Static Program Analysis

Lecture 21: Shape Analysis & Final Remarks

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH AACHEN UNIVERSITY

noll@cs.rwth-aachen.de

http://moves.rwth-aachen.de/teaching/ws-1415/spa/

Winter Semester 2014/15

Outline

The Shape Analysis Approach

- **Goal:** determine the possible shapes of a dynamically allocated data structure at given program point
- **Interesting information:**
  - data types (to avoid type errors, such as dereferencing `nil`)
  - aliasing (different pointer variables having same value)
  - sharing (different heap pointers referencing same location)
  - reachability of nodes (garbage collection)
  - disjointness of heap regions (parallelizability)
  - shapes (lists, trees, absence of cycles, ...)
- **Concrete questions:**
  - Does `x.next` point to a shared element?
  - Does a variable `p` point to an allocated element every time `p` is dereferenced?
  - Does a variable point to an acyclic list?
  - Does a variable point to a doubly-linked list?
  - Can a loop or procedure cause a memory leak?
- **Here:** basic outline; details in [Nielson/Nielson/Hankin 2005, Sct. 2.6]

Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Arithmetic expressions	*AExp*	*a*
Boolean expressions	*BExp*	*b*
Selector names	*Sel*	*sel*
Pointer expressions	*PExp*	*p*
Commands (statements)	*Cmd*	*c*

Context-free grammar:

$$a ::= z \mid x \mid a_1 + a_2 \mid \ldots \mid p \mid \texttt{nil} \in AExp$$
$$b ::= t \mid a_1 = a_2 \mid b_1 \wedge b_2 \mid \ldots \mid \texttt{is-nil}(p) \in BExp$$
$$p ::= x \mid x\,.\,sel$$
$$c ::= [\texttt{skip}]^l \mid [p \texttt{ := } a]^l \mid c_1 ; c_2 \mid \texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \mid$$
$$\texttt{while } [b]^l \texttt{ do } c \mid [\texttt{malloc } p]^l \in Cmd$$

Shape Graphs I

Approach: representation of (infinitely many) concrete heap states by (finitely many) abstract shape graphs

- abstract nodes X = sets of variables
- interpretation: $x \in X$ iff x points to concrete node represented by X
- $\emptyset$ represents all concrete nodes that are not directly addressed by pointer variables
- $x, y \in X$ (with $x \neq y$) indicate aliasing (as x and y point to the same concrete node)
- if $x.sel$ and y refer to the same heap address and if X, Y are abstract nodes with $x \in X$ and $y \in Y$, this yields abstract edge $X \xrightarrow{sel} Y$
- transfer functions transform (sets of) shape graphs

Shape Graphs II

Definition (Shape graph)

A shape graph $G = (S, H)$ consists of

- a set $S \subseteq 2^{Var}$ of abstract locations and
- an abstract heap $H \subseteq S \times Sel \times S$
  - notation: $X \xrightarrow{sel} Y$ for $(X, sel, Y) \in H$

with the following properties:

Disjointness: $X, Y \in S \implies X = Y$ or $X \cap Y = \emptyset$
(a variable can refer to at most one heap location)

Determinacy: $X \neq \emptyset$ and $X \xrightarrow{sel} Y$ and $X \xrightarrow{sel} Z \implies Y = Z$
(target location is unique if source node is unique)

SG denotes the set of all shape graphs.

Remark: the following example shows that determinacy requires $X \neq \emptyset$:

Concrete: $y \longrightarrow \bullet \xleftarrow{sel} \bullet$
$z \longrightarrow \bullet \xleftarrow{sel} \bullet$

Abstract: $\boxed{Y = \{y\}} \xleftarrow{sel} \boxed{X = \emptyset} \xrightarrow{sel} \boxed{Z = \{z\}}$

Shape Graphs and Concrete Heap Properties

Example

Let $G = (S, H)$ be a shape graph. Then the following concrete heap properties can be expressed as conditions on G:

- $\texttt{x} \neq \texttt{nil}$
  $\iff \exists X \in S : \texttt{x} \in X$
- $\texttt{x} = \texttt{y} \neq \texttt{nil}$ (aliasing)
  $\iff \exists Z \in S : \texttt{x}, \texttt{y} \in Z$
- $\texttt{x.sel1} = \texttt{y.sel2} \neq \texttt{nil}$ (sharing)
  $\implies \exists X, Y, Z \in S : \texttt{x} \in X, \texttt{y} \in Y, X \stackrel{\texttt{sel1}}{\longrightarrow} Z \stackrel{\texttt{sel2}}{\longleftarrow} Y$
  (“$\Longleftarrow$” only valid if $Z \neq \emptyset$)

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  - Var, Sel finite $\Longrightarrow$ SG finite $\Longrightarrow$ 2^{SG} finite $\Longrightarrow$ ACC

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Example 21.1 (List reversal; cf. Example 20.4)

- Variables: $\mathit{Var} = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$
- Assumption: $\mathtt{x}$ points to any (finite, non-cyclic) list, $\mathtt{y} = \mathtt{z} = \mathtt{nil}$

$$\Longrightarrow \iota = \left\{ \underbrace{(\emptyset, \emptyset)}_{\text{empty}} \;,\; \underbrace{\boxed{\{\mathtt{x}\}}}_{\text{1 elem.}} \;,\; \underbrace{\boxed{\{\mathtt{x}\}} \xrightarrow{\mathtt{next}} \boxed{\emptyset}}_{\text{2 elem.}} \;,\; \underbrace{\boxed{\{\mathtt{x}\}} \xrightarrow{\mathtt{next}} \boxed{\emptyset}\overset{\mathtt{next}}{\curvearrowright}}_{\geq \text{3 elem.}} \right\}$$

The Transfer Functions

Transfer functions: $\varphi_l : 2^{SG} \to 2^{SG}$ (monotonic)

- Transform each single shape graph into a set of shape graphs:
  $\varphi_l(\{G_1, \ldots, G_n\}) = \bigcup_{i=1}^{n} \varphi_l(G_i)$

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  - $[\texttt{skip}]^l$: $\varphi_l(G) := \{G\}$
  - $[b]^l$: $\varphi_l(G) := \{G\}$
  - $[p \texttt{ := } a]^l$: case-by-case analysis w.r.t. p and a
    - [Nielson/Nielson/Hankin 2005, Sct. 2.6.3]: 12 cases
    - may involve (high degree of) non-determinism
    - see example on following slide
  - $[\texttt{malloc } x]^l$: $\varphi_l(G) := \{(S' \cup \{\{x\}\}, H')\}$ where
    - $S' := \{X \setminus \{x\} \mid X \in S\}$
    - $H' := H \cap S' \times Sel \times S'$
  - $[\texttt{malloc } x.sel]^l$: equivalent to $[\texttt{malloc } t]^{l_1}; [x.sel \texttt{ := } t]^{l_2}; [t \texttt{ := nil}]^{l_3};$
    (with fresh $t \in Var$ and $l_1, l_2, l_3 \in Lab$)

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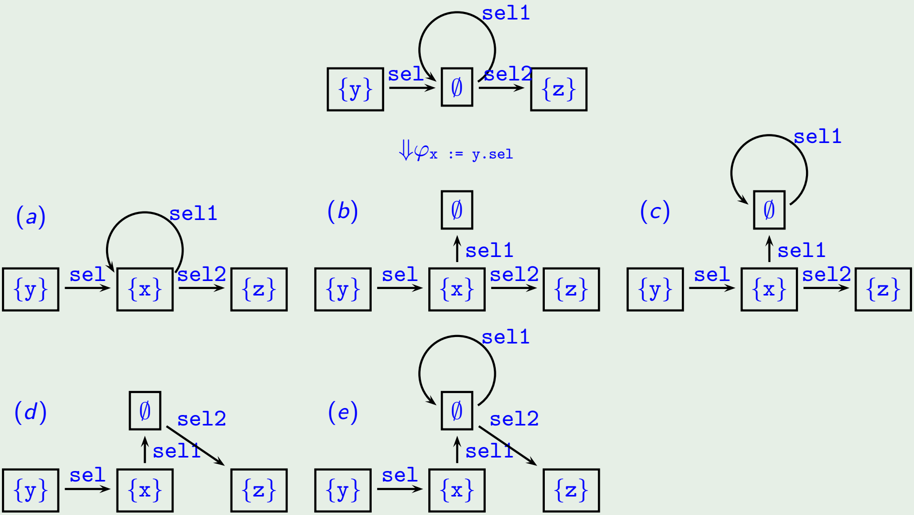
- Crucial for soundness: safety of approximation
  If shape graph G approximates heap h and $h \xrightarrow{B^l} h'$,
  then there exists $G' \in \varphi_l(G)$ such that G' approximates h'

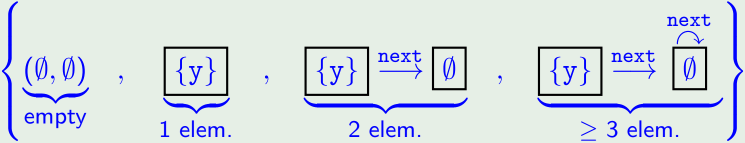
An Example

Example 21.2



Example 21.3 (List reversal; cf. Example 20.4)

Shape analysis of list reversal program yields final result



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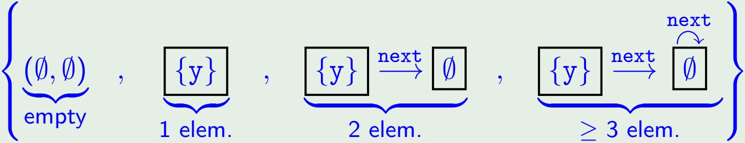

Interpretation:

\+ Result again a finite list

– but potentially cyclic (may be a "lasso", but not a ring)

– also "reversal" property not guaranteed

Outline

Dedicated Algorithms for Pointer Analysis

- `nil` Pointer Analysis: checks whether dereferencing operations possibly involve `nil` pointers
  - with shape analysis: possible for $x \in Var$ if there exists (reachable) $G = (S, H)$ such that $x \notin \bigcup_{X \in S} X$
- Points-To Analysis: yields function pt that for each $x \in Var$ returns set $pt(x)$ of possible pointer targets
  - x and y may be aliases if $pt(x) \cap pt(y) \neq \emptyset$
  - with shape analysis: there exists (reachable) $G = (S, H)$ and $Z \in S$ such that $x, y \in Z$
- Usually faster and sometimes more precise than shape analysis, but less general (only "shallow" properties)
- Fastest algorithms are flow-insensitive (points-to edges only added but never removed)

Graph Grammar Approaches to Pointer Analysis

- e.g., J. Heinen, C. Jansen, J.-P. Katoen, T. Noll: *Verifying Pointer Programs using Graph Grammars*. Science of Computer Programming 97, 157–162, 2015
- idea: specify data structures by graph production rules
- concretization by forward application
- abstraction by backward application
- all pointer operations remain concrete

  $\Longrightarrow$ avoids complicated definition of transfer functions

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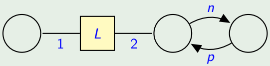

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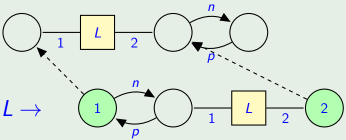

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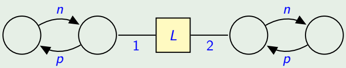

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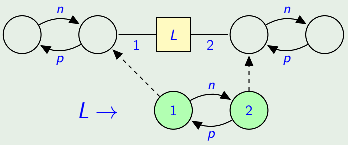

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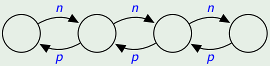

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- **Example:** correctness of Constant Propagation

  Let $c \in Cmd$ with $l_0 = \text{init}(c)$, and let $l \in Lab_c$, $x \in Var$, and $z \in \mathbb{Z}$ such that $\text{CP}_l(x) = z$. Then for all $\sigma_0, \sigma \in \Sigma$ such that $\langle l_0, \sigma_0 \rangle \to^* \langle l, \sigma \rangle$, $\sigma(x) = z$.

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- see [Nielson/Nielson/Hankin 2005, Sct. 2.2]

Outline

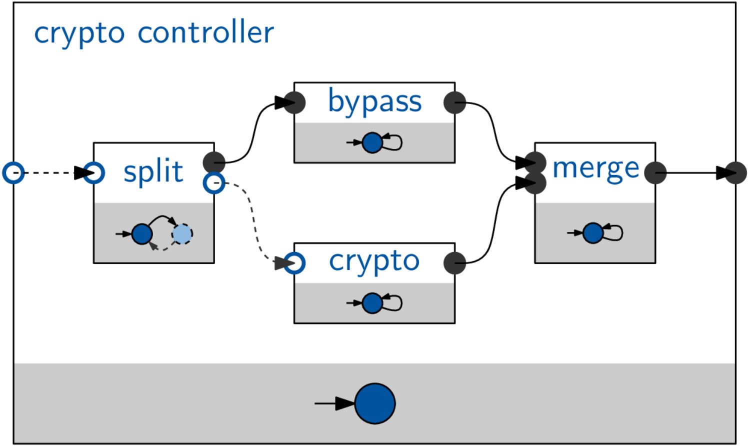
Oral Exams

- Schedule online
  - 12 + 24 March, 8 April
  - see http://moves.rwth-aachen.de/teaching/ws-1415/spa/
- Q&A session on Tuesday, 24 February, 14:00–15:30, AH 6
  - please submit questions beforehand to dehnert@cs.rwth-aachen.de or benjamin.kaminski@cs.rwth-aachen.de
  - contact me in case of unresolved/later questions

Thesis: Analysing Information Flows Using Slicing

- Computer security: system architectures that disallow sensitive information to be "leaked" to unauthorised entities
- Critical: covert channels that expose information
- Requires analysis of information flows within and between architectural components
- Standard approaches (non-interference, slicing) ignore encryption
- Goal: analysis of cryptographically-masked information flows using slicing techniques

Crypto controller



Forthcoming Courses in SS 2015

Introduction to Model Checking [Katoen; V3 Ü2]

1. Labelled transition systems
2. Classification of properties: safety, liveness, fairness
3. Temporal logics LTL and CTL
4. Model checking algorithms
5. Abstraction using (bi-)simulation

Semantics and Verification of Software [Noll; V3 Ü2]

1. The imperative model language WHILE
2. Operational, denotational and axiomatic semantics of WHILE
3. Equivalence of the semantics
4. Applications: compiler correctness, ...
5. Extensions: procedures, non-determinism, concurrency