Theoretical Foundations of the UML

Lecture 11: Safe Realisability

Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group

http://moves.rwth-aachen.de/teaching/ws-1415/uml/

7. Dezember 2014

Outline

1. Safe realisability
2. Closure and inference revisited
3. Characterisation and complexity of safe realisability

Overview

1. Safe realisability
2. Closure and inference revisited
3. Characterisation and complexity of safe realisability

From requirements to implementation

Realisability problem

INPUT: a set of MSCs

OUTPUT: a CFM $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A})$ equals the set of input MSCs.

Questions:

1. Is this possible? (That is, is this decidable?)
2. If so, how complex is it to obtain such CFM?
3. If so, how do such algorithms work?

Problem variants (1)

Realisability problem

INPUT: a set of MSCs

OUTPUT: a CFM $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A})$ equals the set of input MSCs.

Different forms of requirements

- Consider finite sets of MSCs, given as an enumerated set.
- Consider MSGs, that may describe an infinite set of MSCs.
- Consider MSCs whose set of linearisations is a regular word language.
- Consider MSGs that are non-local choice.

Problem variants (2)

Realisability problem

INPUT: a set of MSCs

OUTPUT: a CFM $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A})$ equals the set of input MSCs.

Different system models

- Consider CFMs without synchronisation messages.
- Allow CFMs that may deadlock. Possibly, a realisation deadlocks.
- Forbid CFMs that deadlock. No realisation will ever deadlock.
- Consider CFMs that are deterministic.
- Consider CFMs that are bounded.
- ……

Today's lecture

Today's setting

Realisation of a finite set of MSCs by a deadlock-free weak CFM.

Realisation of a finite set of well-formed words (= language) by a deadlock-free weak CFM.

This is known as safe realisability.

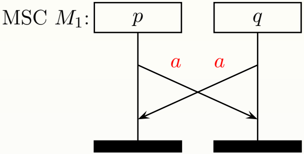
This is the setting of the previous lecture, but now focusing on deadlock-free CFMs

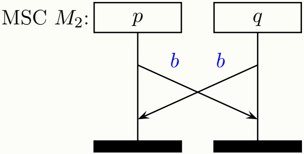
Results:

1. Conditions for realisability of a finite set of MSCs by a deadlock-free weak CFM.
2. Checking safe realisability by deadlock-free CFMs is in P.
   (Realisability for weak CFMs that may deadlock is co-NP complete.)

Safe realisability

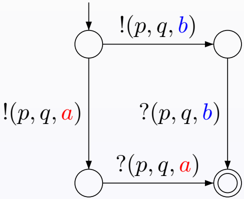
Possibly a set of MSCs is realisable only by a CFM that may deadlock



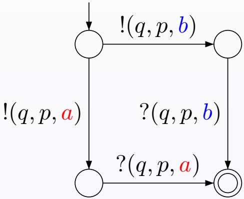


process p and q have to agree on either a or b

Realisation of $\{ M_1, M_2 \}$ by a weak CFM:



process p



process q

Deadlock occurs when, e.g., p sends a and q sends b

Safe realisability

Definition (Safe realisability)

1. MSC M is safely realisable whenever $\{M\} = \mathcal{L}(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.
2. A finite set $\{M_1, \ldots, M_n\}$ of MSCs is safely realisable whenever $\{M_1, \ldots, M_n\} = \mathcal{L}(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.
3. MSG G is safely realisable whenever $\mathcal{L}(G) = \mathcal{L}(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.

Phrased using linearisations

$L \subseteq Act^*$ is safely realisable if $L = Lin(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.

Note:

Safe realisability implies realisability, but the converse does not hold.

Overview

Weak closure

Definition (Inference relation and closure)

For well-formed $L \subseteq Act^*$, and well-formed word $w \in Act^*$, let:

$$L \models w \quad \text{iff} \quad (\forall p \in \mathcal{P}.\, \exists v \in L.\, w \upharpoonright p = v \upharpoonright p)$$

Language L is closed under $\models$ whenever for every $w \in Act^*$, it holds: $L \models w$ implies $w \in L$.

Definition (Weak closure)

Language L is weakly closed under $\models$ whenever for every well-formed prefix w of some word in L, it holds $L \models w$ implies $w \in L$.

Weak closure thus restricts closure under $\models$ to well-formed prefixes in L only. So far, closure was required for all $w \in Act^*$.

Deadlock-free closure

For language L, let $pref(L) = \{w \mid \exists u.\, w{\cdot}u \in L\}$ the set of prefixes of L.

Definition ((Deadlock-free) Inference relation)

For well-formed $L \subseteq Act^*$, and proper word $w \in Act^*$, i.e., w is a prefix of a well-formed word, let:

$$L \models^{df} w \quad \text{iff} \quad (\forall p \in \mathcal{P}.\, \exists v \in pref(L).\, w \upharpoonright p \text{ is a prefix of } v \upharpoonright p)$$

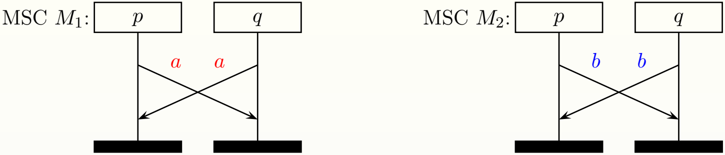
Definition (Closure under $\models^{df}$)

Language L is closed under $\models^{df}$ whenever $L \models^{df} w$ implies $w \in pref(L)$.

Intuition

The closure condition asserts that the set of partial MSCs (i.e., prefixes of L) can be constructed from the projections of the MSCs in L onto individual processes.

Example



Example

$L = Lin(\{M_1, M_2\})$ is not closed under $\models^{df}$:

$$w \;= !(p,q,a)!(q,p,b) \notin \; pref(L)$$

But: $L \models^{df} w$ since w is a proper prefix of a well-formed word, and

- for process p, there exists $u \in L$ with $w \upharpoonright p = !(p,q,a) \in pref(\{u \upharpoonright p\})$, and
- for process q, there exists $v \in L$ with $w \upharpoonright q = !(q,p,b) \in pref(\{v \upharpoonright q\})$.

Note that L is closed under $\models$. So this shows that closure under $\models$ does not imply closure under $\models^{df}$.

Deadlock-free weak CFM are closed under $\models^{df}$

Lemma:

For every deadlock-free weak CFM $\mathcal{A}$, $Lin(\mathcal{A})$ is closed under $\models^{df}$.

Proof.

Similar proof strategy as for the closure of weak CFMs under $\models$ (see previous lecture). Basic intuition is that if $w \upharpoonright p$ is a prefix of $v^p \upharpoonright p$, then from the point of view of process p, w can be prolonged with a word u, say, such that $w \cdot u = v^p$. This applies to all processes, and as the weak CFM is deadlock-free, such continuation is always possible. □

Overview

Characterisation of safe realisability

Theorem: [Alur *et al.*, 2001]

$L \subseteq Act^*$ is safely realisable iff L is weakly closed under $\models$ and closed under $\models^{df}$.

Proof

On the black board.

Corollary

The finite set of MSCs $\{M_1, \ldots, M_n\}$ is safely realisable iff $\bigcup_{i=1}^{n} Lin(M_i)$ is closed under $\models$ and $\models^{df}$.

Characterisation of safe realisability

Theorem

For any well-formed $L \subseteq Act^*$:

L is regular and closed under $\models$
if and only if
$L = Lin(\mathcal{A})$ for some $\forall$-bounded weak CFM $\mathcal{A}$.

Theorem

For any well-formed $L \subseteq Act^*$:

L is regular, weakly closed under $\models$ and closed under $\models^{df}$
if and only if
$L = Lin(\mathcal{A})$ for some $\forall$-bounded deadlock-free weak CFM $\mathcal{A}$.

Complexity of safe realisability

Theorem: [Alur *et al.*, 2001]

The decision problem "is a given set of MSCs safely realisable?" is in P.

Proof

1. For a given finite set of MSCs, safe realisability can be checked in time $\mathcal{O}((n^2 + r) \cdot k)$ where k is the number of processes, n the number of MSCs, and r the number of events in all MSCs together.
2. If the MSCs are not safely realisable, the algorithm returns an MSC which is implied, but not included in the input set of MSCs.

(We skip the details in this lecture.)