

## – Assignment 2 –

### Exercise 1

(2 points)

As presented in the lecture (cf. Lecture slide 4 - p.8), the (weak) concatenation of two MSCs  $M_1$  and  $M_2$  (with  $M_i = \langle \mathcal{P}_i, E_i, \mathcal{C}_i, \ell_i, m_i, \prec_i \rangle$  for  $i \in \{1, 2\}$ ) intuitively is realized by gluing the process lines together such that  $M_1$  is situated on top of MSC  $M_2$  (cf. Figure 1).

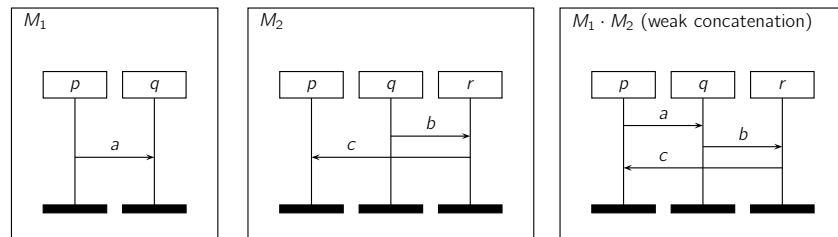


Figure 1: Two MSCs and their weak concatenation

Define the so-called *strong concatenation*  $\bullet_s$  of two MSCs  $M_1$  and  $M_2$ , i.e., all events of MSC  $M_1$  have to be executed before the first event of  $M_2$ . For this purpose determine a structure  $M = M_1 \bullet_s M_2 = \langle \mathcal{P}, E, \mathcal{C}, \ell, m, \prec \rangle$ , that (in terms of  $M_1$  and  $M_2$ ) results from concatenating the two MSCs strongly.

### Exercise 2

(4 points)

The *word language* of a (possibly infinite) set of MSCs  $\mathcal{M} = \{M_1, M_2, \dots\}$  is defined as

$$\mathcal{L}(\mathcal{M}) := \bigcup_{M \in \mathcal{M}} \text{Lin}(M).$$

(Compare with the definition in Lecture slide 4 - p.16).

Show that:

It is decidable whether a *regular* language  $L$  is in a word language for some set of MSCs, i.e., there exists  $\mathcal{M}$  such that  $L \subseteq \mathcal{L}(\mathcal{M})$ .

(Hint: Find a way to show that every word  $w \in L$  is well-formed.)

### Exercise 3

(4 points)

Let  $w \in \text{Act}^*$  be a linearization of an MSC  $M$  and  $p, q \in \mathcal{P}$  be two processes in  $M$ .

The *bound* from  $p$  to  $q$  in  $w$  is defined as:

$$B_w^{pq} := \max_{u \text{ is prefix of } w} \left( \sum_{c \in \mathcal{C}} |u|_{!(p,q,c)} - \sum_{c \in \mathcal{C}} |u|_{?(q,p,c)} \right)$$

where  $|u|_a$  denotes the number of occurrences of action  $a$  in  $u$ . (Compare with the definition in Lecture slide 2 - p.20.) In other words,  $B_w^{pq}$  denotes the maximum number of messages in the channel from  $p$  to  $q$  for the linearization  $w$ .

The *bound*  $B$  for a set of words  $\mathcal{L} \subseteq \text{Act}^*$  is then defined as:

$$B := \max_{w \in \mathcal{L}, p, q \in \mathcal{P}} B_w^{pq}$$

Prove that:

If a word language of a set of MSCs  $\mathcal{M}$  is regular (cf. Exercise 2), then the bound  $B$  for  $\mathcal{L}(\mathcal{M})$  is finite.