

Static Program Analysis

Lecture 20: Wrap-Up Interprocedural DFA & Pointer Analysis

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)



noll@cs.rwth-aachen.de

<http://moves.rwth-aachen.de/teaching/ws-1415/spa/>

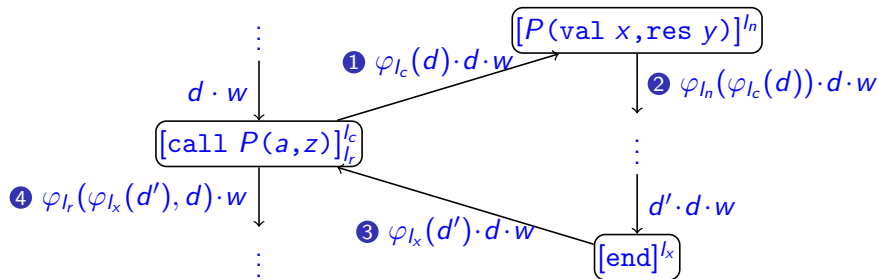
Winter Semester 2014/15

- 1 Recap: Interprocedural Dataflow Analysis – Fixpoint Solution
- 2 Soundness and Completeness
- 3 Context-Sensitive Interprocedural Dataflow Analysis
- 4 Pointer Analysis
- 5 Introducing Pointers
- 6 Shape Graphs

The Interprocedural Extension II

Visualization of

- ① $\hat{\varphi}_{l_c}(d \cdot w) = \varphi_{l_c}(d) \cdot d \cdot w$
- ② $\hat{\varphi}_{l_n}(d' \cdot d \cdot w) = \varphi_{l_n}(d') \cdot d \cdot w$
- ③ $\hat{\varphi}_{l_x}(d' \cdot d \cdot w) = \varphi_{l_x}(d') \cdot d \cdot w$
- ④ $\hat{\varphi}_{l_r}(d' \cdot d \cdot w) = \varphi_{l_r}(d', d) \cdot w$



Formal Definition of Equation System

Dataflow equations:

$$A_l = \begin{cases} \perp & \text{if } l \in E \\ \bigsqcup \{ \hat{\varphi}_{l_c}(A_{l_c}) \mid (l_c, l_n, l_x, l_r) \in \text{iflow} \} & \text{if } l = l_n \\ \bigsqcup \{ f_{l'}(A_{l'}) \mid (l', l) \in F \} & \text{for some } (l_c, l_n, l_x, l_r) \in \text{iflow} \\ & \text{otherwise} \end{cases}$$

(if l not a return label)

Node transfer functions:

$$f_l(w) = \begin{cases} \hat{\varphi}_{l_r}(\hat{\varphi}_{l_x}(F_{l_x}(\hat{\varphi}_{l_c}(w)))) & \text{if } l = l_c \text{ for some } (l_c, l_n, l_x, l_r) \in \text{iflow} \\ \hat{\varphi}_l(w) & \text{otherwise} \end{cases}$$

(if l not an exit or return label)

Procedure transfer functions:

$$F_l(w) = \begin{cases} w & \text{if } l = l_n \\ \bigsqcup \{ f_{l'}(F_{l'}(w)) \mid (l', l) \in F \} & \text{for some } (l_c, l_n, l_x, l_r) \in \text{iflow} \\ & \text{otherwise} \end{cases}$$

(if l occurs in some procedure)

As before: induces monotonic functional on lattice with ACC

\implies least fixpoint effectively computable

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The Fixpoint Iteration

For the fixpoint iteration it is important that the auxiliary functions only operate (at most) on the two topmost elements of the stack:

Lemma 20.1

For every $l \in Lab$, $d \in D$, and $w \in D^*$,

$$f_l(d' \cdot d \cdot w) = f_l(d' \cdot d) \cdot w \text{ and } F_l(d' \cdot d \cdot w) = F_l(d' \cdot d)w$$

Proof.

see J. Knoop, B. Steffen: *The Interprocedural Coincidence Theorem*, Proc. CC '92, LNCS 641, Springer, 1992, 125–140 □

It therefore suffices to consider stacks with **at most two entries**, and so the fixpoint iteration ranges over “finitary objects”.

Soundness and Completeness

The following results carry over from the intraprocedural case:

Theorem 20.2

Let $\hat{S} := (Lab, E, F, (\hat{D}, \hat{\subseteq}), \hat{i}, \hat{\phi})$ be an interprocedural dataflow system.

① (cf. Theorem 6.3)

$$\text{mvp}(\hat{S}) \hat{\subseteq} \text{fix}(\Phi_{\hat{S}})$$

② (cf. Theorem 7.3)

$$\text{mvp}(\hat{S}) = \text{fix}(\Phi_{\hat{S}}) \text{ if all } \hat{\phi}_l \text{ are distributive}$$

Proof.

see J. Knoop, B. Steffen: *The Interprocedural Coincidence Theorem*, Proc. CC '92, LNCS 641, Springer, 1992, 125–140 □

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Context-Sensitive Interprocedural DFA

- **Observation:** MVP and fixpoint solution maintain **proper relationship between procedure calls and returns**
- **But:** do not distinguish between **different procedure calls**

$$AI_l = \begin{cases} \sqcup^{\iota} \{ \hat{\phi}_{l_c}(AI_{l_c}) \mid (l_c, l_n, l_x, l_r) \in \text{iflow} \} & \text{if } l \in E \\ \sqcup \{ f_{l'}(AI_{l'}) \mid (l', l) \in F \} & \text{if } l = l_n \text{ for some} \\ & (l_c, l_n, l_x, l_r) \in \text{iflow} \\ & \text{otherwise} \end{cases}$$

- information about calling states **combined for all call sites**
- procedure body only **analyzed once** using combined information
- resulting information used at **all return points**

⇒ **“context-insensitive”**

- **Alternative:** **context-sensitive** analysis
 - **separate information** for different call sites
 - implementation by **“procedure cloning”** (one copy for each call site)
 - more **precise**
 - more **costly**

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- **So far:** only **static data structures** (variables)
- **Now:** **pointer (variables)** and **dynamic memory allocation** using heaps
- **Problem:**
 - Programs with pointers and dynamically allocated data structures are error prone
 - Identify subtle bugs at compile time
 - Automatically prove correctness
- **Interesting properties of heap-manipulating programs:**
 - No null pointer dereference
 - No memory leaks
 - Preservation of data structures
 - Partial/total correctness

The Shape Analysis Approach

- **Goal:** determine the **possible shapes of a dynamically allocated data structure** at given program point
- **Interesting information:**
 - **data types** (to avoid type errors, such as dereferencing `nil`)
 - **aliasing** (different pointer variables having same value)
 - **sharing** (different heap pointers referencing same location)
 - **reachability** of nodes (garbage collection)
 - **disjointness** of heap regions (parallelizability)
 - **shapes** (lists, trees, absence of cycles, ...)
- **Concrete questions:**
 - Does `x.next` point to a shared element?
 - Does a variable `p` point to an allocated element every time `p` is dereferenced?
 - Does a variable point to an acyclic list?
 - Does a variable point to a doubly-linked list?
 - Can a loop or procedure cause a memory leak?
- **Here:** basic outline; details in [Nielson/Nielson/Hankin 2005, Sct. 2.6]

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Syntactic categories:

Category	Domain	Meta variable
Arithmetic expressions	$AExp$	a
Boolean expressions	$BExp$	b
Selector names	Sel	sel
Pointer expressions	$PExp$	p
Commands (statements)	Cmd	c

Context-free grammar:

$a ::= z \mid x \mid a_1 + a_2 \mid \dots \mid p \mid \text{nil} \in AExp$

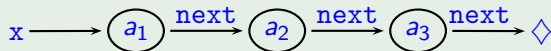
$b ::= t \mid a_1 = a_2 \mid b_1 \wedge b_2 \mid \dots \mid \text{is-nil}(p) \in BExp$

$p ::= x \mid x.sel$

$c ::= [\text{skip}]' \mid [p := a]' \mid c_1 ; c_2 \mid \text{if } [b]' \text{ then } c_1 \text{ else } c_2 \mid$
 $\text{while } [b]' \text{ do } c \mid [\text{malloc } p]' \in Cmd$

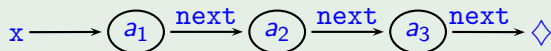
Example 20.3 (List reversal)

Program that reverses list pointed to by x and leaves result in y :



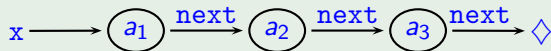
y

z



y → ◇

z



y → ◇

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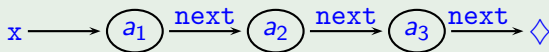
Approach: representation of (infinitely many) concrete heap states by (finitely many) abstract **shape graphs**

- **abstract nodes** X = sets of variables
- interpretation: $x \in X$ iff x points to concrete node represented by X
- \emptyset represents all concrete nodes that are **not directly addressed** by pointer variables
- $x, y \in X$ (with $x \neq y$) indicate **aliasing** (as x and y point to the same concrete node)
- if $x.sel$ and y refer to the same heap address and if X, Y are abstract nodes with $x \in X$ and $y \in Y$, this yields **abstract edge** $X \xrightarrow{sel} Y$
- **transfer functions** transform (sets of) shape graphs

Shape Graphs II

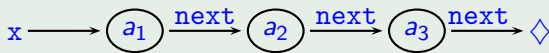
Example 20.4 (List reversal; cf. Example 20.3)

Concrete heap

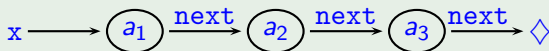


y

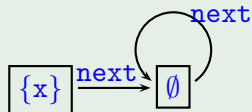
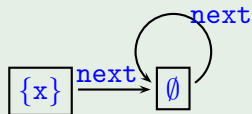
z



z



Shape graph



Definition 20.5 (Shape graph)

A **shape graph** $G = (S, H)$ consists of

- a set $S \subseteq 2^{\text{Var}}$ of **abstract locations** and
- an **abstract heap** $H \subseteq S \times \text{Sel} \times S$
 - notation: $X \xrightarrow{\text{sel}} Y$ for $(X, \text{sel}, Y) \in H$

with the following properties:

Disjointness: $X, Y \in S \implies X = Y$ or $X \cap Y = \emptyset$

(a variable can refer to at most one heap location)

Determinacy: $X \neq \emptyset$ and $X \xrightarrow{\text{sel}} Y$ and $X \xrightarrow{\text{sel}} Z \implies Y = Z$

(target location is unique if source node is unique)

SG denotes the set of all shape graphs.

Remark: the following example shows that determinacy requires $X \neq \emptyset$:

Concrete: $y \longrightarrow \bullet \xleftarrow{\text{sel}} \bullet$ Abstract: $Y = \{y\} \xleftarrow{\text{sel}} X = \emptyset \xrightarrow{\text{sel}} Z = \{z\}$
 $z \longrightarrow \bullet \xleftarrow{\text{sel}} \bullet$

Example 20.6

Let $G = (S, H)$ be a shape graph. Then the following concrete heap properties can be expressed as conditions on G :

- $x \neq \text{nil}$
 $\iff \exists X \in S : x \in X$
- $x = y \neq \text{nil}$ (aliasing)
 $\iff \exists Z \in S : x, y \in Z$
- $x.\text{sel1} = y.\text{sel2} \neq \text{nil}$ (sharing)
 $\implies \exists X, Y, Z \in S : x \in X, y \in Y, X \xrightarrow{\text{sel1}} Z \xleftarrow{\text{sel2}} Y$
(\iff only valid if $Z \neq \emptyset$)