Theoretical Foundations of the UML

Lecture 18: Statecharts Semantics

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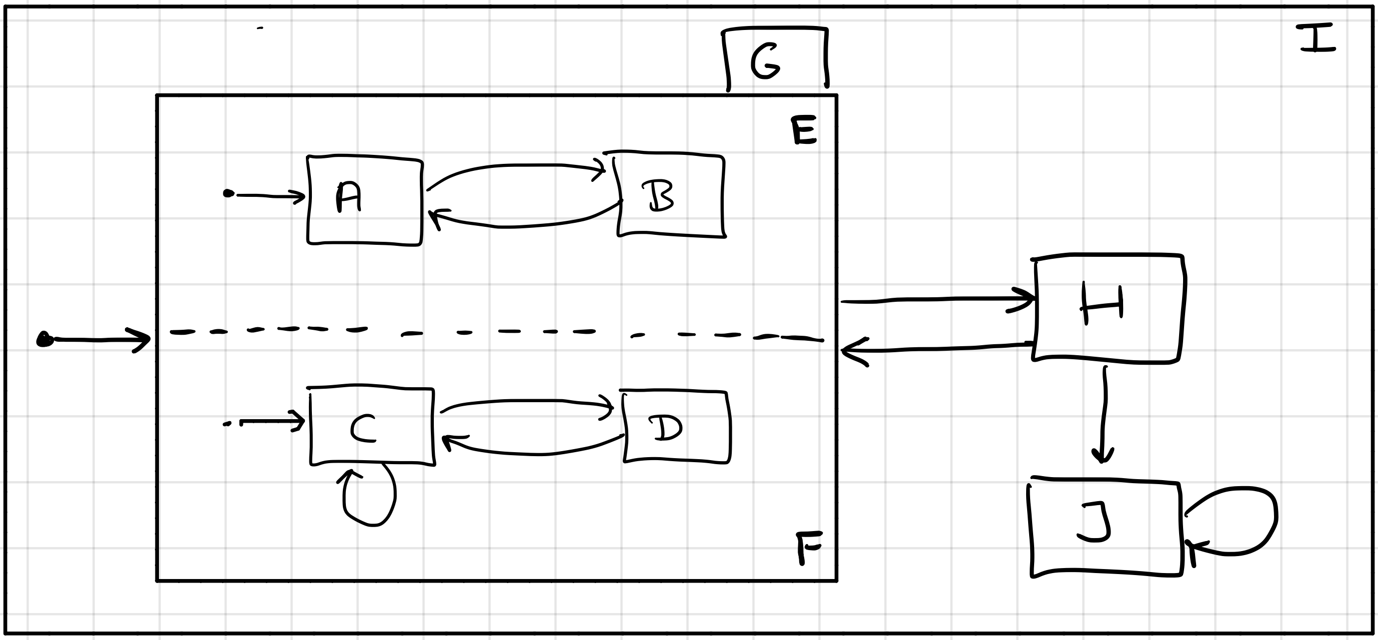
Outline



I = root node

N = set of nodes $= \{A, B, C, \dots, I, J\}$

children : $N \to 2^N$ e.g. children $(H) = \emptyset$

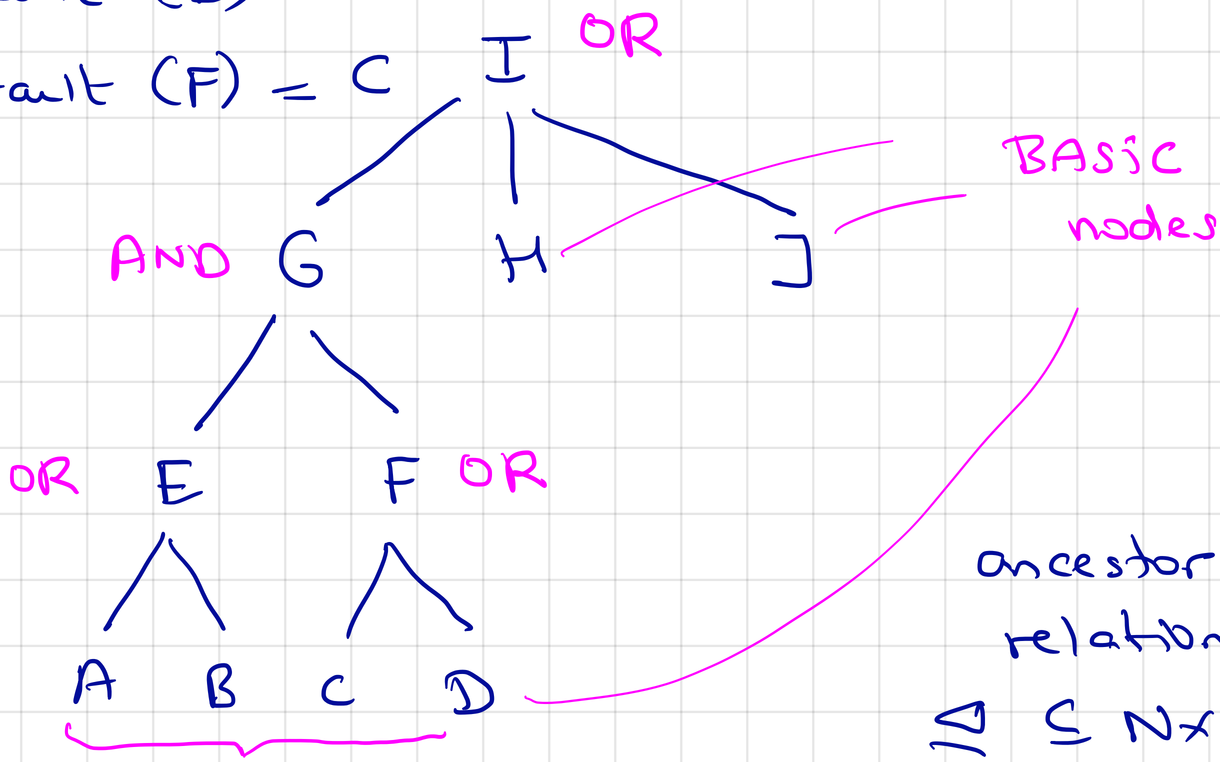
children $(I) = \{G, H, J\}$

children $(G) = \{E, F\}$

default $(I) = G$

default $(E) = A$

default $(F) = C$



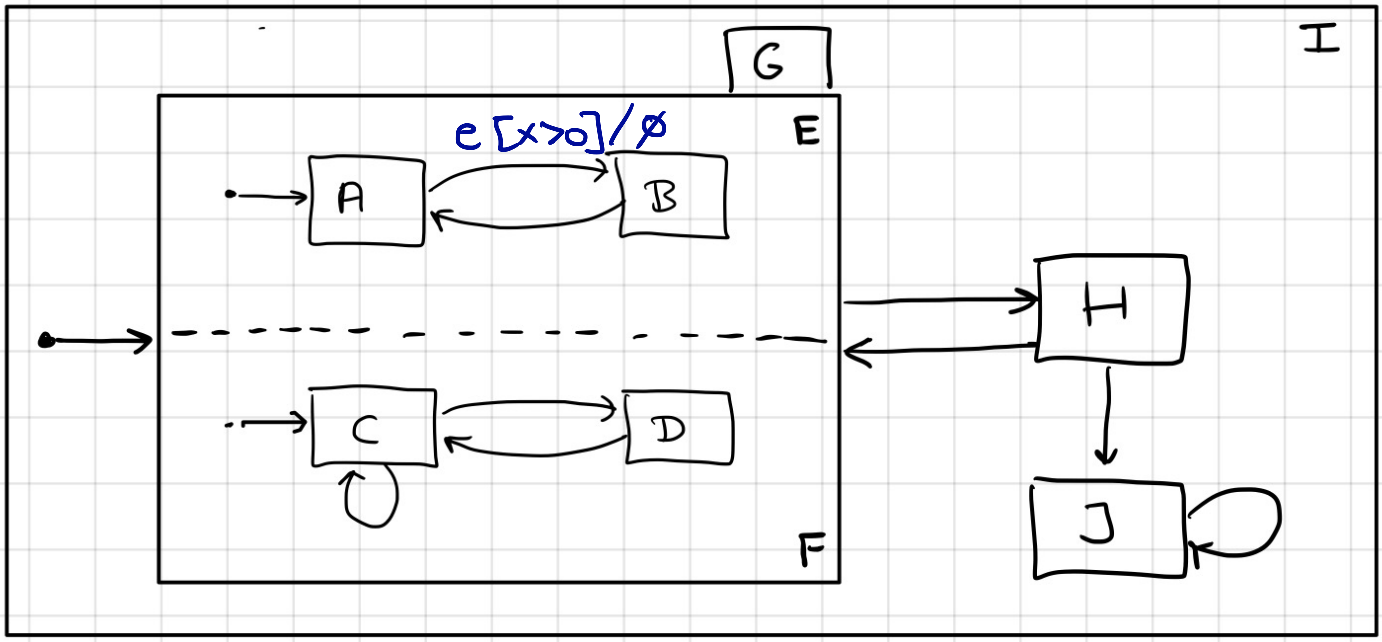
ancestor relation

$\trianglelefteq \subseteq N \times N$

$C \trianglelefteq F \qquad F \trianglelefteq G$

$C \trianglelefteq G$

$H \not\trianglelefteq D$



Edges: $\underset{\subseteq N}{X} \xrightarrow{e[g]/A} \underset{\subseteq N}{Y}$ hyperedge

event e

$\{A\} \xrightarrow{e[x>0]/\emptyset} \{B\}$

guard g

$\{G\} \xrightarrow{e[x=2]/\dots} \{H\}$

A actions

Overview

Towards a Statechart semantics

- Formal semantics: map $(SC_1, \ldots, SC_k)$ onto a single Mealy machine

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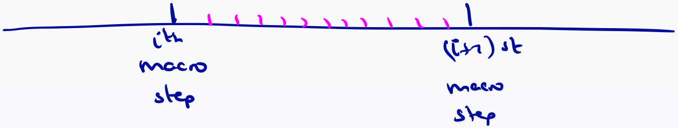
- This is done using a step semantics distinguishing macro and micro steps
- Macro steps are "observable" and are subdivided into a finite number of micro steps that cannot be prolonged



Towards a Statechart semantics

- Formal semantics: map $(SC_1, \ldots, SC_k)$ onto a single Mealy machine
- This is done using a step semantics distinguishing macro and micro steps

send j.e   send j+1, e'

- Macro steps are "observable" and are subdivided into a finite number of micro steps that cannot be prolonged

n   n+1   n+2

- In a macro step, a maximal set of edges is performed

assignments / send j.e

- Events generated in macro step n are only available in macro step $n+1$
  - If such event is not "consumed" in step $n+1$, it dies, and is not available in step $n+2$, $n+3$, …

Assumptions [Eshuis & Wieringa, 2000]

- Input to a macro step is a set of events (and not a queue)

  the order of event generation is ignored, i.e., if e and e' are generated in macro step i, the order in which they are generated is irrelevant in step $i{+}1$

unordered

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- Input to a macro step is a set of events (and not a queue)

  the order of event generation is ignored, i.e., if e and e' are generated in macro step i, the order in which they are generated is irrelevant in step $i{+}1$

- A macro step reacts to all available events

  events can only be used in macro step immediately following their generation

- Instantaneous edges and actions

- Unlimited concurrency

  there is no limit on the number of events that can be consumed in a macro step

- Perfect communication, i.e., messages are not lost

What does a single StateChart mean?

$\underbrace{(SC_1, \ldots, SC_k)}_{\text{several statecharts}} \longmapsto$ single Mealy machine

Intuitive semantics as a transition system:

- State = $\underbrace{\text{a set of nodes}}_{\text{in the statechart}}$ ("current control") + $\underbrace{\text{the values of variables}}$

currently "active" statechart nodes

Intuitive semantics as a transition system:

- State = a set of nodes ("current control") + the values of variables

  ↓ in the statechart

- Edge is enabled if guard holds in current state

$g = x > 0$ – needs to hold in current state

$X \xrightarrow{e\,[g]/A} Y$

enabled

needs to be available

What does a single StateChart mean?

Intuitive semantics as a transition system:

- State = a set of nodes ("current control") + the values of variables
- Edge is enabled if guard holds in current state
- Executing edge $X \xrightarrow{e[g]/A} Y$ = perform actions A, consume event e
  - leave source nodes X and switch to target nodes Y

  $\Rightarrow$ events are unordered, and considered as a set
- Principle: execute as many edges at once (without conflict)

  $\Rightarrow$ the total execution of such maximal set is a macro step

Overview

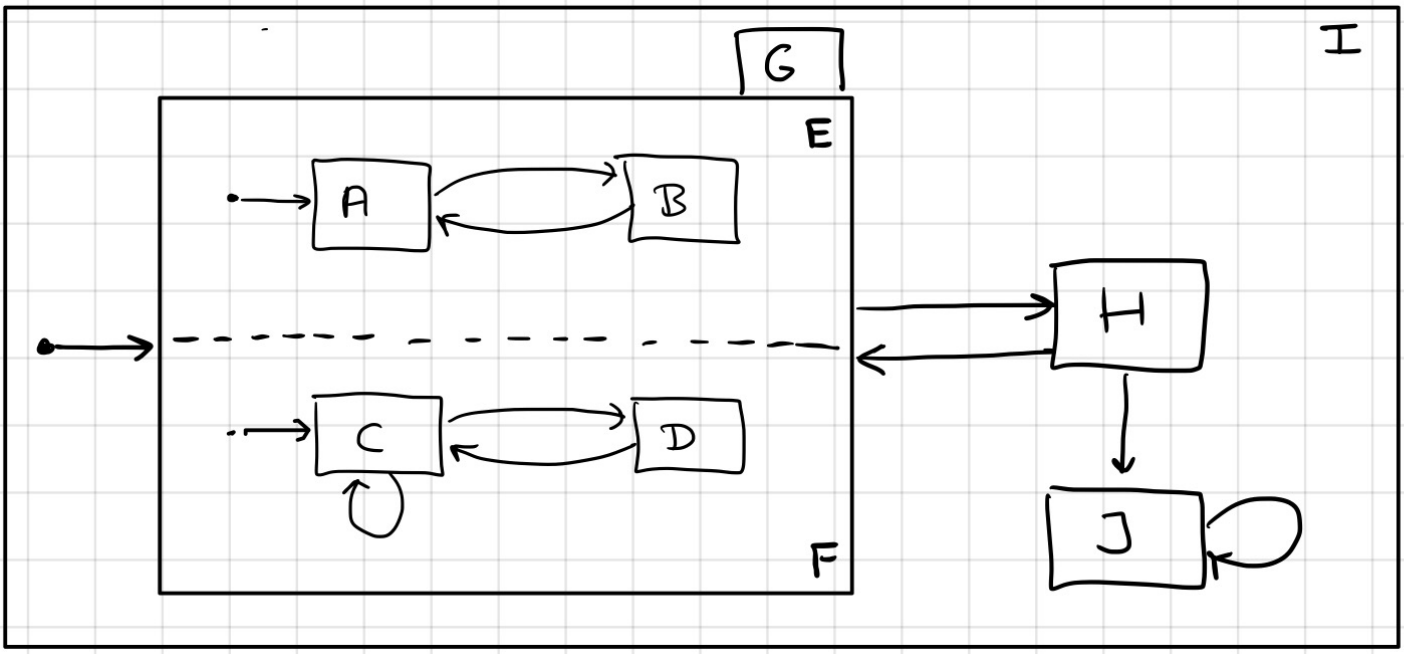
States and configurations

Definition (Configuration)

A configuration of $SC = (N, E, Edges)$ is a set $C \subseteq N$ of nodes satisfying:

- $\text{root} \in C$
- $x \in C$ and $type(x) = \text{OR}$ implies $|children(x) \cap C| = 1$
- $x \in C$ and $type(x) = \text{AND}$ implies $children(x) \subseteq C$

Let $Conf$ denote the set of configurations of SC.



Example configurations

$$C_1 = \{ I, H \}$$

$$C_2 = \{ I, G, E, F, \underline{A}, D \}$$

$$C_3 = \{ I, G, E, F, B, C \} \quad \text{etc.}$$

States $= \quad (C_1, \emptyset, \{x = 0, y = 0\})$

$$(C_2, \{e, e'\}, \{x = 1, y = 0\})$$

etc.

Enabledness: $\underline{\underline{\{A\}}} \xrightarrow{e\,[true]\,/\,\emptyset} \{B\}$

is enabled in $(\underline{\underline{C_2}}, \{e\}, V)$

States and configurations

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Let $Conf$ denote the set of configurations of SC.

Definition (State)

State of $SC = (N, E, Edges)$ is a triple $\underline{(C, I, V)}$ where

- C is a configuration of SC
- $I \subseteq V$ is the set of events to be processed ("available" events)
- V is a valuation of the variables. e.g $x=3$, $y=17$, $c=$"A"

Overview

how to define the transition relation between states?

$$C_1, I_1, V_1 \longrightarrow C_2, I_2, V_2$$

Enabling of an edge

$x>0 \wedge y<10$

g is a global Boolean statement, covering the entire statecharts

Definition (Enabledness)

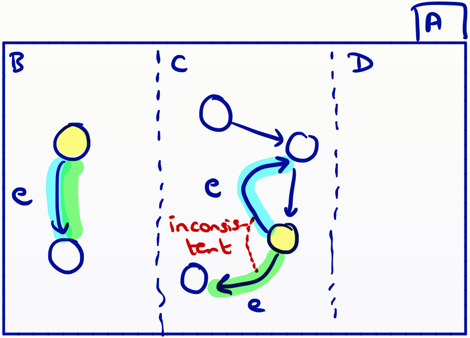
Edge $X \xrightarrow{e[g]/A} Y$ is enabled in state (C, I, V) whenever: $sc_1, \ldots, sc_n$

- $X \subseteq C$, i.e. all source nodes are in configuration C
- $(\underbrace{(C_1, \ldots, C_n)}_{\text{configurations}}, \underbrace{(V_1, \ldots, V_n)}_{\text{variable valuations}}) \models g$, i.e., guard g is satisfied in all statecharts
- either $e \neq \perp$ implies $e \in I$, or $e = \perp$

Let $En(C, I, V)$ denote the set of enabled edges in state (C, I, V).

Macro steps

- On receiving an input e, several edges in SC may become enabled
- Then, a maximal and consistent set of enabled edges is taken



Macro steps

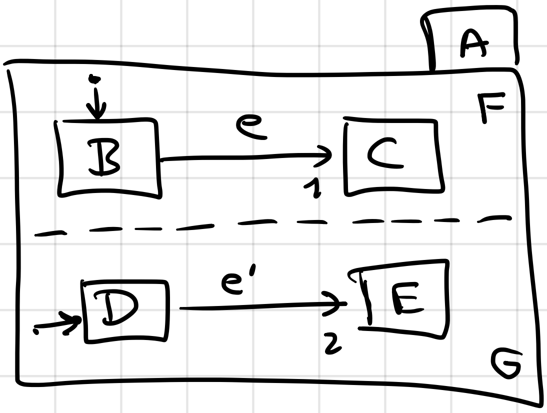
- On receiving an input e, several edges in SC may become enabled ✓
- Then, a maximal and consistent set of enabled edges is taken
- If there are several such sets, choose one nondeterministically
- Edges in concurrent components can be taken simultaneously
- But edges in other components cannot; they are inconsistent
- To resolve nondeterminism (partly), priorities are used

Overview

Consistency: examples

To define consistency formally, we need some auxiliary concepts

1



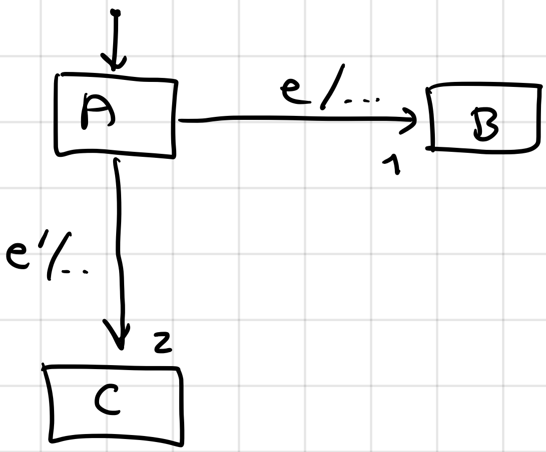
pair of edges (1,2)

intuition: (1,2) are consistent as they can happen in parellel

in state = $(\{A, F, G, B, D\}, \{e, e', \dots\}, V)$

both edges 1, 2 are enabled, and as they are consistent, they can both be taken.

2

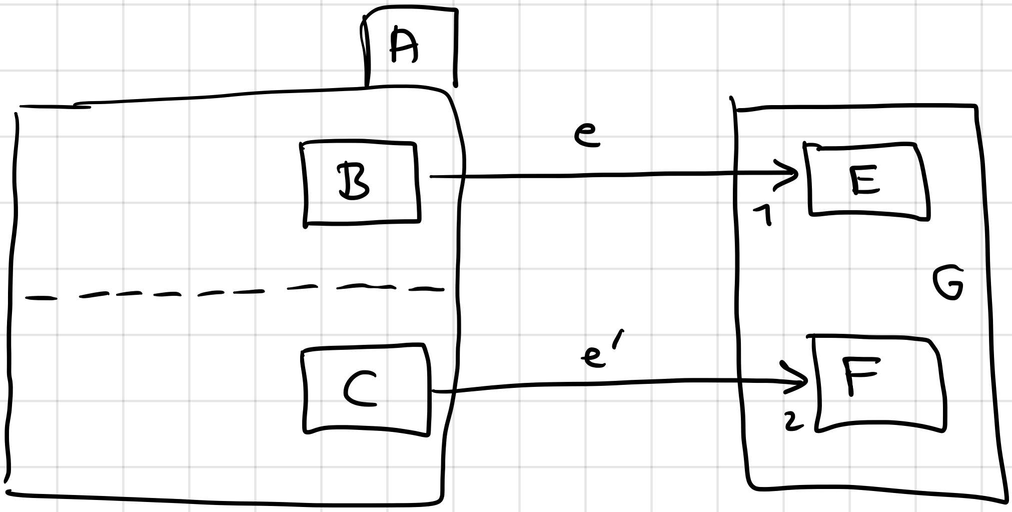


(1,2) is <u>not</u> consistent since from A only one edge can be taken

in state $(\{A, \dots\}, \{e, e', \dots\}, V)$

both 1, 2 are enabled, but only one of them can be taken

3



Edges (1,2) are inconsistent, as the targets E and F are both part of OR-node G.

Definition (Least common ancestor)

For $X \subseteq N$, the least common ancestor, denoted $lca(X)$, is the node $y \in N$ such that:

$$\underline{(\forall x \in X.\, x \trianglelefteq y)} \quad \text{and} \quad \forall z \in N.\, (\underline{\forall x \in X.\, x \trianglelefteq z}) \text{ implies } \underline{y \trianglelefteq z}.$$

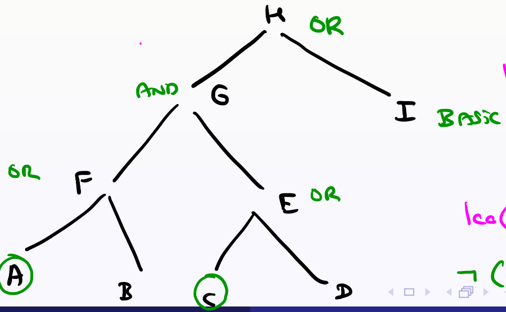
$lca\{A,C\} = G$

$type(G) = AND$

thus $A \perp C$

$F \perp E$

BASIC $\longrightarrow$



$X = \{C, D\}$

$lca(X) = E$

$X' = \{A, C, E\}$

$lca(X') = G$

$\neg(C \perp D)$

Least common ancestor

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For $X \subseteq N$, the least common ancestor, denoted $lca(X)$, is the node $y \in N$ such that:

$$(\forall x \in X.\, x \trianglelefteq y) \quad \text{and} \quad \forall z \in N.\, (\forall x \in X.\, x \trianglelefteq z) \text{ implies } y \trianglelefteq z.$$

Intuition

Node y is an ancestor of any node in X (first clause), and is a descendant of any node which is an ancestor of any node in X (second clause).

Orthogonality of nodes

Definition (Orthogonality of nodes)

Nodes $x, y \in N$ are orthogonal, denoted $x \perp y$, if

$$\neg(x \trianglelefteq y) \quad \text{and} \quad \neg(y \trianglelefteq x) \quad \text{and} \quad \mathit{type}(\mathit{lca}(\{\, x, y \,\})) = \text{AND}.$$

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Orthogonality captures the notion of independence. Orthogonal nodes can execute enabled edges independently, and thus concurrently.

Scope

Definition (Scope of edge)

The scope of edge $X \overset{\cdots}{\longrightarrow} Y$ is the most nested OR-node that is an ancestor of both X and Y.

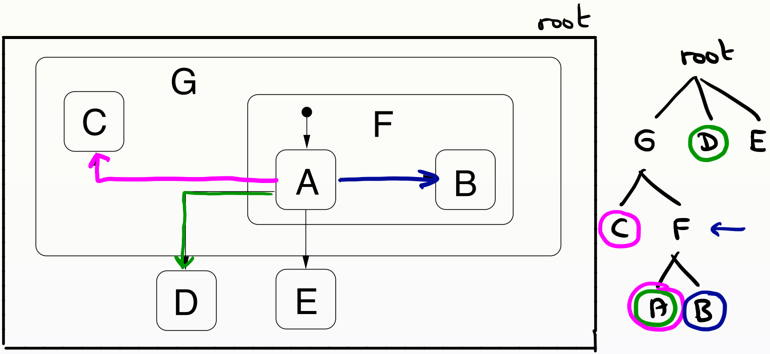
stated differently, A is not left by taking $X \overset{\cdots}{\longrightarrow} Y$.

A

Intuition

The scope of edge $X \overset{\cdots}{\longrightarrow} Y$ is the most nested OR-node that is unaffected by executing the edge $X \overset{\cdots}{\longrightarrow} Y$.

Scope: example



$scope(A \to D) = \text{root}$ and $scope(A \to C) = G$ and $scope(A \to B) = F$

Consistency: formal definition

Definition (Consistency)

1. Edges $ed, ed' \in Edges$ are consistent if:

$$ed = ed' \quad \text{or} \quad scope(ed) \perp scope(ed').$$

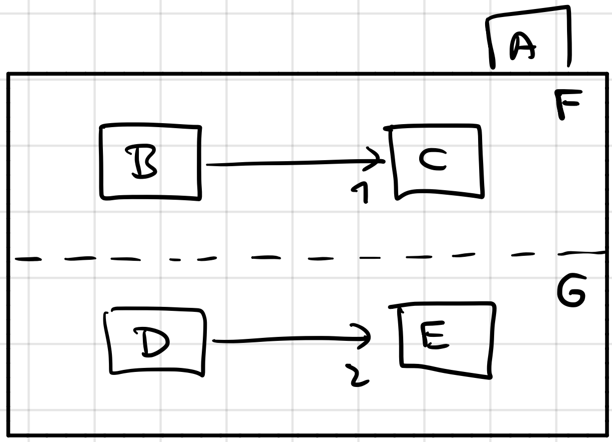
2. $T \subseteq Edges$ is consistent if all edges in T are pairwise consistent. $Cons(T)$ is the set of edges that are consistent with all edges in $T \subseteq Edges$

$$Cons(T) = \{ed \in Edges \mid \forall ed' \in T : ed \text{ is consistent with } ed'\}$$

Example

On the black board.

1



1 & 2 cons, as

$scope(1) = F$

$scope(2) = G$

$F \perp G$

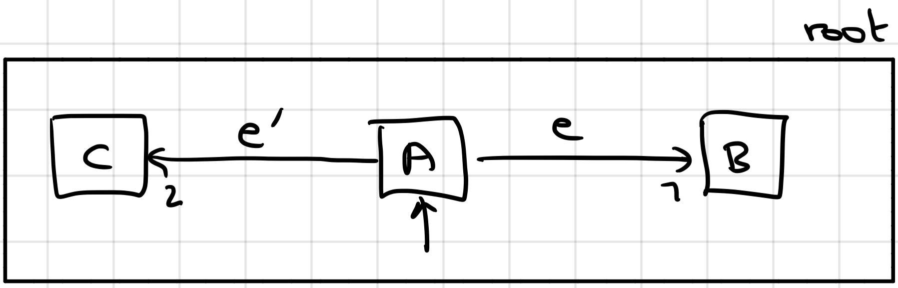
$lca(\{F, G\}) = A$

$type(A) = AND$

So, edges 1 and 2 are

<u>consistent</u>

2



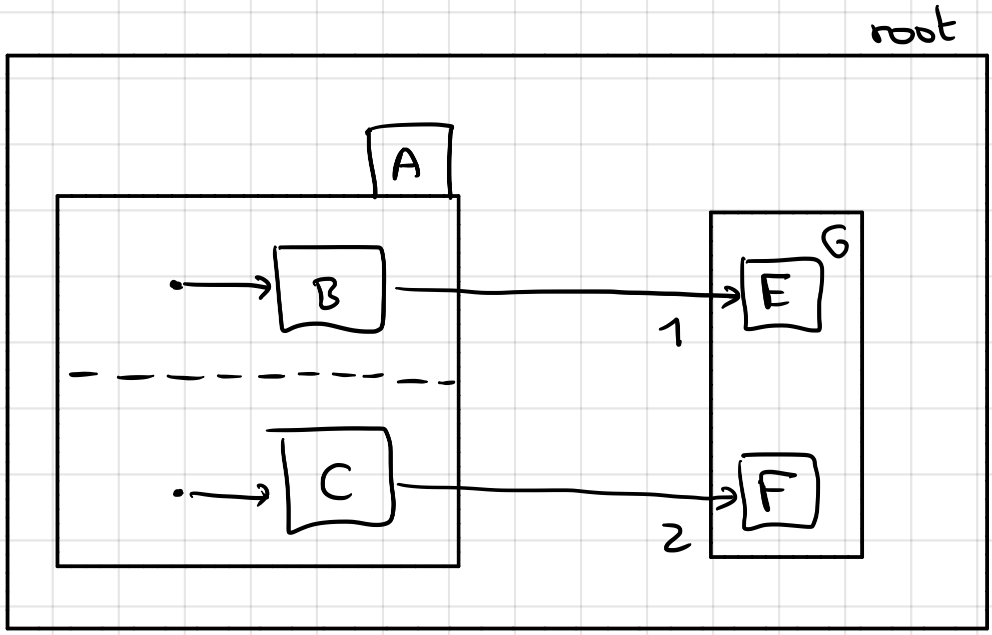
1, 2 is <u>not</u> consistent

$scope(1) = root = scope(2)$

$root \not\trianglelefteq root$

since $root \trianglelefteq root$

3



1, 2 is <u>not</u> consistent

$scope(1) = root$

$= scope(2)$

$root \not\trianglelefteq root$

What is now a macro step?

states $= (C, I, V)$

$$\big\downarrow T$$

(C', I', V')

A macro step is a set T of edges such that:

- all edges in step T are enabled

What is now a macro step?

A macro step is a set T of edges such that:

- all edges in step T are enabled
- all edges in T are pairwise consistent, that is:
  - they are identical or
  - scopes are (descendants of) different children of the same AND-node
- enabled edge ed is not in step T implies

  there exists $ed' \in T$ such that ed is inconsistent with ed', and the priority of ed' is not smaller than ed
- step T is maximal (wrt. set inclusion)

Overview

Priorities

Priorities restrict (but do not abandon) nondeterminism between multiple enabled edges.

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Definition (Priority relation)

The priority relation $\preceq \, \subseteq Edges \times Edges$ is a partial order defined for $ed, ed' \in Edges$ by:

$$\underline{ed \preceq ed'} \quad \text{if} \quad \underline{scope(ed')} \trianglelefteq \underline{scope(ed)}$$

So, ed' has priority over ed if its scope is a descendant of ed's scope.

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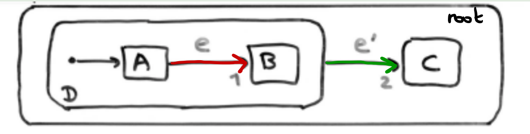
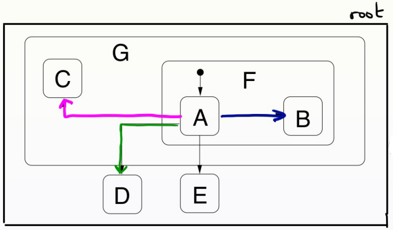
Example:



$2 \preceq 1$ since $scope(1) = D \trianglelefteq scope(2) = \text{root}$.

Priority: examples



$A \to D \qquad A \to C$

$scope(A \to D) = root$

$scope(A \to C) = G$

$G \trianglelefteq root$, so

$A \to D \preceq A \to C$

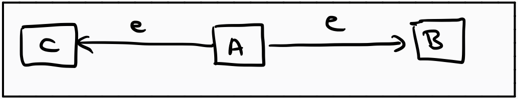
$scope(A \to B) = F$

$F \trianglelefteq G$

$A \to C \preceq A \to B$

Nondeterminism

Priorities rule out some nondeterminism, but not necessarily all.



What is now a macro step?

A macro step is a set T of edges such that:

- all edges in step T are enabled

$$(C, I, V) \xrightarrow{T \subseteq Edges} (C', I', V')$$

What is now a macro step?

A macro step is a set T of edges such that:

- all edges in step T are enabled
- all edges in T are pairwise consistent
  - they are identical or
  - scopes are (descendants of) different children of the same AND-node
- step T is maximal (wrt. set inclusion)
  - T cannot be extended with any enabled, consistent edge
- priorities: enabled edge ed is not in step T implies

  $\exists ed' \in T.\ (\underline{ed \text{ is inconsistent with } ed'} \wedge \underline{\neg(ed' \preceq ed)})$

A macro step — formally

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A macro step is a set T of edges such that:

- enabledness: $T \subseteq En(C, I, V)$

$$(C, I, V) \overset{T}{\rightsquigarrow} (C', I', V')$$

A macro step — formally

A macro step is a set T of edges such that:

- enabledness: $T \subseteq En(C, I, V)$
- consistency: $T \subseteq Cons(T)$
- maximality: $En(C, I, V) \cap Cons(T) \subseteq T$
- priority: $\forall ed \in En(C, I, V) - T$ we have $(\exists ed' \in T.\ (ed$ is inconsistent with $ed' \wedge \neg(ed' \preceq ed)))$ ← (*)

$T = f(T)$

Note:

The first three points yield: $T = En(C, I, V) \cap Cons(T)$. ← (*)

Computing the set T of macro steps in state (C, I, V)

function $\mathit{nextStep}(C, I, V)$

$T := \varnothing$

while $T \subset \mathit{En}(C, I, V) \cap \mathit{Cons}(T)$

do let $ed \in \mathit{High}\,(\underbrace{(\mathit{En}(C, I, V) \cap \mathit{Cons}(T))} - T)$;

$\quad T := T \cup \{ed\}$

not yet in T

od

return T.

where $\mathit{High}(T) \;=\; \{ed \in T \mid \neg(\exists ed' \in T.\, ed \preceq ed')\}$

Correctness

Theorem:

For any state (C, I, V), $nextStep(C, I, V)$ is a macro step.

Proof.

The proof goes in two steps:

1. We prove enabledness, consistency, and maximality by applying some standard results from fixed point theory, in particular Tarski's-Kleene fixpoint theorem;
2. Then we consider priority and use some monotonicity argument.

$\square$

Step execution

$(C, I, V) \xrightarrow{T} (C', I', V')$

What happens in performing a step?

For a single statechart, executing a step results in performing the actions of all the edges in the step, and changing "control" to the target nodes of these edges.

Interference

Actions in statechart SC_j may influence the sets of events of other statecharts, e.g., SC_i with $i \neq j$ if action *send i.e* is performed by SC_j in a step.

Thus:

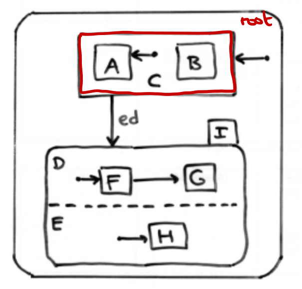
Execution of steps is considered on the system $(SC_1, \ldots, SC_n)$.

Default completion

Definition (Default completion)

The default completion C' of some set C of nodes is the canonical (unique) superset of C such that C' is a configuration. If C' contains an OR-node x and $children(x) \cap C = \varnothing$ implies $default(x) \in C'$.

Example:



AND

1. Default completion of

   $C_1 = \{\text{root}, I\}$ is $C' = C_1 \cup \{D, E, F, H\}$

2. Default completion of

   $C_2 = \{\text{root}, C\}$ is $C' = C_2 \cup \{A\}$.

Step execution by a single statechart

- Let C_j be the current configuration of statechart SC_j
- Let $T_j \subseteq \ Edges_j$ be a step for SC_j
- The next state (C'_j, I'_j, V'_j) of statechart SC_j is given by:
  1. C'_j is the default completion of

$$\bigcup_{X \xrightarrow{e[g]/A} Y \in T_j} \left(Y \ \cup \ \{x \in C_j \mid \forall X \rightarrow Y \in T_j . \underbrace{\neg(x \trianglelefteq scope(X \rightarrow Y))}_{\text{nodes that are unaffected by taking edge } X \rightarrow Y} \} \right)$$

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$$\bigcup_{X \xrightarrow{e[g]/A} Y \in T_j} Y \;\cup\; \{x \in C_j \mid \forall X \to Y \in T_j.\, \neg(x \trianglelefteq \mathit{scope}(X \to Y))\}$$

  2. $I'_j = \underbrace{\bigcup_{k=1}^{n}}_{\text{all } SC_k\text{'s}} \{e \mid \exists X \xrightarrow{e[g]/A} Y \in T_k.\, \mathit{send}\ j.e \in A\}$

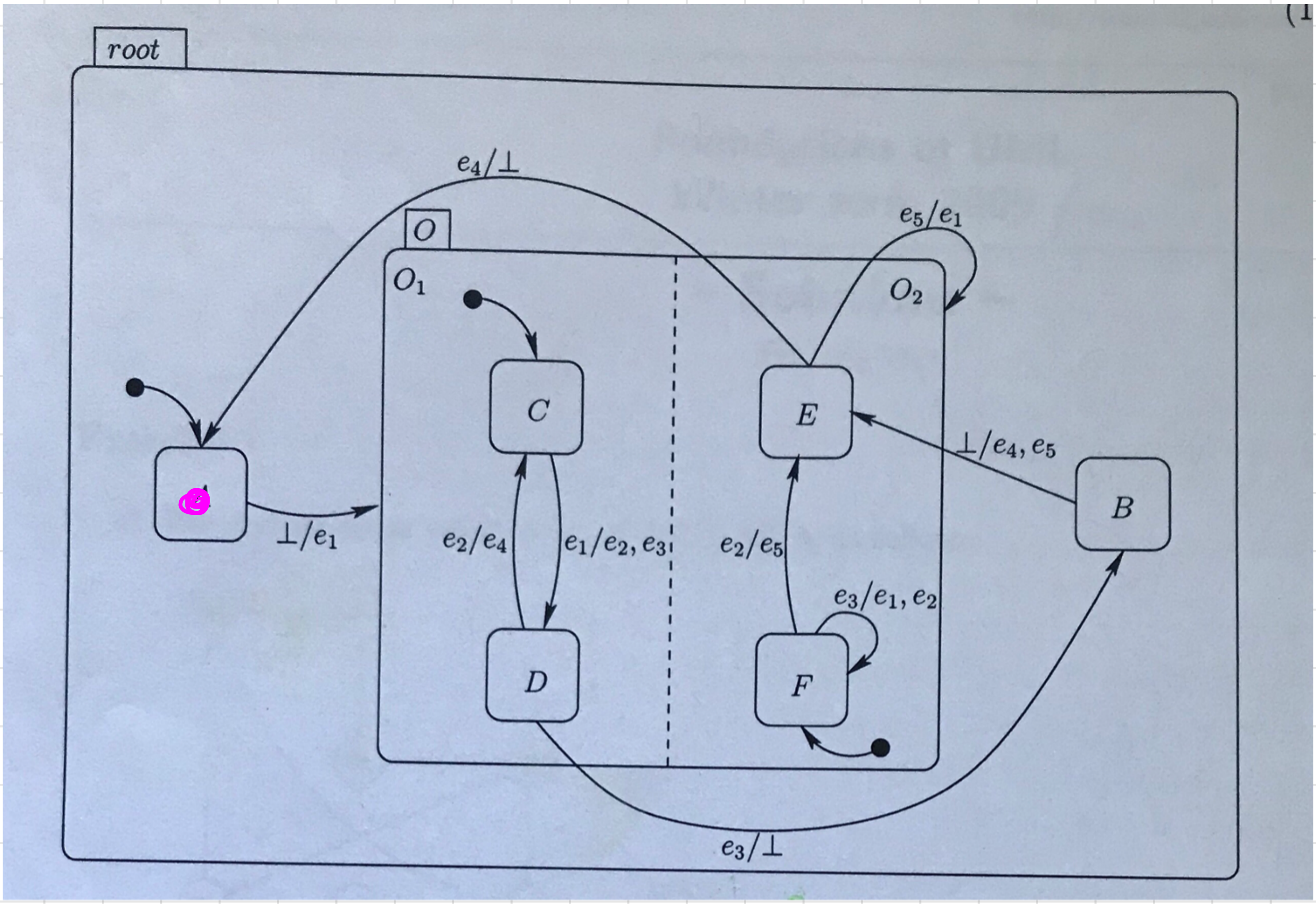
set of events available for the next macro steps

Step execution by a single statechart

- Let C_j be the current configuration of statechart SC_j
- Let $T_j \subseteq \mathit{Edges}_j$ be a step for SC_j
- The next state (C'_j, I'_j, V'_j) of statechart SC_j is given by:
  1. C'_j is the default completion of

$$\bigcup_{X \xrightarrow{e[g]/A} Y \in T_j} Y \;\cup\; \underline{\{x \in C_j \mid \forall X \rightarrow Y \in T_j.\, \neg(x \trianglelefteq \mathit{scope}(X \rightarrow Y))\}}$$

  2. $I'_j = \bigcup_{k=1}^{n} \{e \mid \exists X \xrightarrow{e[g]/A} Y \in T_k.\, \mathit{send}\ j.e \in A\}$
  3. $V'_j(v) = \begin{cases} V_j(v) & \text{if } \forall X \xrightarrow{e[g]/A} Y \in T_j.\, v := \ldots \notin A \\ \underline{\mathit{val}(\text{expr})} & \text{if } \exists X \xrightarrow{e[g]/A} Y \in T_j.\, v := \text{expr} \in A \end{cases}$



Variables are omitted $(C, I, V) \longrightarrow (C, I)$, where C = conf and $I \subseteq E$

default completion of root

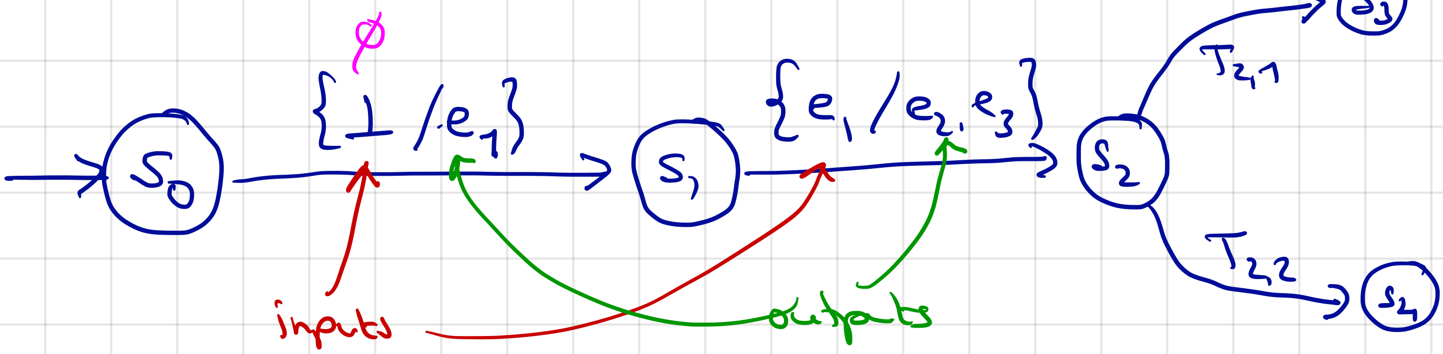
$n = 1$

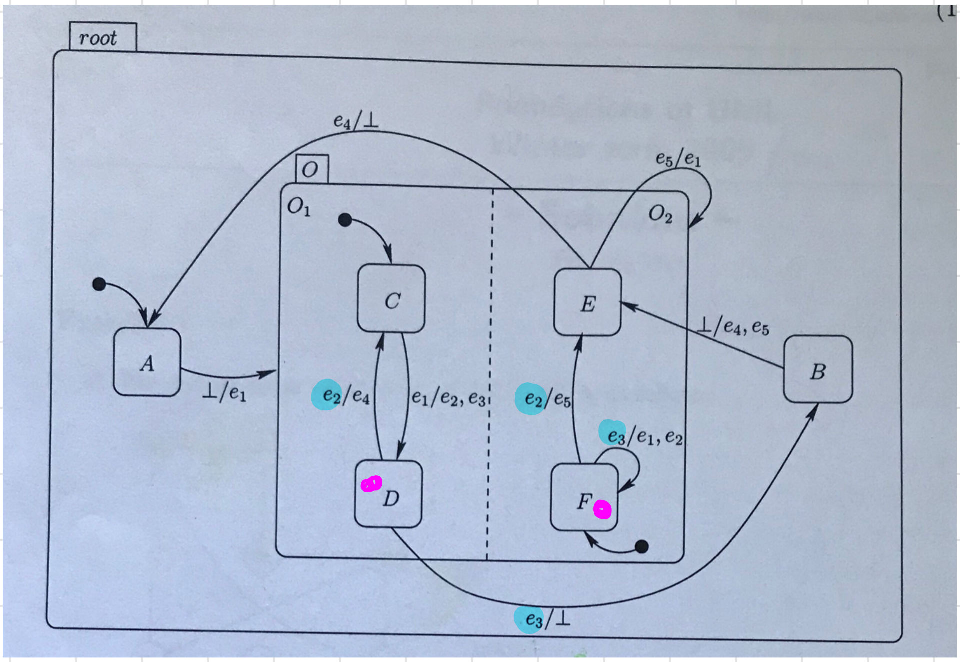
$$s_0 = (\{root, A\}, \emptyset)$$

$$En(s_0) = \{ A \xrightarrow{\perp/e_1} O \} = Step(s_0) = T_0$$

$$s_1 = (\{O, root, O_1, O_2, C, F\}, \{e_1\})$$

$$scope(A \to O) = root$$





$$s_1 = \left(\{ root, O, O_1, O_2, C, F \}, \{ \ \} \right)$$

$$step(s_1) = \{ C \xrightarrow{e_1/e_2, e_3} \underline{D} \}$$

$$s_2 = \left(\{ D, root, O, O_1, O_2, F \}, \{ e_2, e_3 \} \right)$$

$$En(s_2) = \{ \underbrace{D \to C, D \to B}_{\text{inconsistent}}, \underbrace{F \to F, F \to E}_{\text{inconsistent}} \}$$

$$step(s_2) = \{ \{ D \to C, F \to F \}, \{ D \to C, F \to E \} \}$$

the edge $D \to B$ has a lower priority than $D \to C$

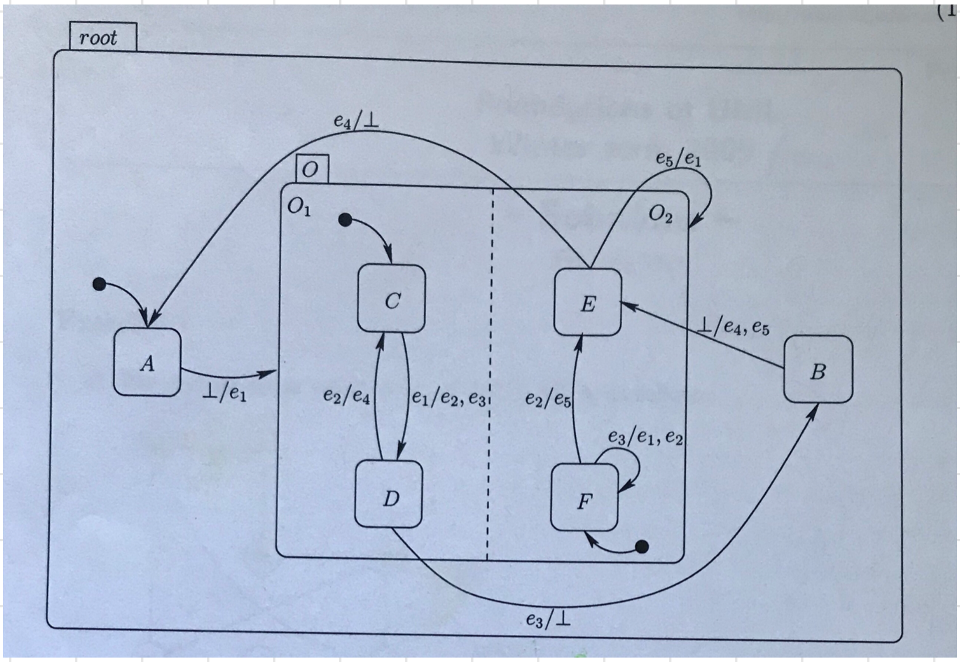
as $scope(D \to B) = root$, $scope(D \to C) = O_1$

$O_1 \trianglelefteq root$, thus $D \to B \preceq D \to C$

so $step(s_2) = \{ T_{2,1}, T_{2,2} \}$

Take $T_{2,1}$ in state s_2 yields the state

$$s_3 = \left(\{ C, F, root, O, O_1, O_2 \}, \{ e_1, e_2, e_4 \} \right)$$

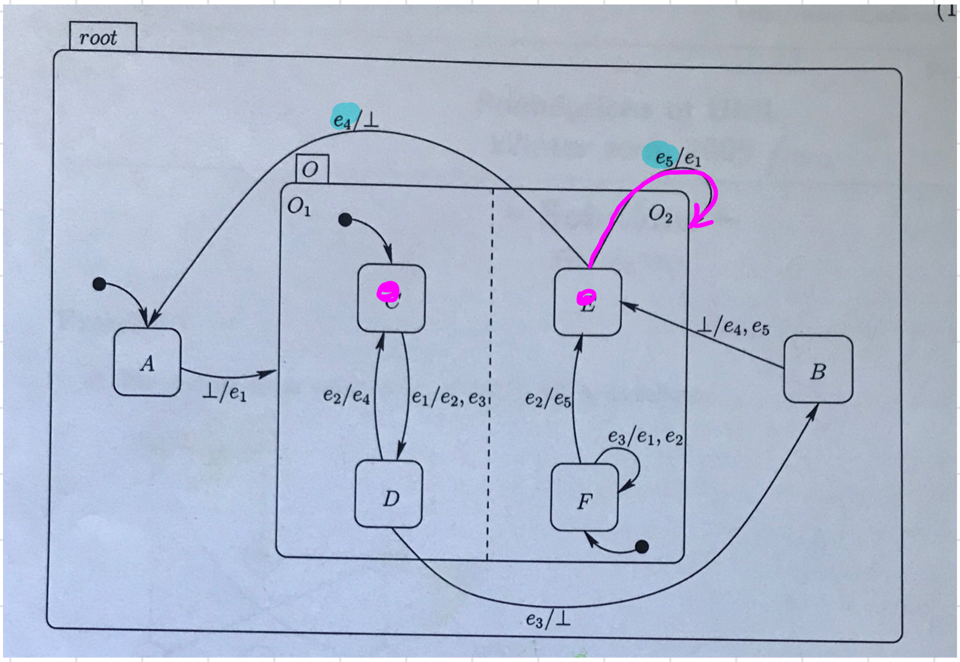


Take $T_{2,2}$ in state s_2:

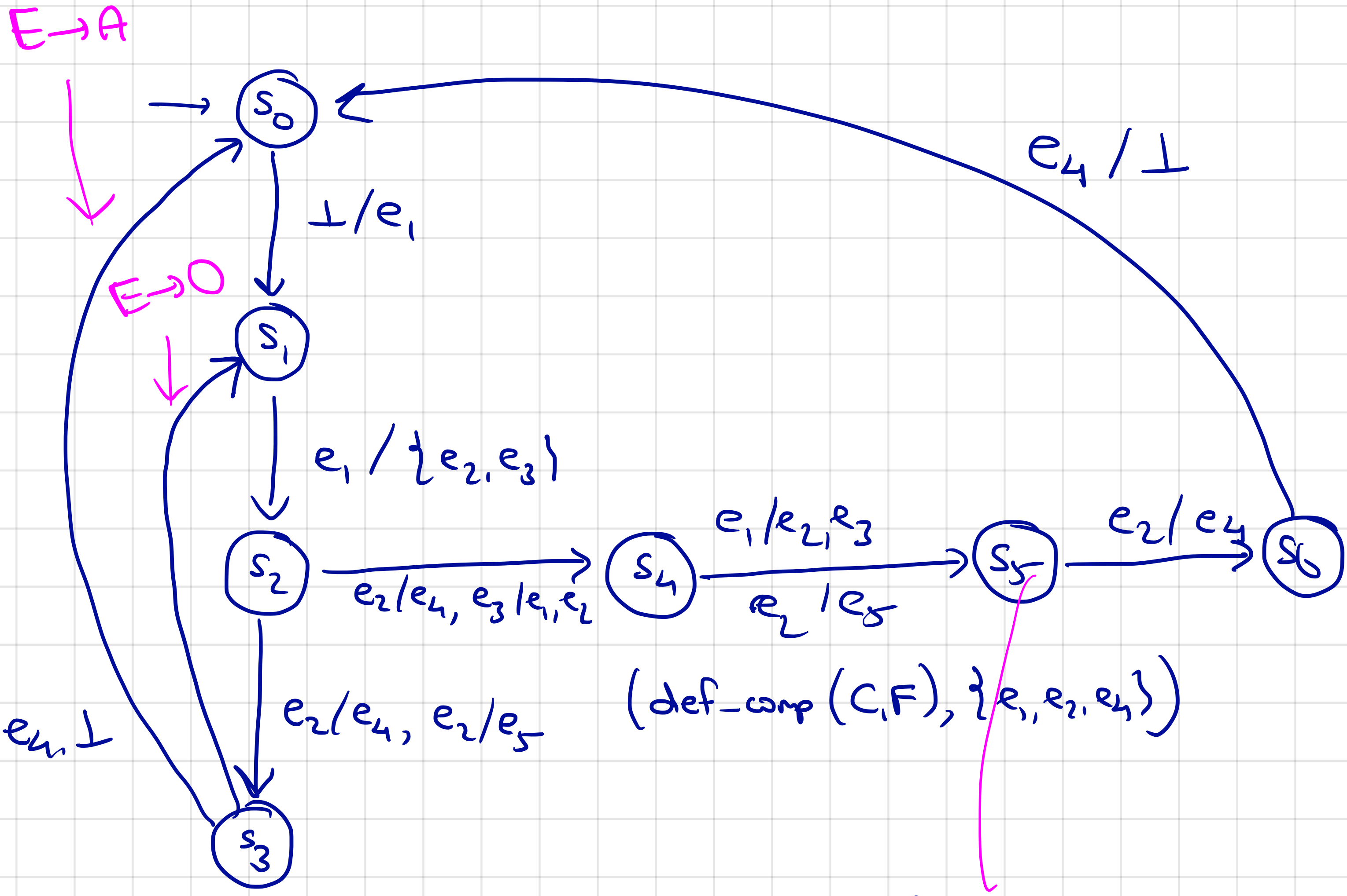
$\hookrightarrow = \{(D \to C, F \to E)\}$

yields the state

$$(\{C, E, root, O, O_1, O_2\}, \{e_4, e_5\}) = S_4$$



Semantics of this statechart is



(def-compl (C,E), $\{e_4, e_5\}$)

(def-comp (D,E), $\{e_1, e_3, e_5\}$)

$s_6 =$ (def-compl (C,E), $\{e_4\}$)

Mealy machines [Mealy, 1953]

Definition (Mealy machine)

A Mealy machine $\mathcal{A} = (Q, q_0, \Sigma, \Gamma, \delta, \omega)$ with:

- Q is a finite set of states with initial state $q_0 \in Q$
- Σ is the input alphabet
- Γ is the output alphabet
- $\delta : Q \times \Sigma \rightarrow Q$ is the deterministic (input) transition function, and
- $\omega : Q \times \Sigma \rightarrow \Gamma$ is the output function

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Intuition

A Mealy machine (or: finite-state transducer) is a finite-state automaton that produces output on a transition, based on current input and state.

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Intuition

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Moore machines

In a Moore machine $\omega : Q \to \Gamma$, output is purely state-based.

From statecharts to a Mealy machine (1)

$n=1$ $\quad q = (C, I, V)$

States

A state q is a tuple of the (local) states of SC_1 through SC_n.

Input and output events

Any input is a set of events, and any output is a set of events.

Next-state function δ

Defines the effect of executing a step.

Output function ω

Defines all events sent to some SC outside the system $(SC_1, \ldots, SC_n)$.

From statecharts to a Mealy machine (2)

States

A state q is a tuple of the (local) states of SC_1 through SC_k.

Formally:

- $Q = \prod_{k=1}^{n} (Conf_k \times 2^{E_k} \times Val_k)$ is the set of states
  - where $Conf_k$ is the set of configurations of SC_k,
  - E_k is the set of the events of SC_k,
  - and Val_k is the set of variable valuations of SC_k

From statecharts to a Mealy machine (2)

States

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Formally:

- $Q = \prod_{k=1}^{n} (\mathit{Conf}_k \times 2^{E_k} \times \mathit{Val}_k)$ is the set of states
  - where Conf_k is the set of configurations of SC_k,
  - E_k is the set of the events of SC_k,
  - and Val_k is the set of variable valuations of SC_k
- $q_0 = \prod_{k=1}^{n} (C_{0,k}, \varnothing, \mathit{Val}_{0,k})$ is the initial state
  - where $C_{0,k}$ is the default completion of the set $\{\text{root}\}$
  - the initial set of events is empty
  - $\mathit{Val}_{0,k}$ is the initial variable valuation of SC_k

From statecharts to a Mealy machine (3)

Input and output events

Any input is a set of events, and any output is a set of events.

Formally,

- Input alphabet: $\Sigma = 2^E - \{ \varnothing \}$
  - where $E = \bigcup_{k=1}^{n} E_k$ is the set of events in all statecharts
- Output alphabet: $\Gamma = 2^{E'}$
  - with $E' = \underbrace{\left\{ send\ j.e \in \bigcup\limits_{k=1}^{n} SC_k \mid j \notin \{1, \ldots, n\} \right\}}_{\text{all outputs that cannot be consumed}}$

From statecharts to a Mealy machine (4)

Next-state function δ

Defines the effect of executing a step.

Formally,

- $(s'_1, \ldots, s'_n) \in \delta((s_1, \ldots, s_n), E)$ where
  - $s''_i = (C'_i, I''_i, V'_i)$ is the next state after executing some $T_i = \mathit{nextStep}(C_i, I_i, V_i)$
  - and $s'_i = (C'_i, I''_i \cup (E \cap E_i), V'_i)$

From statecharts to a Mealy machine (5)

Output function ω

Defines all events sent to some SC outside the system $(SC_1, \ldots, SC_n)$.

Formally,

- $\omega((s_1, \ldots, s_n), E) =$
$$\left\{ \text{send } j.e \mid j \notin \{1, \ldots, n\} \wedge \exists i. \exists X \xrightarrow{e[g]/\text{send } j.e} Y \in nextStep(C_i, I_i, V_i) \right\}$$