Theoretical Foundations of the UML

Lecture 8: Bounded MSCs and CFMs

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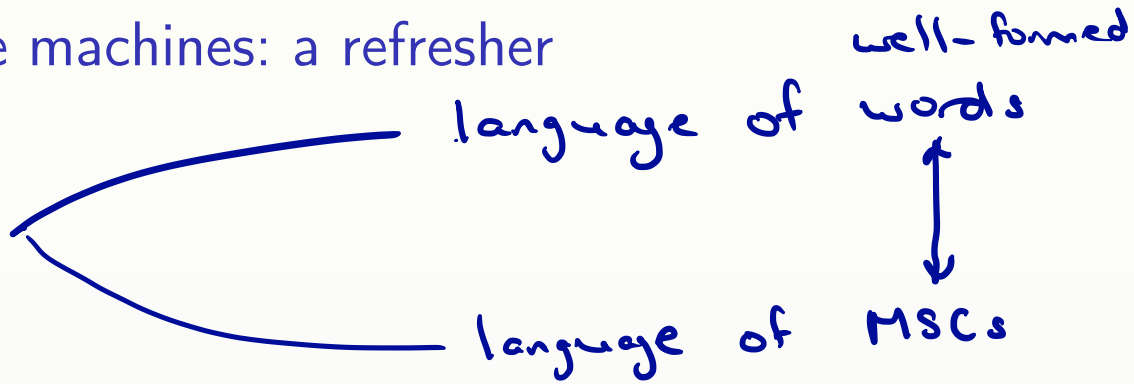
Outline

1. Communicating finite-state machines: a refresher
2. Well-formedness of CFMs
3. Bounded CFMs
   - Bounded words ✓
   - Bounded MSCs ✓
   - Bounded CFMs ✓



bounded = "bound the capacity of the communication channels of a CFM"

Overview

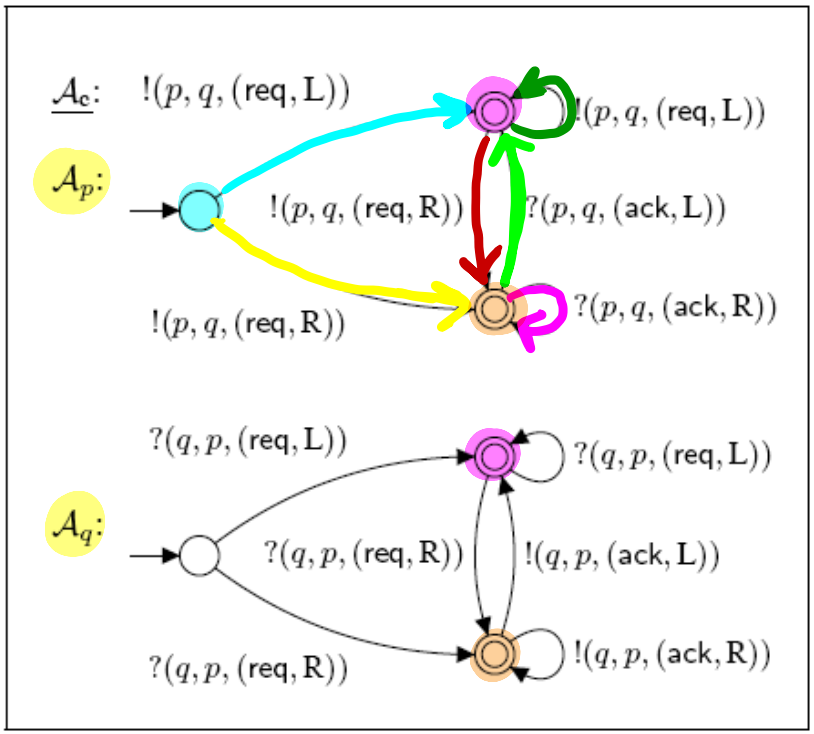
1. Communicating finite-state machines: a refresher
2. Well-formedness of CFMs
3. Bounded CFMs
   - Bounded words
   - Bounded MSCs
   - Bounded CFMs

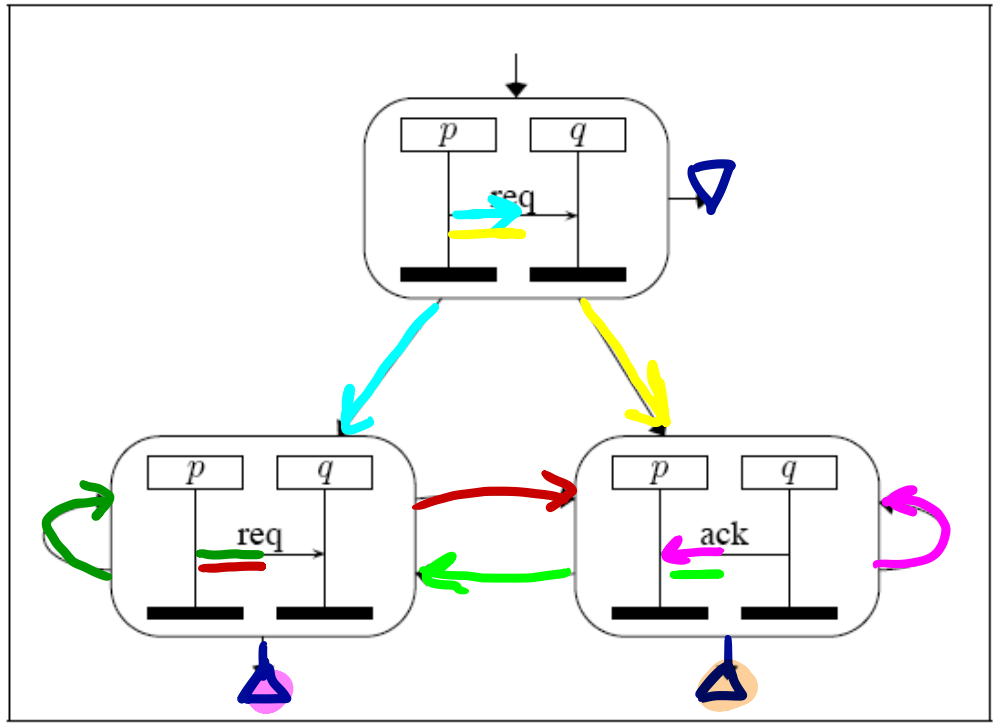
Communicating finite-state machines

- A communicating finite-state machine (CFM) is a collection of finite-state machines, one for each process
- Communication between these machines takes place via (a priori) unbounded reliable FIFO channels
- The underlying system architecture is parametrised by the set $\mathcal{P}$ of processes and the set $\mathcal{C}$ of messages
- Action $!(p, q, m)$ puts message m at the end of the channel (p, q)
- Action $?(q, p, m)$ is enabled only if m is at head of buffer, and its execution by process q removes m from the channel (p, q)
- Synchronisation messages are used to avoid deadlocks

Example communicating finite-state machine

CFM "implements" MSG





final

This CFM accepts if $\mathcal{A}_p$ and $\mathcal{A}_q$ are in some local state, and (as usual) all channels are empty

Formal definition

Definition (What is a CFM?)

A communicating finite-state machine (CFM) over $\mathcal{P}$ and $\mathcal{C}$ is a tuple

$$\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$$

local automata

sync data $(\mathbb{L}, \mathbb{R})$

global initial state

global final states

where

- for each $p \in \mathcal{P}$:

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- for each $p \in \mathcal{P}$:
  - S_p is a non-empty finite set of local states (the S_p are disjoint)
  - $\Delta_p \subseteq S_p \times Act_p \times \mathbb{D} \times S_p$ is a set of local transitions

$(s, \sigma, m, s') \in \Delta_p$

$s \xrightarrow{\sigma, m}_p s'$

!(p,q,m)
?(p,q,m)

In sequel, let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

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- $F \subseteq S_{\mathcal{A}}$ is the set of global final states

In sequel, let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Formal semantics of CFMs

Definition (Configuration)

Configurations of $\mathcal{A}$: $Conf_{\mathcal{A}} := S_{\mathcal{A}} \times \{\eta \mid \eta : Ch \to (\mathcal{C} \times \mathbb{D})^*\}$

global state

content of all channels

$\eta((p,q)) =$ (ack, R) . (ack, L) (req, R) (req, L)

Formal semantics of CFMs

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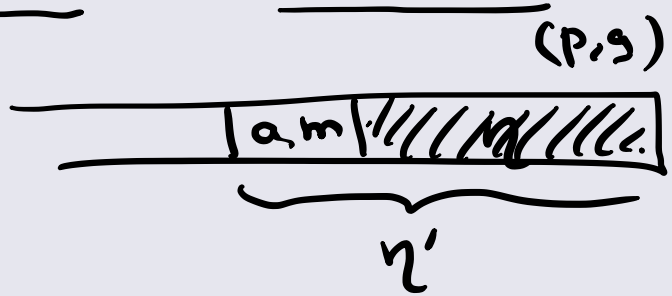
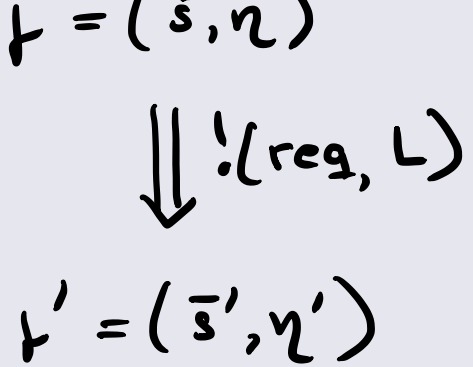
Definition (Transitions between configurations)

$\Longrightarrow_{\mathcal{A}} \subseteq Conf_{\mathcal{A}} \times Act \times \mathbb{D} \times Conf_{\mathcal{A}}$ is defined as follows:

- sending a message: $((\overline{s}, \eta), !(p,q,a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], !(p,q,a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta' = \eta[(p,q) := (a,m) \cdot \eta((p,q))]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$





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  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$
- receipt of a message: $((\overline{s}, \eta), ?(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], ?(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta((q, p)) = w \cdot (a, m) \neq \epsilon$ and $\eta' = \eta[(q, p) := w]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

Linearizations of a CFM

Definition ((Accepting) Runs)

A run of $\mathcal{A}$ on $\sigma_1 \dots \sigma_n \in Act^*$ is a sequence $\rho = \gamma_0\, m_1\, \gamma_1 \dots \gamma_{n-1}\, m_n\, \gamma_n$ such that

- $\gamma_0 = (s_{init}, \eta_\varepsilon)$ with η_ε mapping any channel to ε
- $\gamma_{i-1} \overset{\sigma_i, m_i}{\Longrightarrow}_{\mathcal{A}} \gamma_i$ for any $i \in \{1, \dots, n\}$

previous slide

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- $\gamma_{i-1} \xRightarrow{\sigma_i, m_i}_{\mathcal{A}} \gamma_i$ for any $i \in \{1, \dots, n\}$

Run ρ is accepting if $\gamma_n \in F \times \{\eta_\varepsilon\}$.

↓

is a final global state

Linearizations of a CFM

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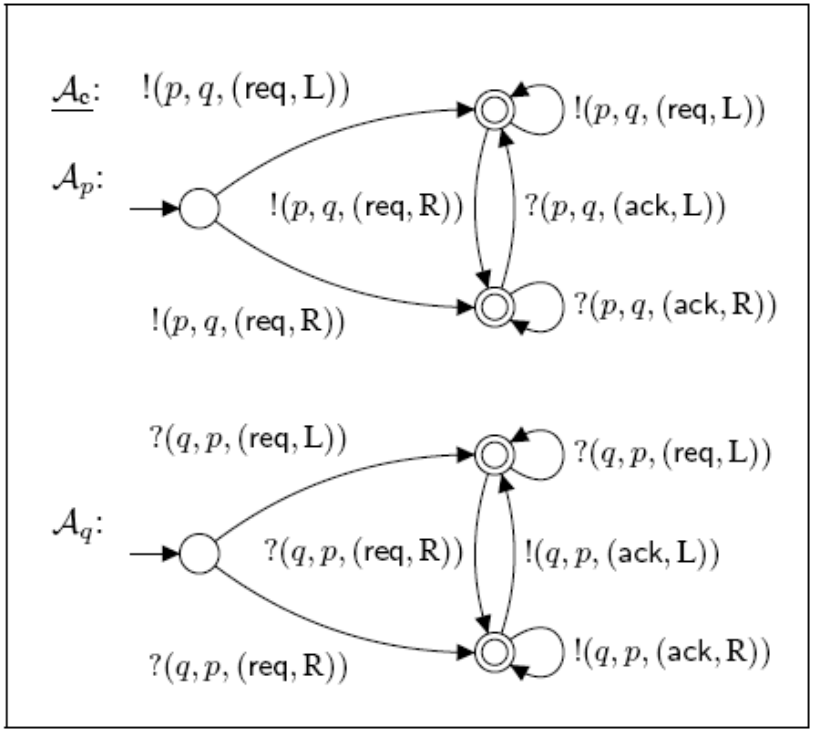
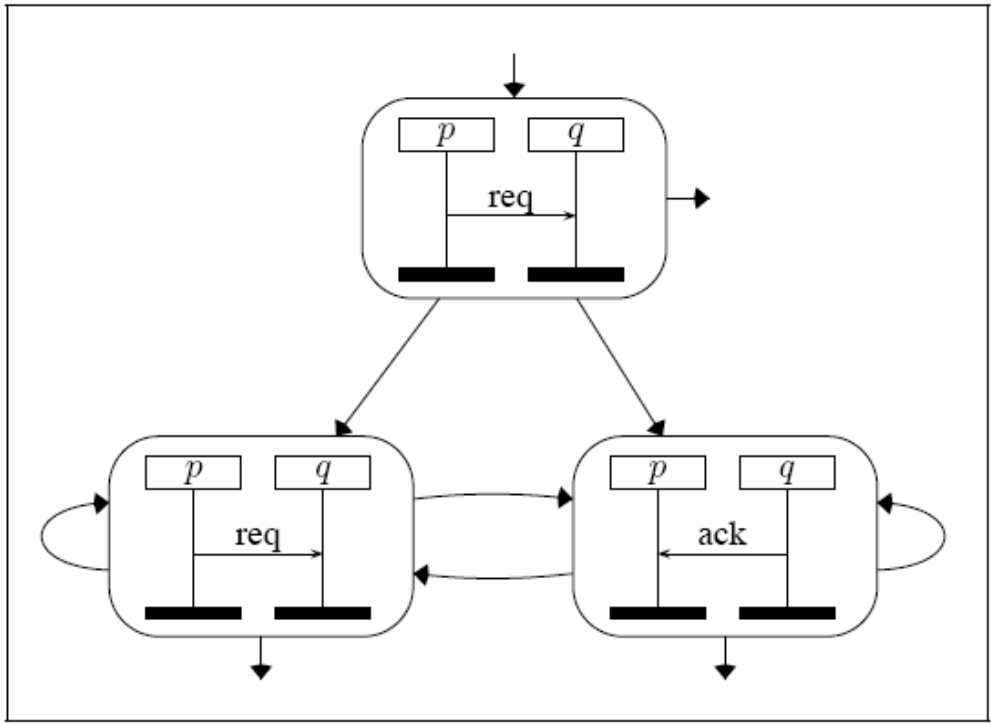
Definition (Linearizations)

The set of linearizations of CFM $\mathcal{A}$:

$Lin(\mathcal{A}) := \{w \in Act^* \mid \text{there is an accepting run of } \mathcal{A} \text{ on } w\}$

Example communicating finite-state machine





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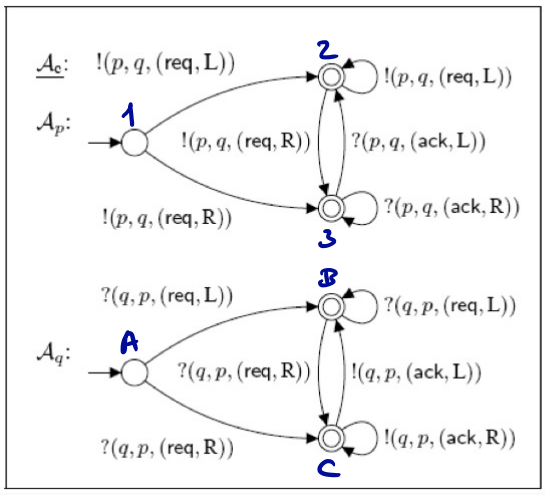

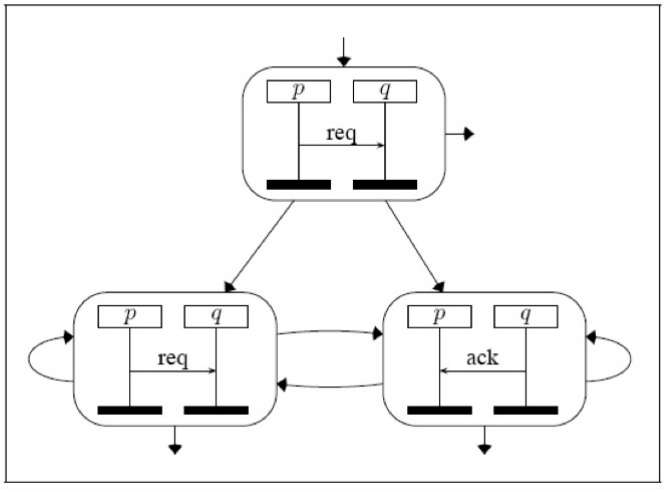

initial configuration:

$\eta((p,q))$ $\eta((q,p))$

$((1, A), \varepsilon, \varepsilon)$

$\Downarrow$!(req, R)

$((3, A), (req, R), \varepsilon)$

$\Downarrow$?(req, R)

$((3, C), \varepsilon, \varepsilon)$

$\Downarrow$!(ack, L)

$((3, B), \varepsilon, (ack, L))$

non-accepting run

Overview

Well-formedness (reminder)

Let $Ch := \{(p, q) \mid p \neq q\,,\ p, q \in \mathcal{P}\}$ be a set of channels over $\mathcal{P}$.

We call $w = a_1 \ldots a_n \in Act^*$ proper if

1. every receive in w is preceded by a corresponding send, i.e.:
   $\forall (p, q) \in Ch$ and prefix u of w, we have:

$$\underbrace{\sum_{m \in \mathcal{C}} |u|_{!(p,q,m)}}_{\#\text{ sends from } p \text{ to } q} \geqslant \underbrace{\sum_{m \in \mathcal{C}} |u|_{?(q,p,m)}}_{\#\text{ receipts by } q \text{ from } p}$$

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   where $|u|_a$ denotes the number of occurrences of action a in u

2. the FIFO policy is respected, i.e.:
   $\forall 1 \leqslant i < j \leqslant n$, $(p,q) \in Ch$, and $a_i = !(p,q,m_1)$, $a_j = ?(q,p,m_2)$:

$$\sum_{m \in \mathcal{C}} |a_1 \ldots a_{i-1}|_{!(p,q,m)} = \sum_{m \in \mathcal{C}} |a_1 \ldots a_{j-1}|_{?(q,p,m)} \quad \text{implies} \quad m_1 = m_2$$

sends from p→q # receives at q from p

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A proper word w is well-formed if $\sum_{m \in \mathcal{C}} |w|_{!(p,q,m)} = \sum_{m \in \mathcal{C}} |w|_{?(q,p,m)}$

Well-formedness and CFMs

{ $w \in Act^*$ | there is an accepting run of $\mathcal{A}$ on w }

Lemma

For any CFM $\mathcal{A}$ and $w \in Lin(\mathcal{A})$, w is well-formed.

Recall that there is a strong correspondence between well-formed linearizations and MSCs.

From linearizations to partial orders (reminder)

Associate to $w = a_1 \dots a_n \in Act^*$ an Act-labelled poset

$$M(w) = (E, \preceq, \ell)$$

$\ell : E \to Act$

such that:

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  - $i \prec_p j$ if and only if $i < j$ for any $i, j \in E_p$
  - $i \prec_{\text{msg}} j$ if for some $(p, q) \in Ch$ and $m \in \mathcal{C}$ we have:

$$\ell(i) = !(p, q, m) \text{ and } \ell(j) = ?(q, p, m) \text{ and}$$
$$\sum_{m \in \mathcal{C}} |a_1 \dots a_{i-1}|_{!(p,q,m)} = \sum_{m \in \mathcal{C}} |a_1 \dots a_{j-1}|_{?(q,p,m)}$$

CFMs and well-formed words

Relating well-formed words to MSCs

For any well-formed word $w \in Act^*$, $M(w)$ is an MSC.

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Definition (MSC language of a CFM)

For CFM $\mathcal{A}$, let $\mathcal{L}(\mathcal{A}) = \{ M(w) \mid w \in Lin(\mathcal{A}) \}$.

MSCs

equivalent

two views on a CFM

- CFM accepts finite words ($Lin(\mathcal{A})$)

- CFM accepts MSCs ($\mathcal{L}(\mathcal{A})$)

CFMs and well-formed words

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Relating well-formed words to CFMs

For any well-formed words u and v with $M(u)$ is isomorphic to $M(v)$:

for any CFM $\mathcal{A}$: $u \in \mathcal{L}(\mathcal{A})$ iff $v \in \mathcal{L}(\mathcal{A})$.

$M(u) \in \mathcal{L}(A)$ iff $M(v) \in \mathcal{L}(A)$

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   - Bounded words
   - Bounded MSCs
   - Bounded CFMs

Emptiness problem is undecidable for CFMs

Theorem: [Brand & Zafiropulo 1983]

The following (emptiness) problem:

INPUT: CFM $\mathcal{A}$ over processes $\mathcal{P}$ and message contents $\mathcal{C}$
QUESTION: Is $\mathcal{L}(\mathcal{A})$ empty?

is undecidable.

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The following (emptiness) problem:

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is undecidable. (Even if $\mathcal{C}$ is a singleton set).

Restrictions on CFMs

- So: most elementary problems for CFMs are undecidable.
- This is (very) unsatisfactory.
- Main cause: presence of channels with unbounded capacity
- Consider restricted versions of CFMs by bounding the channel capacities. (size)
- Thus: we fix the channel capacities a priori.

Restrictions on CFMs

- So: most elementary problems for CFMs are undecidable.
- This is (very) unsatisfactory.
- Main cause: presence of channels with unbounded capacity
- Consider restricted versions of CFMs by bounding the channel capacities.
- Thus: we fix the channel capacities a priori.
- This yields:
  - universally bounded CFMs: all runs need a finite buffer capacity
  - existentially bounded CFMs: some runs need a finite buffer capacity possibly, some runs may still need unbounded buffers.

We define bounded CFMs, by first considering bounded words and bounded MSCs. Bounded CFMs will then "generate" bounded MSCs.

aka: accept

Bounded words

Definition (B-bounded words)

Let $B \in \mathbb{N}$ and $B > 0$. A word $w \in Act^*$ is called B-bounded if for any prefix u of w and any channel $(p, q) \in Ch$:

$$0 \leqslant \sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \leqslant B$$

FIFO

$\sum_{a \in \mathcal{C}} |u|_{!(p,q,a)}$ = # sends from $p \to q$ in prefix u

$\sum_{a \in \mathcal{C}} |u|_{?(q,p,a)}$ = # receives at q from p in prefix u

= #sends from $p \to q$ in u that have not been received yet in u

there are at most B pending sends from p to q in prefix u

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Intuition

Word w is B-bounded if for any pair of processes (p, q), the number of sends from p to q cannot be more than B ahead of the number of receipts by q from p (for every message a).

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this prefix violates (*) for B=1

Example

$!(1,2,a)\ !(1,2,b)\ ?(2,1,a)\ ?(2,1,b)$ is 2-bounded but not 1-bounded.

2 1 0

Example

$$w = \underbrace{!(p,q,a)\ !(p,q,a)\ ?(q,p,a)}_{1}\ \underbrace{!(p,q,a)}_{2}\ !(q,p,b)\ ?(q,p,a)$$

$$\underbrace{!(p,q,a)}_{1}\ \underbrace{!(p,q,a)}_{2}$$

2	(p,q)	1 (p,q)
1	(q,p)	1 (q,p)

Claim: w is 3 bounded $\quad B = 3$

prefix u

- $\varepsilon =$ ✓
- $!(p,q,a) = 1$
- $!(p,q,a)\ !(p,q,a) = 2$
- $!(p,q,a)\ !(p,q,a)\ ?(q,p,a) = 1$

Claim w is 2-bounded

Proposition if w is B-bounded

then w is $(B+1)$-bounded

Typically, we are interested in determining the minimal B for which w is B-bounded.

Bounded MSCs

Definition (Universally bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called universally B-bounded ($\forall B$-bounded, for short) if

$$Lin(M) = Lin^B(M)$$

all linearizations of MSC M are B-bounded

where $Lin^B(M) := \{w \in Lin(M) \mid w \text{ is } B\text{-bounded}\}$.

Bounded MSCs

Definition (Universally bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called universally B-bounded ($\forall B$-bounded, for short) if

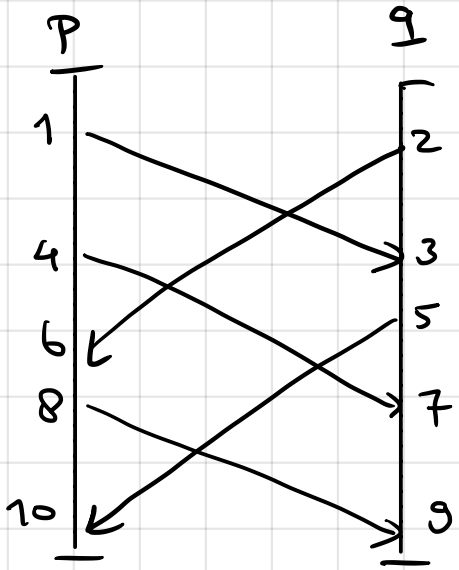
$$Lin(M) \;=\; Lin^{B}(M)$$

where $Lin^{B}(M) := \{w \in Lin(M) \mid w \text{ is } B\text{-bounded}\}$.

Intuition

MSC M is $\forall B$-bounded if all its linearizations are B-bounded.

MSC M



Claim MSC M is

∀3-bounded

∃2-bounded

∃3-bounded

Lin (M) = { 1 2 3 4 6 5 8 7 10 9 , 1 4 2 6 8 3 5 10 7 9 , ... }

2-bounded

3-bounded

(p,q)	0	1	1	0	1	1	1	(2)	1	1	0
		1	2	3	4	6	5	8	7	10	9
(q,p)	0	0	1	1	1	0	1	1	1	0	0

(p,q)	0	1	2	2	2	(3)	2	2	2	1	0
		1	4	2	6	8	3	5	10	7	9
(q,p)	0	0	0	1	0	0	0	1	0	0	0

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Intuition

MSC M is $\forall B$-bounded if all its linearizations are B-bounded.

So: if M is $\forall B$-bounded, then a buffer capacity B is sufficient for all possible runs of MSC M.

Bounded MSCs

Definition (Existentially bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called existentially B-bounded ($\exists B$-bounded, for short) if $Lin(M) \cap Lin^B(M) \neq \varnothing$.

B-bounded linearisations

Proposition if MSC M is $\exists B$-bounded, then it is $\exists(B+1)$-bounded.

Similarly for $\forall B$-bounded MSCs

Bounded MSCs

Definition (Existentially bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called existentially B-bounded ($\exists B$-bounded, for short) if $Lin(M) \cap Lin^B(M) \neq \varnothing$.

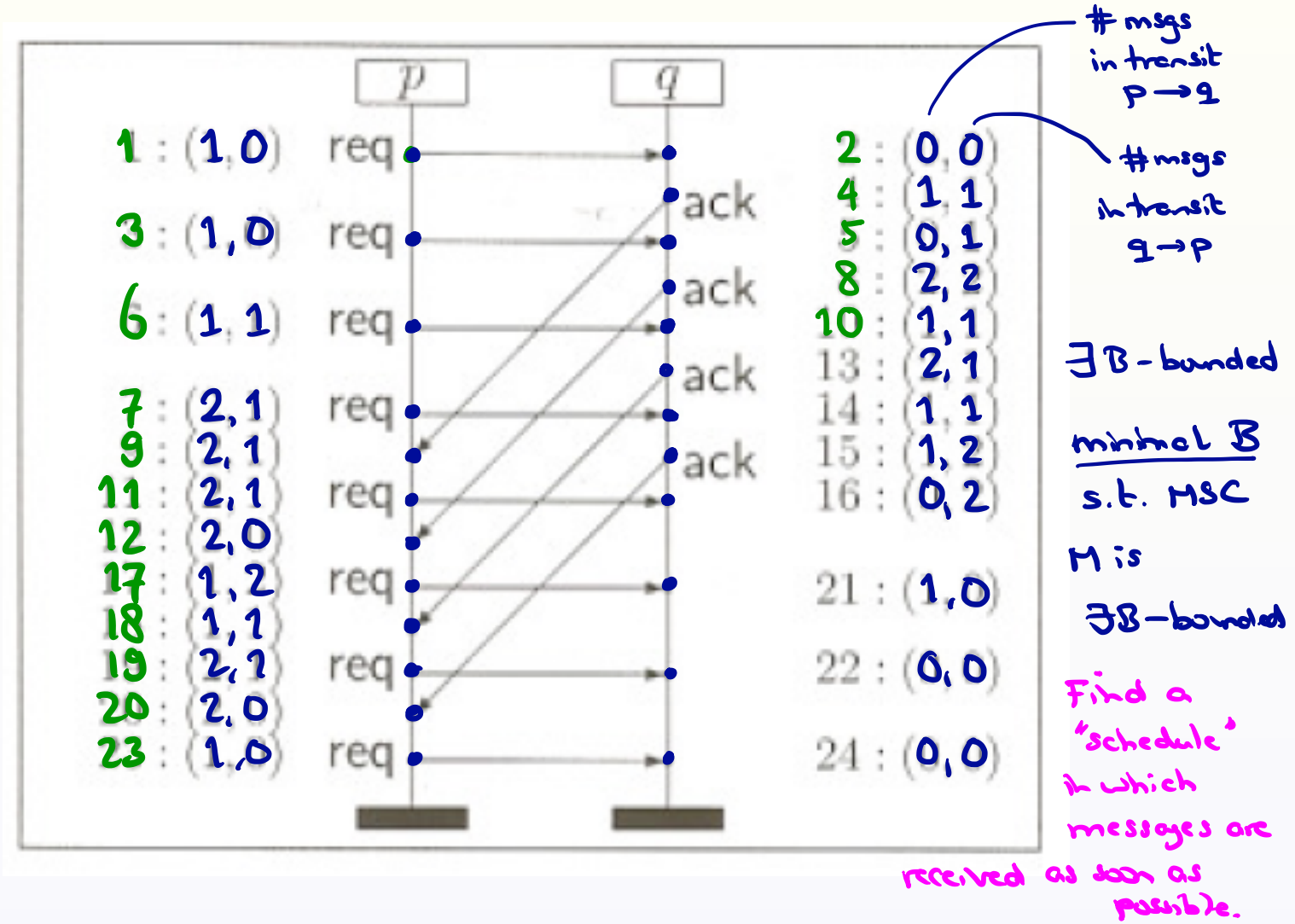
Intuition

MSC M is $\exists B$-bounded if at least one linearization of M is B-bounded.

Consequence

The events of an $\exists B$-bounded MSC M can be "scheduled" in such a way that no channel ever contains more than B messages.

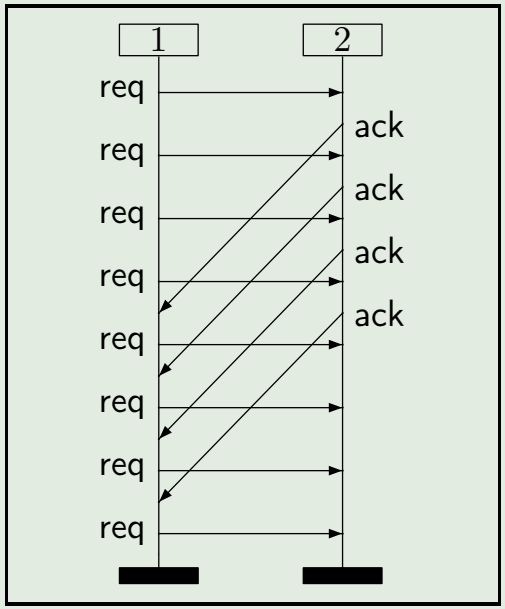
Bounded MSCs

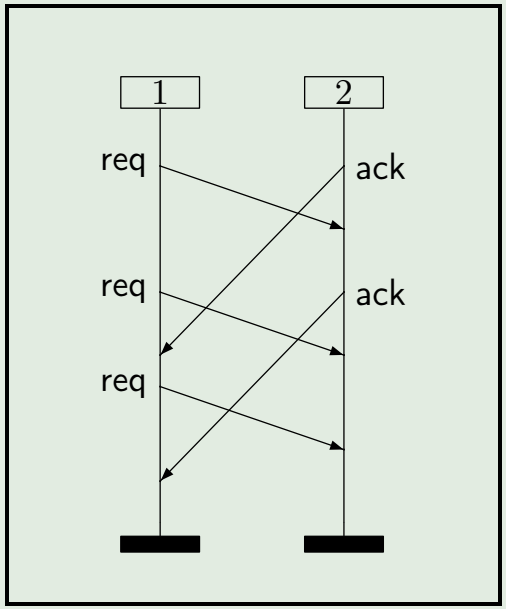


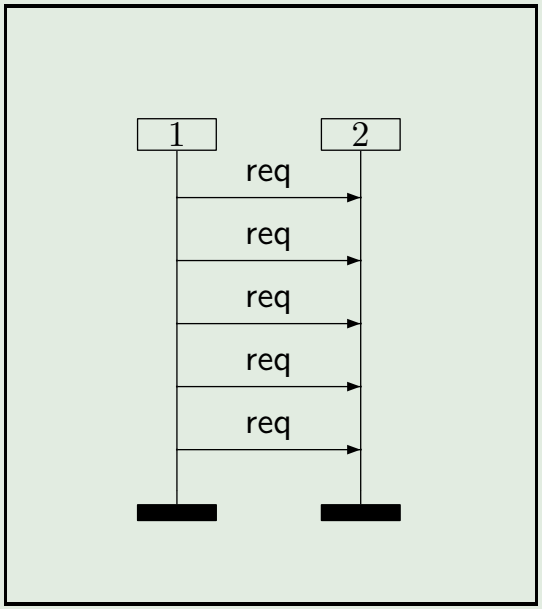
An $\exists 2$-bounded MSC with a corresponding justification

Bounded MSCs

Example





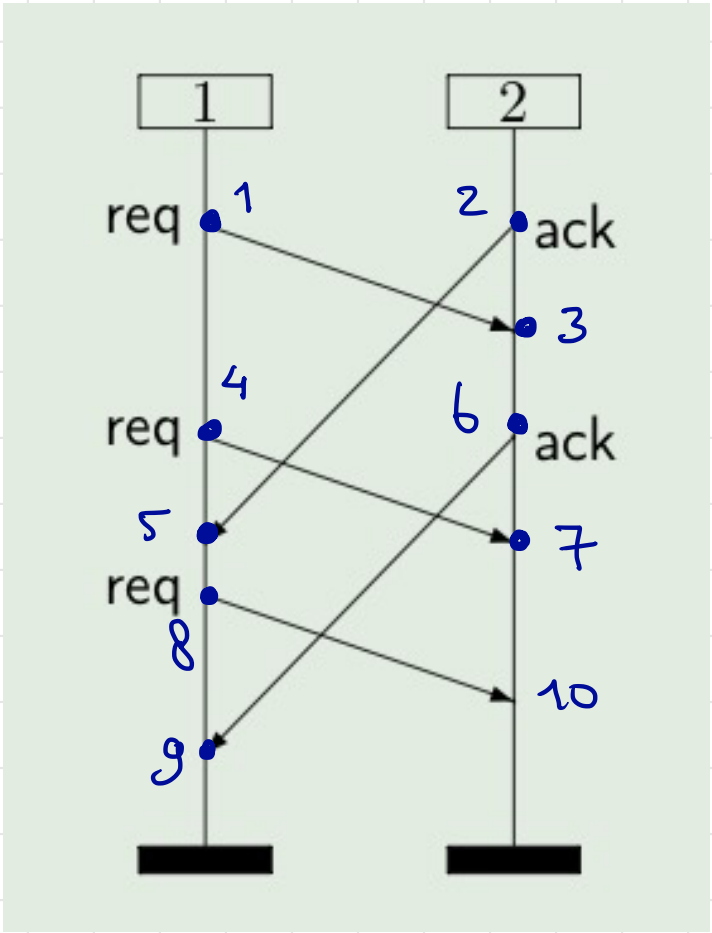


(*)

What is the minimal $\mathbb{B}$ s.t.

M is $\exists\mathbb{B}$-bounded ?

$\mathbb{B} = 1$



(1,0) (1,1) (0,1) (1,1) (1,0) (0,1) (1,1) (0,0) (1,0)

"Scheduling" strategy of the events is:

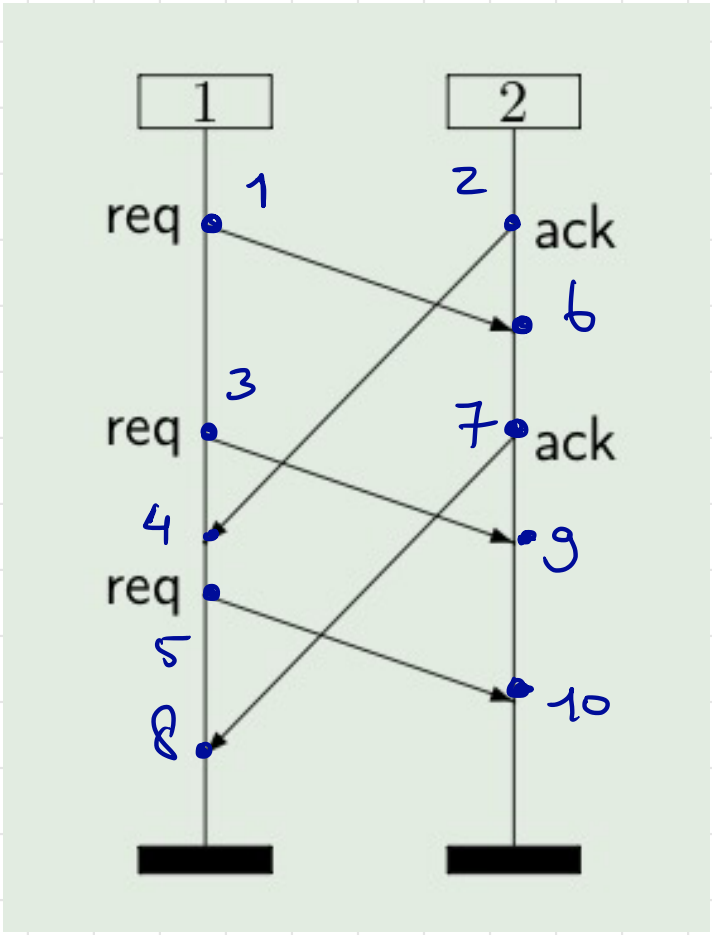
- receive messages as soon as possible
- postpone sending messages as long as possible

What is the minimal $\mathbb{B}$ s.t.

M is $\forall \mathbb{B}$-bounded ?

$=$

$\mathbb{B} = 3$



(1,0) (2,1) (2,0) (3,0) — (2,0)

(1,1) (2,0) (2,1) (1,0) (0,0)

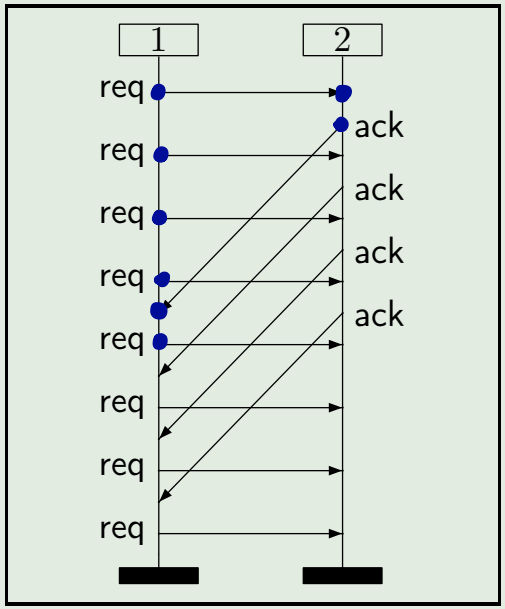
"Scheduling" strategy:

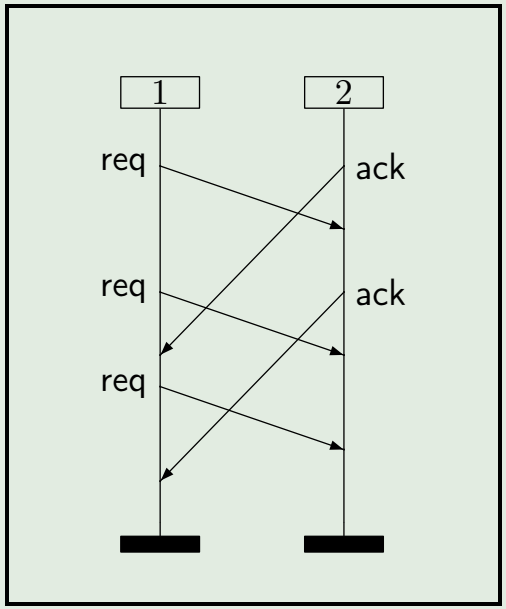
- defer receiving messages as long as possible
- send messages as soon as possible

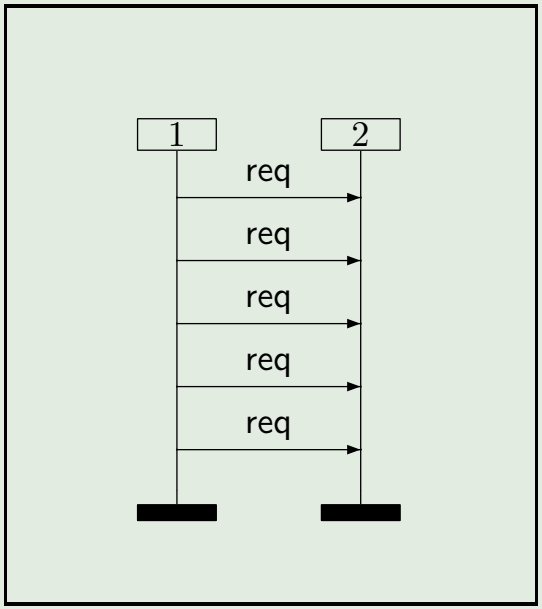
$\Rightarrow$ in order to find the maximal capacity needed for all possible linearisations

Bounded MSCs

Example



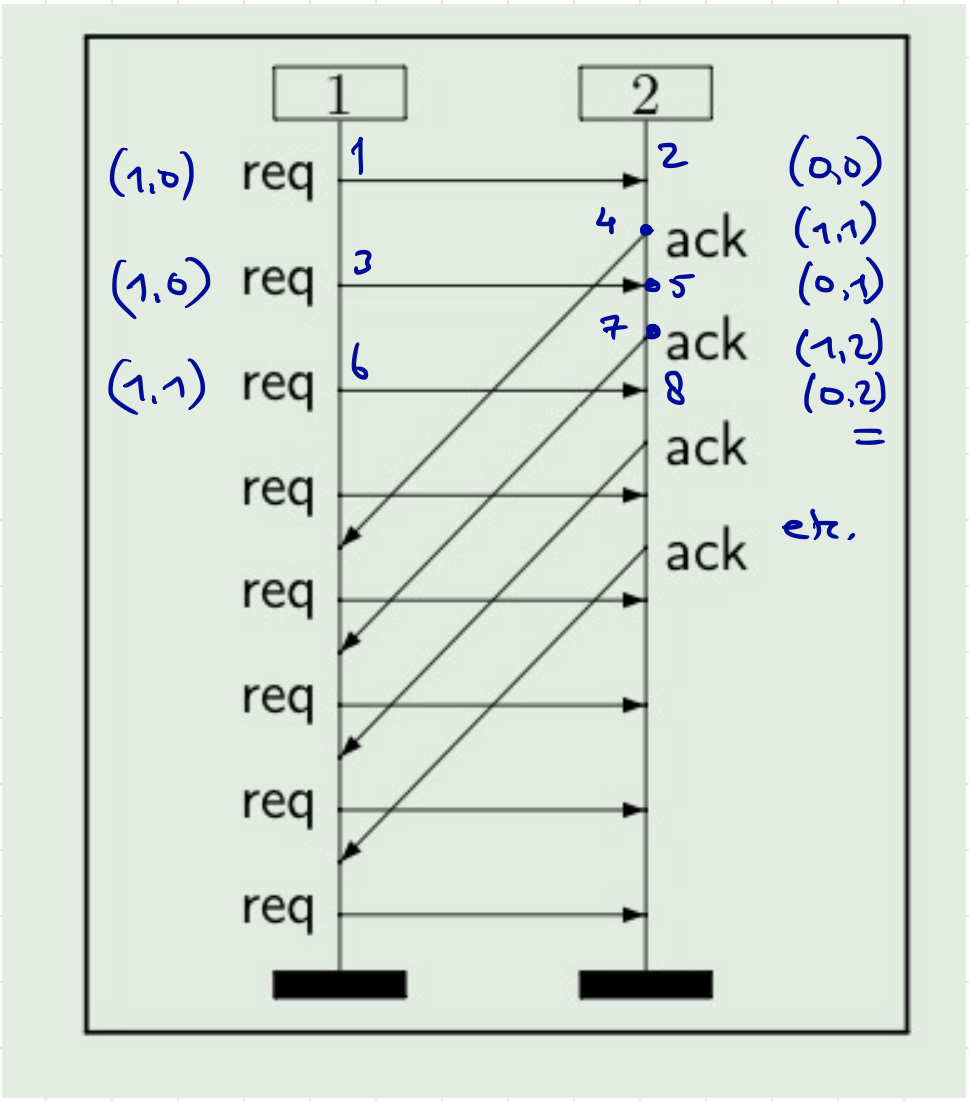




$\forall$4-bounded

$\exists$2-bounded

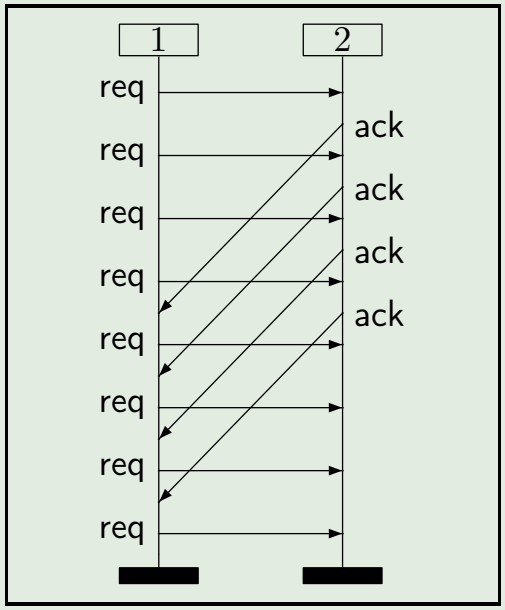
not $\exists$1-bounded


1
2
(1,0) req
(1,0) req
(1,1) req
req
req
req
req
req
req
1
2
3
4
5
6
7
8
ack
ack
ack
(0,0)
(1,1)
(0,1)
(1,2)
(0,2)
=
etc.

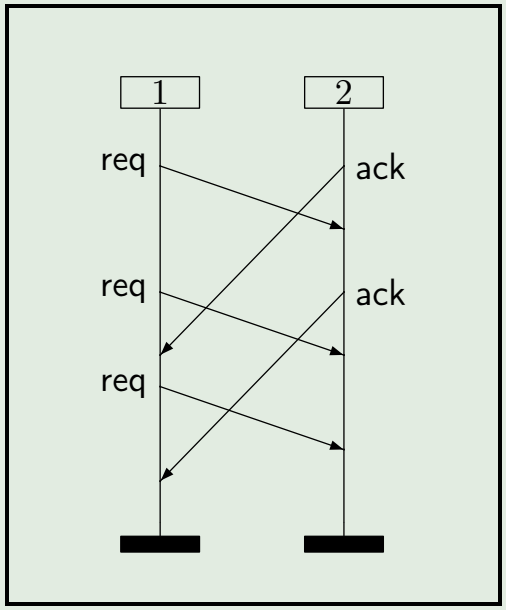
∃2-bounded.

Bounded MSCs

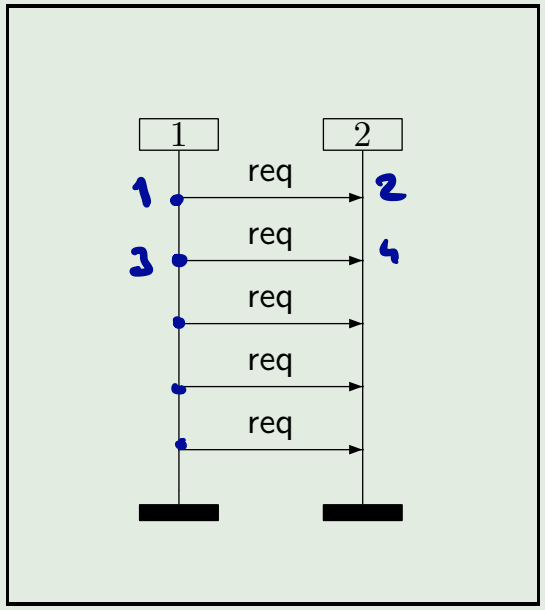
Example



∀4-bounded
∃2-bounded
not ∃1-bounded



∀3-bounded
∃1-bounded



∀5-bounded
∃1-bounded

Bounded CFMs

Definition (Universally bounded CFM)

1. Let $B \in \mathbb{N}$ and $B > 0$. CFM $\mathcal{A}$ is *universally B-bounded* if each MSC in $\mathcal{L}(\mathcal{A})$ is $\forall B$-bounded.
2. CFM $\mathcal{A}$ is *universally bounded* if it is $\forall B$-bounded for some $B \in \mathbb{N}$ and $B > 0$.

Proposition every $\forall B$-banded CFM has finitely many configurations.

(emptiness problem for $\forall B$-bounded CFMs is —obviously— decidable)

Bounded CFMs

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Definition (Existentially bounded CFM)

1. Let $B \in \mathbb{N}$ and $B > 0$. CFM $\mathcal{A}$ is *existentially B-bounded* if each MSC in $\mathcal{L}(\mathcal{A})$ is $\exists B$-bounded.
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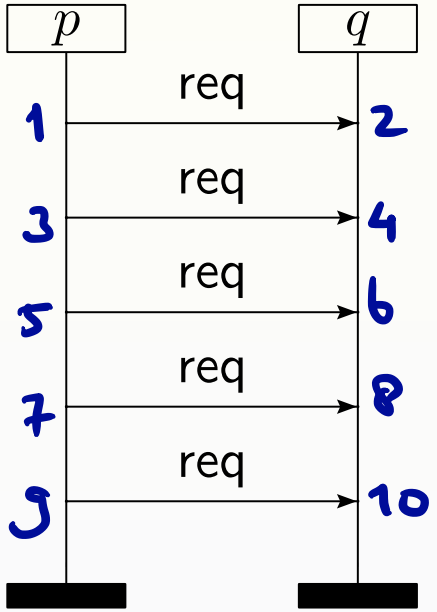
Example (1)

CFM

process p: process q:



∃1-bounded

∀5-bounded

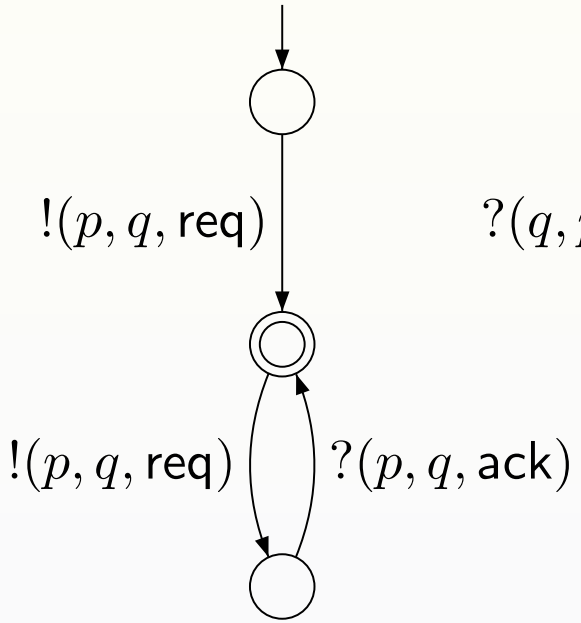
∃1-bounded, but not $\forall B$-bounded for any B

so, not ∀-bounded.

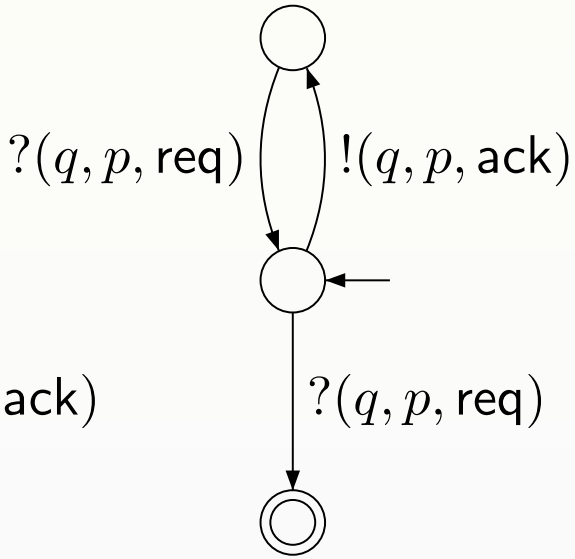
Example (2)

CFM $\mathcal{A}$

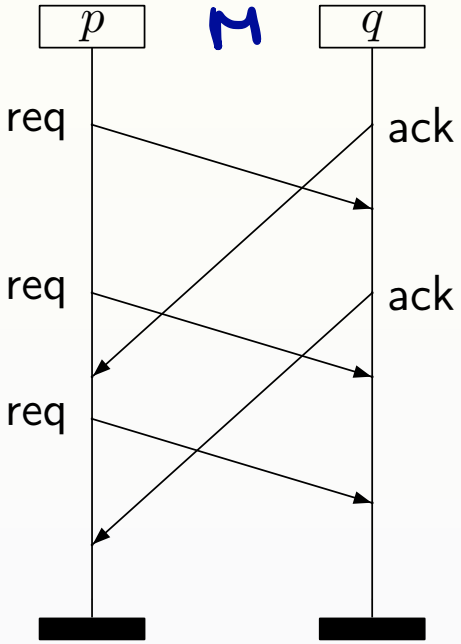
process p:



process q:



Yannakakis



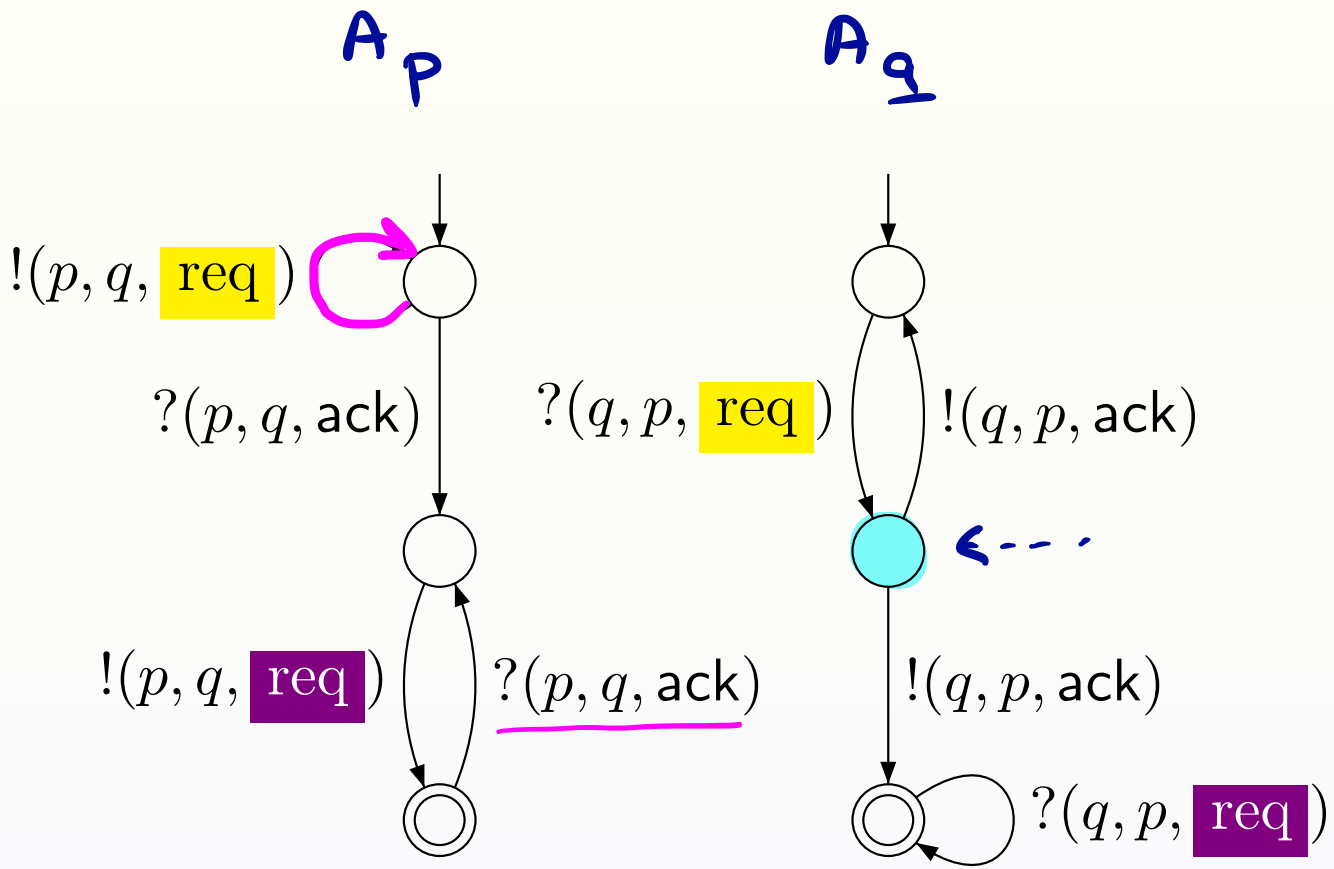
$\exists$1-bounded

$\forall$3-bounded

$\exists$1-bounded, and $\forall$3-bounded

$M \in \mathcal{L}(\mathcal{A})$

Example (3)



n = # yellow messages

$\exists \lceil \frac{n}{2} \rceil$-bounded, but not $\forall B$-bounded for any B

p can send arbitrarily many messages to q

Justification

- Phase 1: process p sends n messages to q
  - messages of phase 1 are tagged with data req
- ... and waits for the first acknowledgement of q
- Phase 2: each ack is directly answered by p by another message
  - messages of phase 2 are tagged with data req
- So, p sends $2n$ reqs to q and q sends n acks
  - existentially $\lceil \frac{n}{2} \rceil$-bounded
  - q starts to send acks after $\lceil \frac{n}{2} \rceil$ requests have been sent by p
  - after n sends, process p receives the first ack; then phase 2 starts
  - in phase 2, process p and q keep sending and receiving messages "in sync"
- Note: the CFM is also non-deterministic, and may deadlock.

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- Note: the CFM is also non-deterministic, and may deadlock. Why?

Emptiness is decidable for ∃-bounded CFMs

Theorem: [Genest *et. al*, 2006]

For any ∃-bounded CFM, the emptiness problem is decidable (and is PSPACE-complete).

$L(A) = \emptyset$?

Emptiness is decidable for ∃-bounded CFMs

Theorem: [Genest *et. al*, 2006]

For any ∃-bounded CFM, the emptiness problem is decidable (and is PSPACE-complete).

Note:

This decision problem is undecidable for arbitrary CFMs, and is obviously decidable for ∀-bounded CFMs, as ∀-bounded CFMs have finitely many configurations, and thus one can check whether a configuration (s, η_ε) with $s \in F$ is reachable by a simple graph analysis.

Some (un)decidability results

Undecidable

The following problems on CFM $\mathcal{A}$ are all undecidable:

1. For $B \in \mathbb{N}$ and $B > 0$, is CFM $\mathcal{A}$ $\forall B$-bounded?
2. Is CFM $\mathcal{A}$ universally bounded?
3. For $B \in \mathbb{N}$ and $B > 0$, is CFM $\mathcal{A}$ $\exists B$-bounded?
4. Is CFM $\mathcal{A}$ existentially bounded?

the proofs of all these facts are left as an exercise

Deadlocks

Deadlock-free CFMs

$(\overline{s}, \eta) \in Conf_{\mathcal{A}}$ is a deadlock configuration of CFM $\mathcal{A}$ if there is no "accepting" configuration $(\overline{s}', \eta') \in F \times \{\eta_\varepsilon\}$ with $(\overline{s}, \eta) \Longrightarrow_{\mathcal{A}}^{*} (\overline{s}', \eta')$.

CFM $\mathcal{A}$ is deadlock-free whenever it has no reachable deadlock configuration.

Checking deadlock-freeness is undecidable

The decision problem: Is CFM $\mathcal{A}$ deadlock free? is undecidable.

Checking B-boundedness for deadlock-free CFMs is decidable

The decision problem: for deadlock-free CFM $\mathcal{A}$ and $B \in \mathbb{N}$ with $B > 0$, is $\mathcal{A}$ $\forall B$-bounded? is decidable.