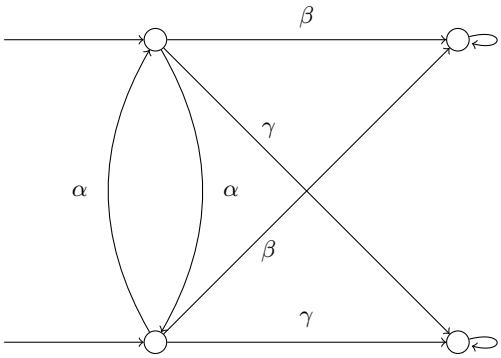
Software Modeling and Verification Chair | RWTH AACHEN UNIVERSITY

— Master's Thesis —

Equivalences of Strategies with Memory for MDPs and Nondeterministic Probabilistic Programs

What is it all about?

Markov Decision Processes (MDPs) [BBKW24] are models of nondeterministic and probabilistic systems. An example of an MDP is:



The nondeterminism is modeled by different actions α, β, γ. Uncertainty is modeled by probabilisties on transtitions. In order to resolve the nondeterminism, strategies are considered. The easiest form of a strategy is a mapping from the states to the actions: $S \to \{\alpha, \beta, \gamma\}$. These are called memoryless strategies. Strategies with memory can make a decision based on the entire history of visited states and are thus more powerful.
On the other hand side, we have nondeterministic probabilistic programs [BKWZ25] that extend deterministic programs by a nondeterministic choice about which code branch is executed next. They can be defined by the following grammar:

$$\mathrm{c} \coloneqq \mathrm{skip} \mid x \coloneqq \mathrm{a} \mid \{\mathrm{c}\}[\mathrm{p}]\{\mathrm{c}\} \mid \{\mathrm{c}\}[]\{\mathrm{c}\} \mid \mathrm{c};\mathrm{c} \mid \mathrm{if\ b\ then\ c\ else\ c\ end} \mid \mathrm{while\ b\ do\ c\ end}.$$

The nondeterministic choice in a probabilistic programs, needs to be resolved by a strategy too. In this thesis, we want to compare different kinds of strategies for a) MDPs and b) nondeterministic probabilistic programs and establish a formal relationship (equivalence) between the two different notions.

What is to be done?

The goals of this project are:

1. Define a notion of memory for strategies on program level that corresponds to strategies with memory on MDP level
2. Define a determinization operator for strategies with memory turning a MDP to a MC (Markov chain)
3. Prove equivalence of different strategy classes

This list is of course non-exhaustive! The above suggestions may be changed, shortened and/or extended while we work on our project and gain more insights on how difficult the topic is.

What we expect:

- Solid background in theoretical computer science and maths – ideally you have already taken theoretical CS electives
- Passion and endurance for solving theoretical problems

What you can expect:

- Get a chance to work on relevant problems of both theoretical and practical nature
- You can work in the student room at our chair – we have a coffee machine, lots of tea and sometimes cookies :)

Apply

- Daniel Zilken (daniel.zilken@cs.rwth-aachen.de)
  Please introduce yourself briefly and say why you're interested in this topic!

References

[BBKW24] Kevin Batz, Tom Jannik Biskup, Joost-Pieter Katoen, and Tobias Winkler. Programmatic strategy synthesis: Resolving nondeterminism in probabilistic programs. *Proc. ACM Program. Lang.*, 8(POPL), January 2024.

[BKWZ25] Kevin Batz, Joost-Pieter Katoen, Tobias Winkler, and Daniel Zilken. Verifying sampling algorithms via distributional invariants, 2025.