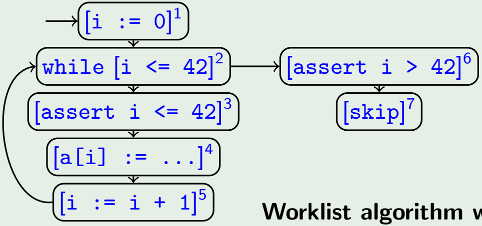

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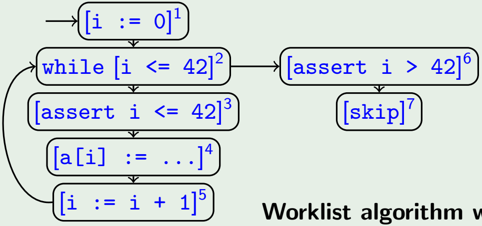

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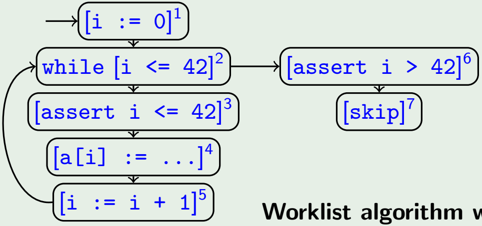

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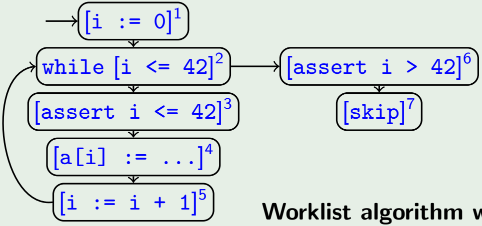

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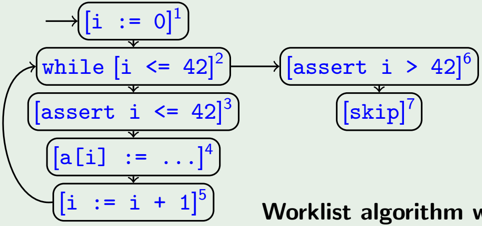
Interval Analysis with Assertions IV

Example 9.2 (Interval analysis for array index; cf. Example 7.6)



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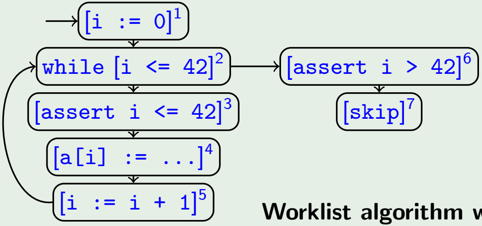

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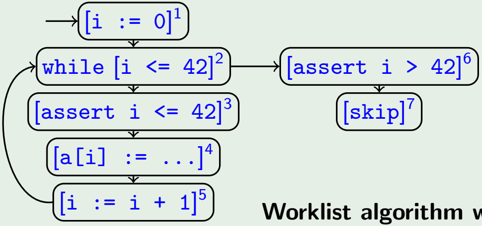
$52, 26, 67$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$\emptyset$	$\emptyset$
$26, 67$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$\emptyset$	$\emptyset$

Example 9.2 (Interval analysis for array index; cf. Example 7.6)



$$\begin{aligned}
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\end{aligned}$$

Worklist algorithm with widening:

W	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
12, 23, 34, 45, 52, 26, 67	$[-\infty,+\infty]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
23, 34, 45, 52, 26, 67	$[-\infty,+\infty]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
34, 45, 52, 26, 67	$[-\infty,+\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
45, 52, 26, 67	$[-\infty,+\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$
52, 26, 67	$[-\infty,+\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
23, 26, 67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
34, 26, 67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
45, 26, 67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,0]$	$\emptyset$	$\emptyset$
52, 26, 67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$\emptyset$	$\emptyset$
26, 67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$\emptyset$	$\emptyset$
67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$\emptyset$

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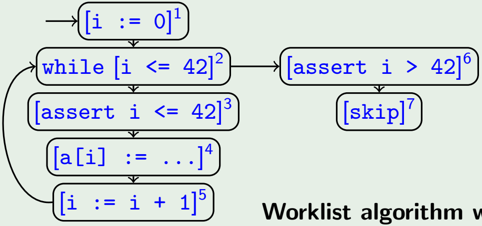

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34, 45, 52, 26, 67	$[-\infty,+\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
45, 52, 26, 67	$[-\infty,+\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$
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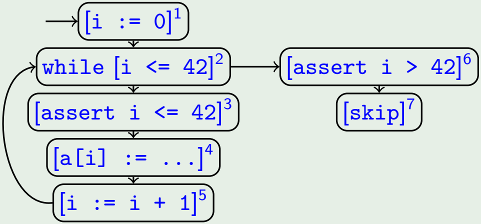
67	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$\emptyset$
ε	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$

Interval Analysis with Assertions V

Example 9.2 (Interval analysis for array index; continued)



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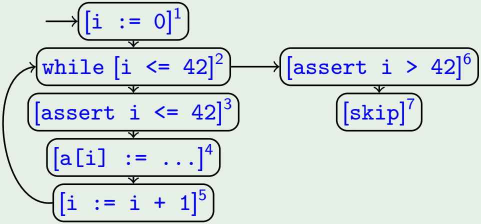

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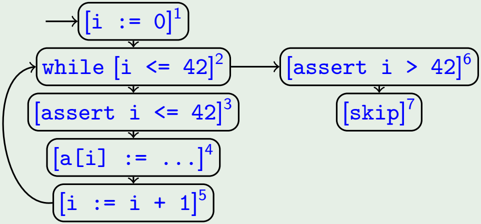

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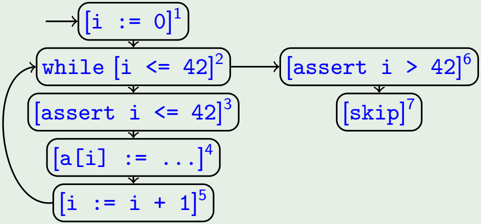
$$

Narrowing:

	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$\mathrm{fix}^{\nabla}(\Phi_S)$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[43, +\infty]$

Example 9.2 (Interval analysis for array index; continued)



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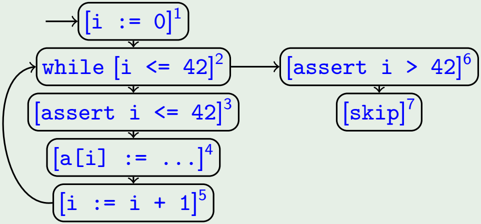
$\mathrm{fix}^{\nabla}(\Phi_S)$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[43, +\infty]$
$\Phi_S(\mathrm{fix}^{\nabla}(\Phi_S))$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, 42]$	$[0, +\infty]$	$[0, +\infty]$	$[43, +\infty]$

Example 9.2 (Interval analysis for array index; continued)



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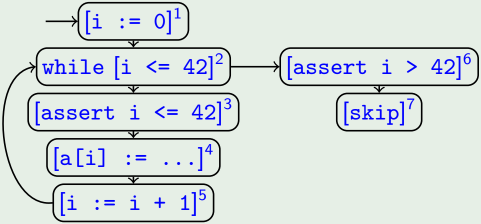
$\Phi_S(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^2(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$

Example 9.2 (Interval analysis for array index; continued)



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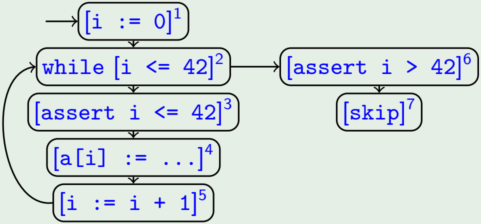
$\text{fix}^{\nabla}(\Phi_S)$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^2(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^3(\text{fix}^{\nabla}(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$

Example 9.2 (Interval analysis for array index; continued)



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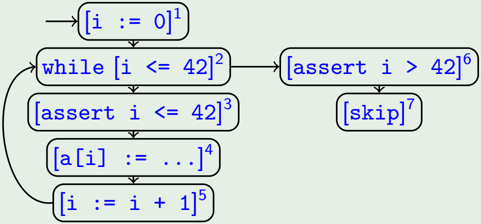
	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$\mathrm{fix}^\nabla(\Phi_S)$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^2(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^3(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^4(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,+\infty]$

Interval Analysis with Assertions V

Example 9.2 (Interval analysis for array index; continued)



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	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$\mathrm{fix}^\nabla(\Phi_S)$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[43,+\infty]$
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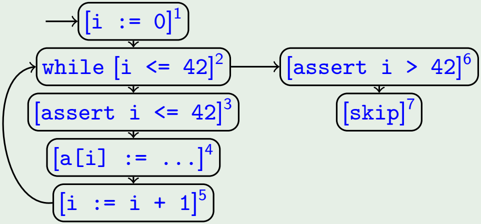
$\Phi_S^3(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,+\infty]$	$[0,42]$	$[0,42]$	$[0,+\infty]$	$[43,+\infty]$
$\Phi_S^4(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,+\infty]$
$\Phi_S^5(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,43]$

Interval Analysis with Assertions V

Example 9.2 (Interval analysis for array index; continued)



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$\Phi_S^4(\mathsf{fix}^\nabla(\Phi_S))$	$[-\infty,+\infty]$	$[0,43]$	$[0,43]$	$[0,42]$	$[0,42]$	$[0,43]$	$[43,+\infty]$
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Outline

Java Bytecode

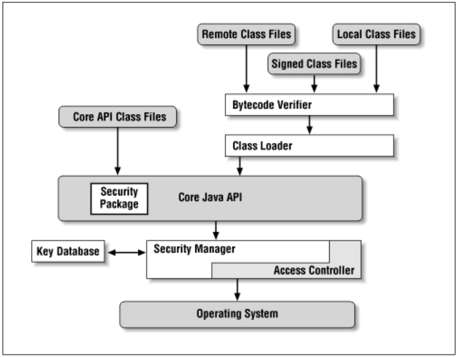
- Intermediate language between high-level language and machine code
- Execution on Java Virtual Machine (JVM)

Java Bytecode

- Intermediate language between high-level language and machine code
- Execution on Java Virtual Machine (JVM)
- **Advantages:**
  - architecture independency (especially for web applications)
  - faster than pure (i.e., source code) interpretation
- **Problem:** security issues
  - destruction of data
  - modification of data
  - disclosure of personal information
  - modification of other programs

Java Security: the Sandbox

- Insulation layer providing indirect access to system resources
- Hardware access via API classes and methods
- Bytecode verification upon uploading
  - well-typedness
  - proper object referencing
  - proper control flow



The Java Virtual Machine

- Conventional stack-based abstract machine
- Supports object-oriented features: classes, methods, etc.
- Stack for intermediate results of expression evaluations
- Registers for source-level local variables and method parameters
- Both part of method activation record
  (and thus preserved across method calls)
- Method entry point specifies required number of registers (m_r) and stack slots (m_s; for memory allocation)
- (Most) instructions are typed

Example: Factorial Function

Example 9.3 (Factorial function)

Java source code:

```
static int factorial(int n)
{ int res;
  for (res = 1; n > 0; n--) res = res * n;
  return res; }
```

Example: Factorial Function

Example 9.3 (Factorial function)

Java source code:

```
static int factorial(int n)
{ int res;
  for (res = 1; n > 0; n--) res = res * n;
  return res; }
```

Corresponding JVM bytecode:

```
method static int factorial(int), 2 registers, 2 stack slots
   1: istore 0     // store n in register 0
   2: iconst_1     // push constant 1
   3: istore 1     // store res in register 1
   4: iload 0      // push n
   5: ifle 12      // if <= 0, go to end
   6: iload 1      // push res
   7: iload 0      // push n
   8: imul         // res * n on top of stack
   9: istore 1     // store in res
  10: iinc 0, -1   // decrement n
  11: goto 4       // go to loop header
  12: iload 1      // push res
  13: ireturn      // return res to caller
```

JVM Instruction Set (excerpt)

`iload` n: push integer from register n

`istore` n: pop integer into register n

`iconst_`z: push integer z

`aconst_null`: push null reference

`iadd`: add two topmost integers on stack and push sum

`getfield` C f τ: pops reference to object (of class C) and pushes value of field f (of type τ)

`putfield` C f τ: pops value v (of type τ) and reference to object o (of class C) and assigns v to field f of o

`new` C: creates new object (of class C) and pushes reference

`invoke` C M $\tau_0(\tau_1, \ldots, \tau_n)$: pops values $v_1, \ldots, v_n$ (of type $\tau_1, \ldots, \tau_n$) and reference to object (of class C), calls method M with parameters $v_1, \ldots, v_n$, and pushes return value (of type τ_0)

`if_icmpeq` l: pop two topmost integers from stack and jump to line l if equal

`ireturn`: return to caller with integer result on top of stack

($\approx$ 200 instructions in total)

Malicious Bytecode

Example 9.4 (Malicious bytecode)

```
1: iconst_5
2: iconst_1
3: putfield A f int
```

interprets second stack entry (5) as reference to object of class `A` and assigns first stack entry (1) to field `f` of this object

Outline

Correctness of Bytecode

Conditions to ensure proper operation:

Type correctness: arguments of instructions always of expected type

No stack over-/underflow: never push to full stack or pop from empty stack

Code containment: PC must always point into the method code

Register initialization: load from non-parameter register only after store

Object initialization: constructor must be invoked before using class instance

Access control: operations must respect visibility modifiers (`private`/`protected`/`public`)

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Access control: operations must respect visibility modifiers (`private`/`protected`/`public`)

Options:

- dynamic checking at execution time ("defensive JVM approach")
  - expensive, slows down execution
- static checking at loading time (here)
  - verified code executable at full speed without extra dynamic checks

The Java Bytecode Verifier

Summary: dataflow analysis applied to type-level abstract interpretation of JVM

1. Association of type information with register and stack contents
   - set of types forms a complete lattice
2. Simulation of execution of instructions at type level
3. Use dataflow analysis to cover all concrete executions
4. Modularity: analysis proceeds method per method

(see X. Leroy: *Java Bytecode Verification: Algorithms and Formalizations*, Journal of Automated Reasoning 30(3-4), 2003, 235–269)

Outline

Types

The set of types, *Typ*, is composed of

- Primitive types:
  - *int* (covering *boolean*, *byte*, *char*, *short*)
  - *long*
  - *float*
  - *double*

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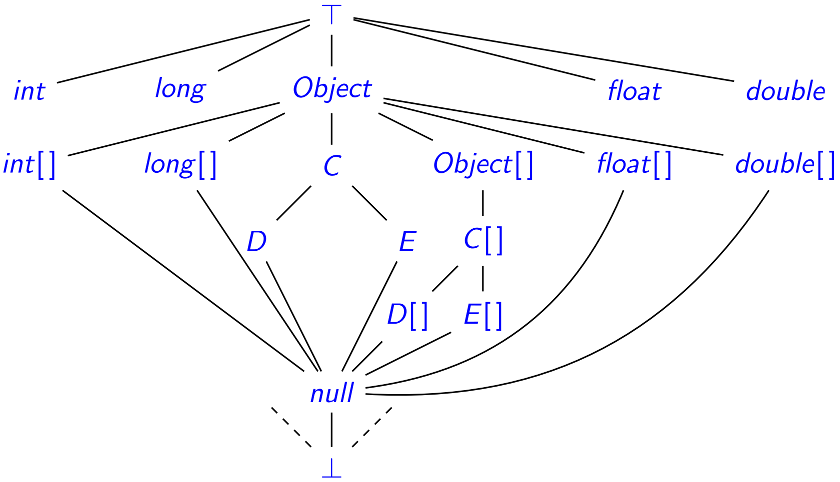
- Special types:
  - *null* (null reference)
  - *Object* (any object)
  - $\top$ (contents of uninitialized registers, i.e., any value)
  - $\bot$ (absence of any value)

The Subtyping Relation (excerpt)

(C, D, E user-defined classes; D, E extending C)



Notation: $\tau_1 \sqsubseteq_t \tau_2$

The Type-Level Abstract Interpreter I

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- Errors (type mismatch, stack over-/underflow, ...) denoted by absence of transition

The Type-Level Abstract Interpreter II

Some transition rules:

`iconst_z`: $(S, R) \rightarrow (int.S, R)$ if $|S| < m_s$

`aconst_null`: $(S, R) \rightarrow (null.S, R)$ if $|S| < m_s$

`iadd`: $(int.int.S, R) \rightarrow (int.S, R)$

`if_icmpeq` l: $(int.int.S, R) \rightarrow (S, R)$

`iload` n: $(S, R) \rightarrow (int.S, R)$
if $0 \leq n < m_r, R(n) = int, |S| < m_s$

`aload` n: $(S, R) \rightarrow (R(n).S, R)$
if $0 \leq n < m_r, R(n) \sqsubseteq_t Object, |S| < m_s$

`istore` n: $(int.S, R) \rightarrow (S, R[n \mapsto int])$ if $0 \leq n < m_r$

`astore` n: $(\tau.S, R) \rightarrow (S, R[n \mapsto \tau])$
if $0 \leq n < m_r, \tau \sqsubseteq_t Object$

`getfield` C f τ: $(D.S, R) \rightarrow (\tau.S, R)$ if $D \sqsubseteq_t C$

`putfield` C f τ: $(\tau'.D.S, R) \rightarrow (S, R)$
if $\tau' \sqsubseteq_t \tau, D \sqsubseteq_t C$

`invoke` C M σ: $(\tau'_n \ldots \tau'_1.\tau'.S, R) \rightarrow (\tau_0.S, R)$
if $\sigma = \tau_0(\tau_1, \ldots, \tau_n), \tau'_i \sqsubseteq_t \tau_i$ for $1 \leq i \leq n, \tau' \sqsubseteq_t C$

Some Theoretical Properties

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3. *(Soundness) If* $i : (S, R) \to (S', R')$, *then for all concrete states* (s, r) *matching* (S, R), *the defensive JVM will not stop with a run-time type exception when applying* i *to* (s, r) *(but rather change to some* (s', r') *matching* (S', R')*).*

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Proof.

see X. Leroy: *Java Bytecode Verification: Algorithms and Formalizations* □