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Exercise 1 (Reaching Definitions Analysis using Abstract Interpretation): (8 Points)

We will now consider the Reaching Definitions Analysis (RDA). Given a labeled WHILE-program, this analysis computes for every program location and every variable all other program locations in which the variable might have been most recently written (i.e. written without being re-written in between). As an example, consider the following program.

```
[x := 2]^1;
[x := 3]^2;
while [y < 10]^3;
    [y := y + 1]^4;
[x := y * 2]^5;
```

$(x, 2)$ is a reaching definition at label 4, because there is a path reaching label 4 such that x is most recently written at label 2. On the other hand $(x, 1)$ is not a reaching definition at label 4. If the most recent definition of a variable is "before the program", this is indicated by a question mark as the label information. For example, for label 5 we have the reaching definitions $\{(y, 4), (y, ?), (x, 2)\}$.

Your task is to implement this analysis in the abstract interpretation framework. Assume that you are given a **labeled** WHILE-program.

a) Describe informally why you need to extend the concrete semantics of the WHILE-language presented in the lecture.

b) Formally (re-)define the parts of the concrete semantics of WHILE that need to be changed.

c) Formally adapt the abstract semantics of WHILE.

d) Build the abstract transition system for your abstraction and the following WHILE-program.

```
[x := 2]^1;
[y := x + 2]^2;
while [x > 3]^3
    [y := y + x]^4;
[x := y]^5;
```

e) What do you ultimately have to do with the abstract transition system to derive the desired output of the analysis?

Exercise 2 (Lattice): (2 Points)

Consider the following predicates q_1 and q_2. Calculate $q_1 \sqcup q_2$ and $q_1 \sqcap q_2$.

1. $q_1 := p_1 \wedge p_2 \wedge p_3$, $q_2 := \neg p_3$
2. $q_1 := p_1 \wedge \neg p_3$, $q_2 := p_1 \wedge p_2 \wedge \neg p_3$
3. $q_1 := \neg p_1 \wedge \neg p_2 \wedge \neg p_3$, $q_2 := p_1 \wedge \neg p_2$