Static Program Analysis

Lecture 20: Wrap-Up Interprocedural DFA & Pointer Analysis

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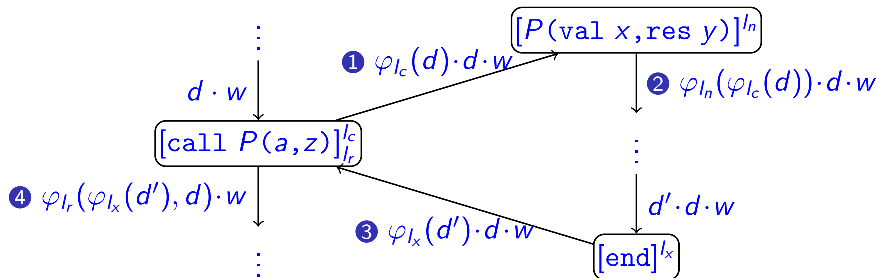
Winter Semester 2014/15

Outline

1. Recap: Interprocedural Dataflow Analysis – Fixpoint Solution
2. Soundness and Completeness
3. Context-Sensitive Interprocedural Dataflow Analysis
4. Pointer Analysis
5. Introducing Pointers
6. Shape Graphs

Visualization of

1. $\hat{\varphi}_{l_c}(d \cdot w) = \varphi_{l_c}(d) \cdot d \cdot w$
2. $\hat{\varphi}_{l_n}(d' \cdot d \cdot w) = \varphi_{l_n}(d') \cdot d \cdot w$
3. $\hat{\varphi}_{l_x}(d' \cdot d \cdot w) = \varphi_{l_x}(d') \cdot d \cdot w$
4. $\hat{\varphi}_{l_r}(d' \cdot d \cdot w) = \varphi_{l_r}(d', d) \cdot w$



Formal Definition of Equation System

Dataflow equations:

$$\mathrm{AI}_l = \begin{cases} \iota & \text{if } l \in E \\ \bigsqcup\{\hat{\varphi}_{l_c}(\mathrm{AI}_{l_c}) \mid (l_c, l_n, l_x, l_r) \in \mathrm{iflow}\} & \text{if } l = l_n \\ & \text{for some } (l_c, l_n, l_x, l_r) \in \mathrm{iflow} \\ \bigsqcup\{f_{l'}(\mathrm{AI}_{l'}) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$$

(if l not a return label)

Node transfer functions:

$$f_l(w) = \begin{cases} \hat{\varphi}_{l_r}(\hat{\varphi}_{l_x}(F_{l_x}(\hat{\varphi}_{l_c}(w)))) & \text{if } l = l_c \text{ for some } (l_c, l_n, l_x, l_r) \in \mathrm{iflow} \\ \hat{\varphi}_l(w) & \text{otherwise} \end{cases}$$

(if l not an exit or return label)

Procedure transfer functions:

$$F_l(w) = \begin{cases} w & \text{if } l = l_n \\ & \text{for some } (l_c, l_n, l_x, l_r) \in \mathrm{iflow} \\ \bigsqcup\{f_{l'}(F_{l'}(w)) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$$

(if l occurs in some procedure)

As before: induces monotonic functional on lattice with ACC

$\Longrightarrow$ least fixpoint effectively computable

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The Fixpoint Iteration

For the fixpoint iteration it is important that the auxiliary functions only operate (at most) on the two topmost elements of the stack:

Lemma 20.1

For every $l \in Lab$, $d \in D$, *and* $w \in D^*$,

$$f_l(d' \cdot d \cdot w) = f_l(d' \cdot d) \cdot w \text{ and } F_l(d' \cdot d \cdot w) = F_l(d' \cdot d)w$$

Proof.

see J. Knoop, B. Steffen: *The Interprocedural Coincidence Theorem*, Proc. CC '92, LNCS 641, Springer, 1992, 125–140 □

It therefore suffices to consider stacks with at most two entries, and so the fixpoint iteration ranges over "finitary objects".

Soundness and Completeness

The following results carry over from the intraprocedural case:

Theorem 20.2

Let $\hat{S} := (Lab, E, F, (\hat{D}, \hat{\sqsubseteq}), \hat{\iota}, \hat{\varphi})$ *be an interprocedural dataflow system.*

1. *(cf. Theorem 6.3)*

$$\mathsf{mvp}(\hat{S}) \mathrel{\hat{\sqsubseteq}} \mathsf{fix}(\Phi_{\hat{S}})$$

2. *(cf. Theorem 7.3)*

$$\mathsf{mvp}(\hat{S}) = \mathsf{fix}(\Phi_{\hat{S}}) \text{ if all } \hat{\varphi}_l \text{ are distributive}$$

Proof.

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  - information about calling states combined for all call sites
  - procedure body only analyzed once using combined information
  - resulting information used at all return points

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- **Alternative:** context-sensitive analysis
  - separate information for different call sites
  - implementation by "procedure cloning" (one copy for each call site)
  - more precise
  - more costly

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  - Programs with pointers and dynamically allocated data structures are error prone
  - Identify subtle bugs at compile time
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- **Problem:**
  - Programs with pointers and dynamically allocated data structures are error prone
  - Identify subtle bugs at compile time
  - Automatically prove correctness
- **Interesting properties of heap-manipulating programs:**
  - No null pointer dereference
  - No memory leaks
  - Preservation of data structures
  - Partial/total correctness

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- **Interesting information:**
  - data types (to avoid type errors, such as dereferencing `nil`)
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  - sharing (different heap pointers referencing same location)
  - reachability of nodes (garbage collection)
  - disjointness of heap regions (parallelizability)
  - shapes (lists, trees, absence of cycles, ...)

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- **Concrete questions:**
  - Does `x.next` point to a shared element?
  - Does a variable `p` point to an allocated element every time `p` is dereferenced?
  - Does a variable point to an acyclic list?
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- **Here:** basic outline; details in [Nielson/Nielson/Hankin 2005, Sct. 2.6]

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Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Arithmetic expressions	*AExp*	*a*
Boolean expressions	*BExp*	*b*
Selector names	*Sel*	*sel*
Pointer expressions	*PExp*	*p*
Commands (statements)	*Cmd*	*c*

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Context-free grammar:

$$
\begin{aligned}
a &::= z \mid x \mid a_1 + a_2 \mid \ldots \mid p \mid \texttt{nil} \in AExp \\
b &::= t \mid a_1 = a_2 \mid b_1 \wedge b_2 \mid \ldots \mid \texttt{is-nil}(p) \in BExp \\
p &::= x \mid x.sel \\
c &::= [\texttt{skip}]^l \mid [p \texttt{ := } a]^l \mid c_1 ; c_2 \mid \texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \mid \\
&\quad\ \texttt{while } [b]^l \texttt{ do } c \mid [\texttt{malloc } p]^l \in Cmd
\end{aligned}
$$

An Example

Example 20.3 (List reversal)

Program that reverses list pointed to by x and leaves result in y:

```
[y := nil]1;
while [¬is-nil(x)]2 do
  [z := y]3;
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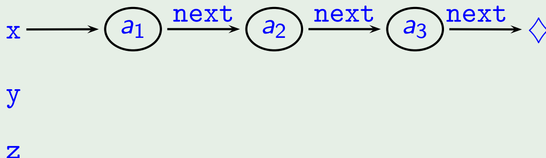

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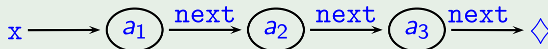
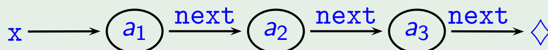


y

z



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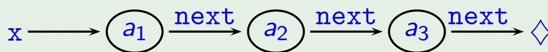

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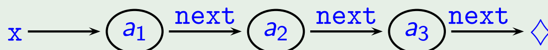

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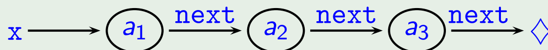

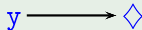

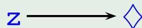

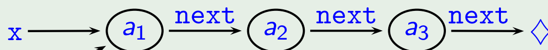

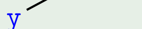

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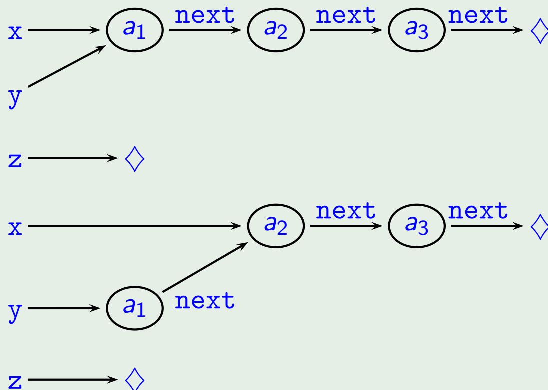

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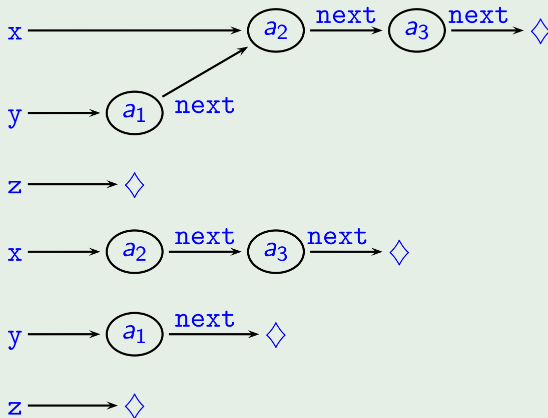

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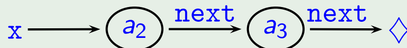
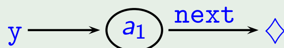
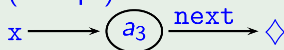
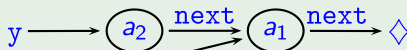




(4 steps)





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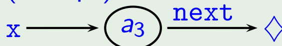

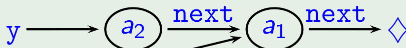

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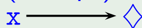

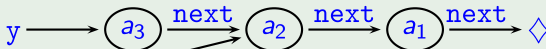

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(4 steps)

x → ◇

y → a_3 —next→ a_2 —next→ a_1 —next→ ◇

z → a_2

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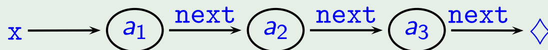
Approach: representation of (infinitely many) concrete heap states by (finitely many) abstract shape graphs

- abstract nodes X = sets of variables
- interpretation: $x \in X$ iff x points to concrete node represented by X
- $\emptyset$ represents all concrete nodes that are not directly addressed by pointer variables
- $x, y \in X$ (with $x \neq y$) indicate aliasing (as x and y point to the same concrete node)
- if $x.sel$ and y refer to the same heap address and if X, Y are abstract nodes with $x \in X$ and $y \in Y$, this yields abstract edge $X \xrightarrow{sel} Y$
- transfer functions transform (sets of) shape graphs

Example 20.4 (List reversal; cf. Example 20.3)

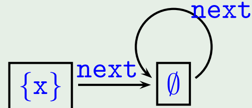
Concrete heap



y

z

Shape graph



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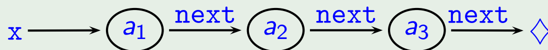
Concrete heap | **Shape graph**



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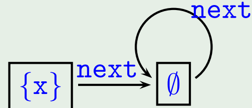
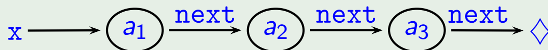
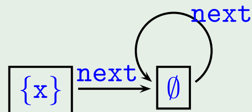




y → ◇

z



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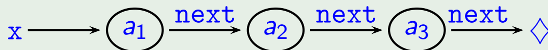
Concrete heap | **Shape graph**



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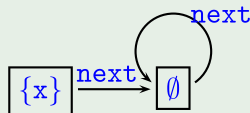

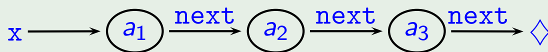

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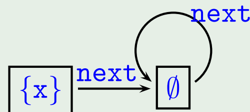

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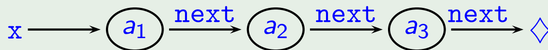
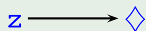
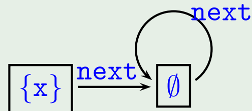
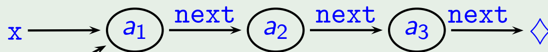
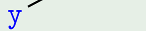
Shape graph

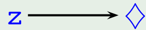


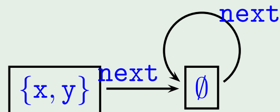






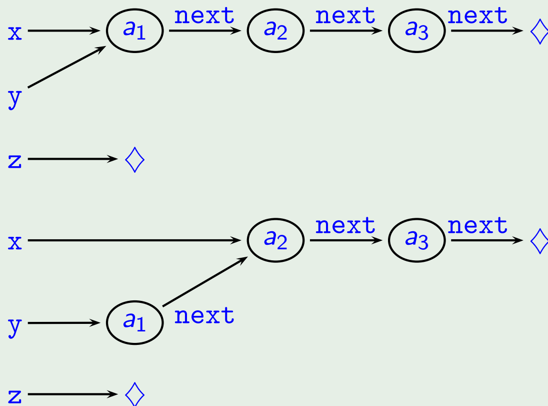




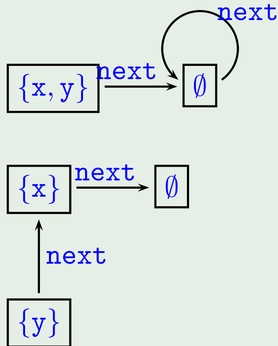


Example 20.4 (List reversal; cf. Example 20.3)

Concrete heap

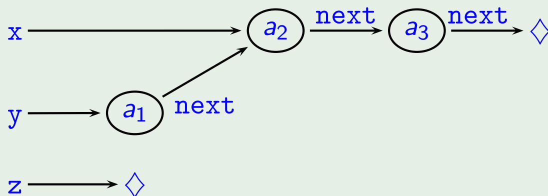


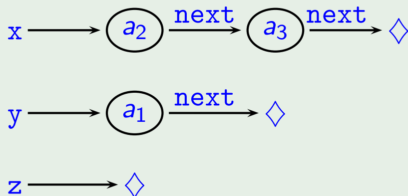
Shape graph



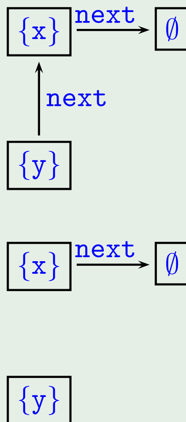
Example 20.4 (List reversal; cf. Example 20.3)

Concrete heap





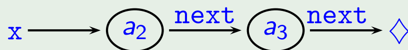
Shape graph

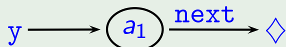


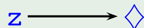
Shape Graphs II

Example 20.4 (List reversal; cf. Example 20.3)

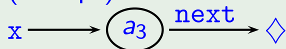
Concrete heap

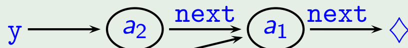




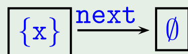


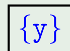
(4 steps)

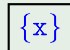


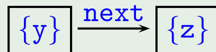


Shape graph





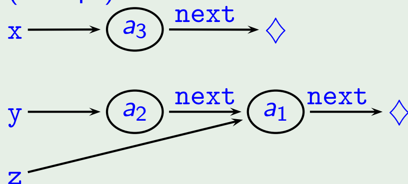


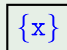


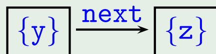
Example 20.4 (List reversal; cf. Example 20.3)

Concrete heap | **Shape graph**

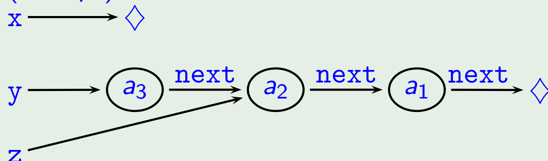
(4 steps)

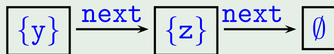






(4 steps)





Example 20.4 (List reversal; cf. Example 20.3)

Concrete heap

Shape graph

(4 steps)

Shape Graphs III

Definition 20.5 (Shape graph)

A shape graph $G = (S, H)$ consists of

- a set $S \subseteq 2^{Var}$ of abstract locations and
- an abstract heap $H \subseteq S \times Sel \times S$
  - notation: $X \xrightarrow{sel} Y$ for $(X, sel, Y) \in H$

with the following properties:

Disjointness: $X, Y \in S \implies X = Y$ or $X \cap Y = \emptyset$
(a variable can refer to at most one heap location)

Determinacy: $X \neq \emptyset$ and $X \xrightarrow{sel} Y$ and $X \xrightarrow{sel} Z \implies Y = Z$
(target location is unique if source node is unique)

SG denotes the set of all shape graphs.

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(target location is unique if source node is unique)

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Remark: the following example shows that determinacy requires $X \neq \emptyset$:

Concrete: $y \longrightarrow \bullet \xleftarrow{sel} \bullet$ $\quad$ Abstract: $\boxed{Y = \{y\}} \xleftarrow{sel} \boxed{X = \emptyset} \xrightarrow{sel} \boxed{Z = \{z\}}$

$z \longrightarrow \bullet \xleftarrow{sel} \bullet$

Shape Graphs and Concrete Heap Properties

Example 20.6

Let $G = (S, H)$ be a shape graph. Then the following concrete heap properties can be expressed as conditions on G:

- $\mathtt{x} \neq \mathtt{nil}$
  $\iff \exists X \in S : \mathtt{x} \in X$
- $\mathtt{x} = \mathtt{y} \neq \mathtt{nil}$ (aliasing)
  $\iff \exists Z \in S : \mathtt{x}, \mathtt{y} \in Z$
- $\mathtt{x.sel1} = \mathtt{y.sel2} \neq \mathtt{nil}$ (sharing)
  $\implies \exists X, Y, Z \in S : \mathtt{x} \in X, \mathtt{y} \in Y, X \stackrel{\mathtt{sel1}}{\longrightarrow} Z \stackrel{\mathtt{sel2}}{\longleftarrow} Y$
  ($\Longleftarrow$ only valid if $Z \neq \emptyset$)