Theoretical Foundations of the UML

Lecture 7: Communicating Finite-State Machines

Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group

moves.rwth-aachen.de/teaching/ss-20/fuml/

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Outline

Overview

1. Introduction
2. Communicating Finite-State Machines
3. Semantics of Communicating Finite-State Machines
4. Emptiness Problem for CFMs

Specification to implementation

- consider (C)MSGs as <u>complete</u> specifications of a system
- MSG g, $L(g)$ = set of MSCs = set of possible scenarios

finite

countably infinite (e.g. CMSG for the Yannakakis example)

<u>Central question</u>: can we obtain a system <u>"realisation"</u> that exhibits all possible scenarios in $L(g)$

First question: how do such system "realisations" look like?

- model the behavior of each process by a finite automaton ("local" automaton)
- processes can communicate via unbounded FIFO channels

Specification to implementation

- Consider an MSGs as complete system specifications
  - they describe a full set of possible system scenarios

$L(G)$ = set of all possible scenarios

Specification to implementation

- Consider an MSGs as complete system specifications
  - they describe a full set of possible system scenarios

- Can we obtain “realisations“ that exhibit precisely these scenarios?

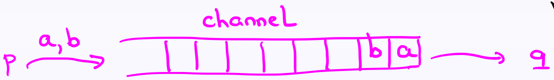
central question in the next 3-4 lectures

Specification to implementation

- Consider an MSGs as complete system specifications
  - they describe a full set of possible system scenarios
- Can we obtain “realisations“ that exhibit precisely these scenarios?
- Map MSGs, i.e., scenarios onto an "executable model"
  - model each process by a finite-state automaton
  - that communicate via unbounded directed FIFO channels

$p \longrightarrow q$

(c) MSG $\longmapsto$ communicating finite-state machine (CFM)



Specification to implementation

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  - they describe a full set of possible system scenarios
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- Map MSGs, i.e., scenarios onto an executable model
  - model each process by a finite-state automaton
  - that communicate via unbounded directed FIFO channels

⇒ This yields Communicating Finite-state Machines

Brand & Zafiropoulou

Intuition

Example 1

process p "realisation" process q

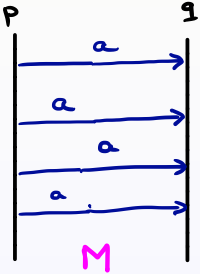
"local" automaton of p

"local" automaton of q

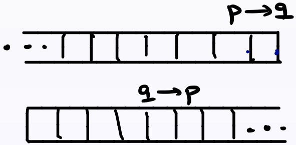
global initial state = (1, A)

global final states = { (1, A) }

CMSC



possible behavior of the CFM:



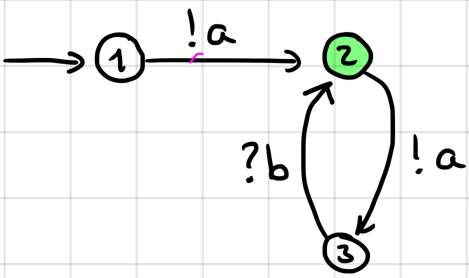
not used

CFM accepts if

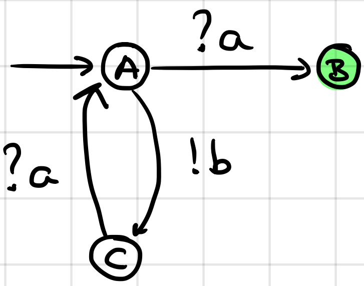
1. all channels are empty
2. we are in state (1, A)

Example 2

process p

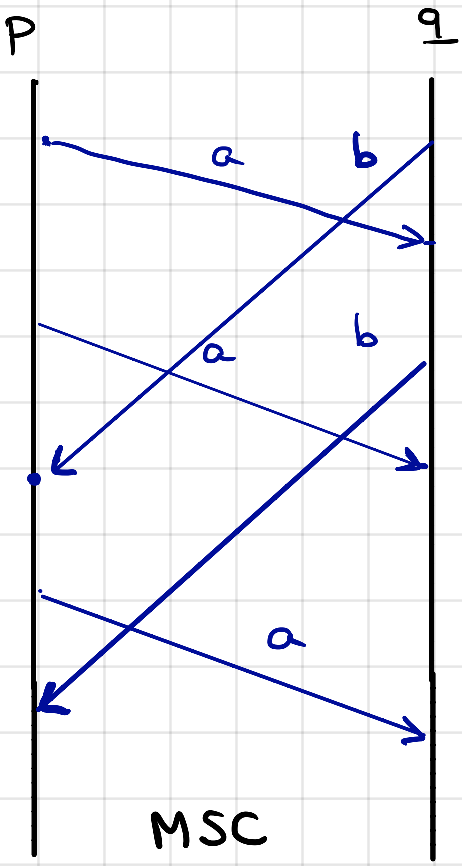


process q

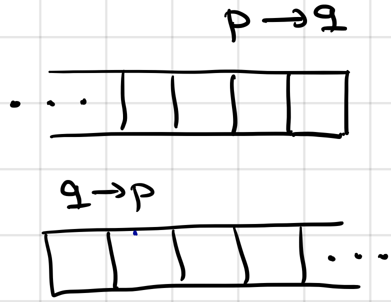


global initial state = $(1, A)$

global final states = $\{(2, B)\}$



MSC

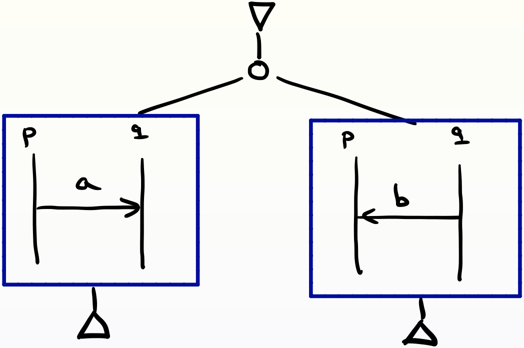


is "accepted" by the example CFM

(Yannakakis example)

The need for synchronisation messages

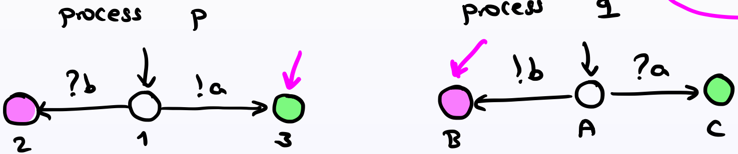
Suppose we want to realise

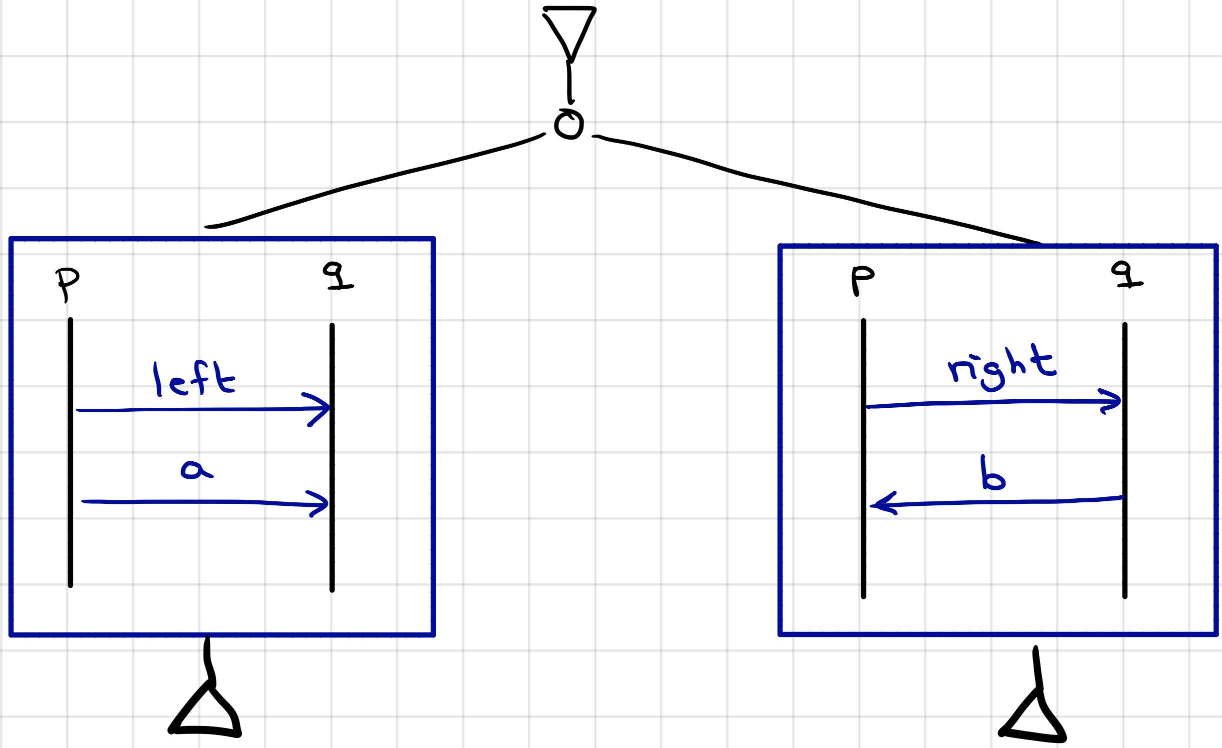


final states
$\{(2,B),(3,C)\}$

CFM has a deadlock

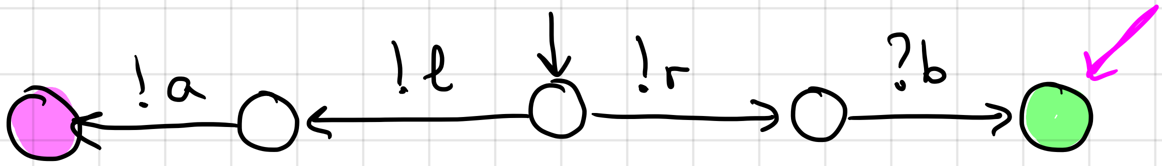
CFM:



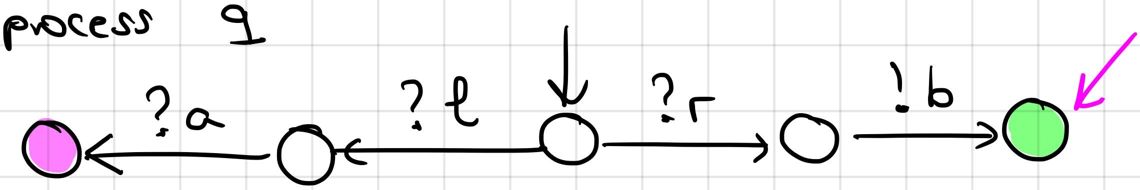


Process p informs process q whether to go "left" or "right"

Automaton for process p: $F = \{(\circ,\circ),(\circ,\circ)\}$



For process q



A deadlock like in the previous example cannot occur

Overview

Preliminaries

Definition

Let

- $\mathcal{P}$ be a finite set of at least two (sequential) processes
- $\mathcal{C}$ be a finite set of message contents

a, b, c

Preliminaries

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Definition (communication actions, channels)

- $Act_p^! := \{!(p, q, a) \mid q \in \mathcal{P} \setminus \{p\}, \ a \in \mathcal{C}\}$
  the set of send actions by process p

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- $Act_p := Act_p^! \cup Act_p^?$

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- $Act := \bigcup_{p \in \mathcal{P}} Act_p$
- $Ch := \{(p,q) \mid p,q \in \mathcal{P},\ p \neq q\}$ "channels"

ordered

(p,q)

(q,p)

Definition

A communicating finite-state machine (CFM) over $\mathcal{P}$ and $\mathcal{C}$ is a structure

set of global final states

$$\mathcal{A} = (\underbrace{((S_p, \Delta_p))_{p\in\mathcal{P}}}_{\text{"local" automaton for each process}}, \mathbb{D}, s_{init}, F)$$

where

global initial state

synchronisation messages (e.g. left, right)

Communicating finite-state machines

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A communicating finite-state machine (CFM) over $\mathcal{P}$ and $\mathcal{C}$ is a structure

$$\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \underline{\mathbb{D}}, s_{init}, F)$$

where

e.g.
left, right

- $\mathbb{D}$ is a nonempty finite set of synchronization messages (or data)

We often write $s \xrightarrow{\sigma, m}_p s'$ instead of $(s, \sigma, m, s') \in \Delta_p$

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- $\mathbb{D}$ is a nonempty finite set of synchronization messages (or data)
- for each $p \in \mathcal{P}$:
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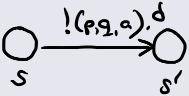
  - $\Delta_p \subseteq \underbrace{S_p \times Act_p \times \mathbb{D} \times S_p}$ is a set of local transitions

$$(s, !(p,q,a), d, s') \in \Delta_p$$



$\in Act_p$

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- $s_{init} \in S_{\mathcal{A}}$ is the global initial state
  - where $S_{\mathcal{A}} := \prod_{p \in \mathcal{P}} S_p$ is the set of global states of $\mathcal{A}$

p, q, r

(p, q, r)

We often write $s \xrightarrow{\sigma,m}_p s'$ instead of $(s, \sigma, m, s') \in \Delta_p$

Communicating finite-state machines

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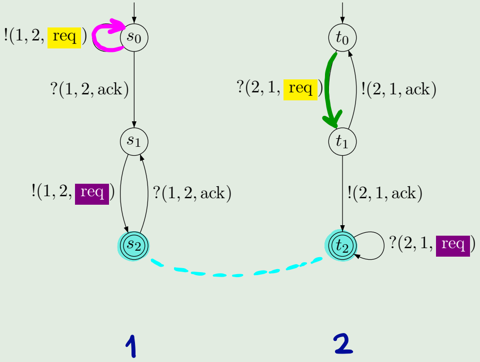
- $F \subseteq S_{\mathcal{A}}$ is the set of global final states

We often write $s \xrightarrow{\sigma,m}_p s'$ instead of $(s, \sigma, m, s') \in \Delta_p$

Example



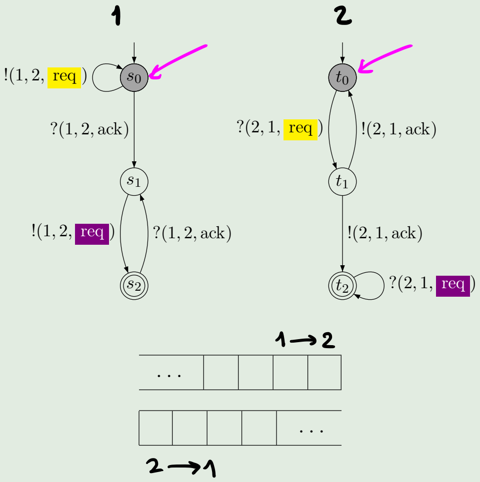
CFM $\mathcal{A}$ over $\mathcal{P} = \{1, 2\}$ and $\mathcal{C} = \{\text{req}, \text{ack}\}$

- $\mathbb{D} = \{\ , \ , \ \}$
- $S_1 = \{s_0, s_1, s_2\}$
- $S_2 = \{t_0, t_1, t_2\}$
- Δ_1: $s_0 \xrightarrow{!(1,2,\text{req})}_1 s_0 \ \ldots$

  Δ_2: $t_0 \xrightarrow{?(2,1,\text{req})}_2 t_1 \ \ldots$
- $s_{init} = (s_0, t_0)$
- $F = \{(s_2, t_2)\}$

Communicating finite-state machines

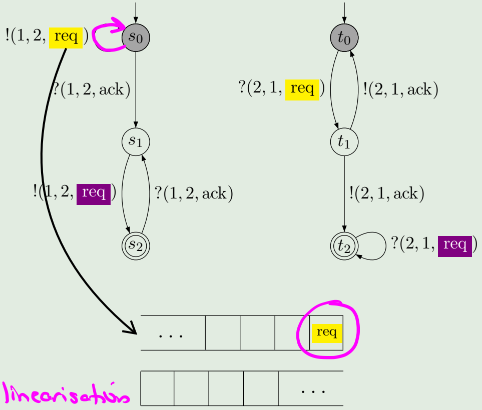
Example

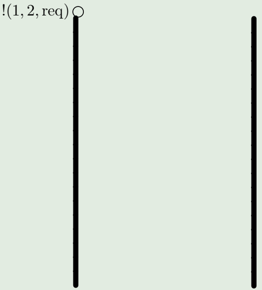


Communicating finite-state machines

Example

CMSC

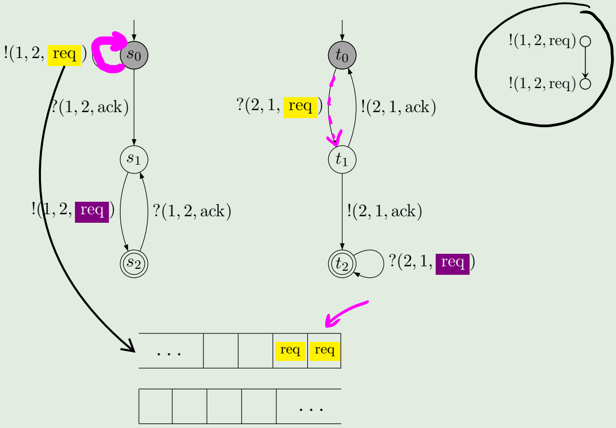




$!(1,2,\text{req})$

Communicating finite-state machines

Example



$!(1,2,\text{req})$ $!(1,2,\text{req})$

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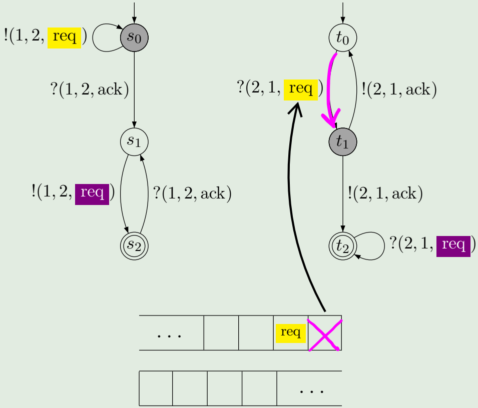
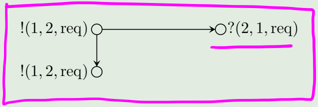
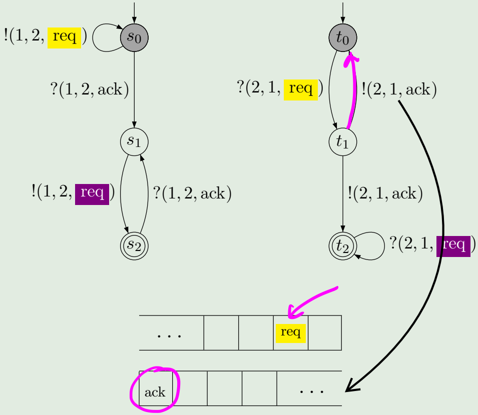




$!(1, 2, \mathrm{req})\ !(1, 2, \mathrm{req})\ ?(2, 1, \mathrm{req})$

Communicating finite-state machines

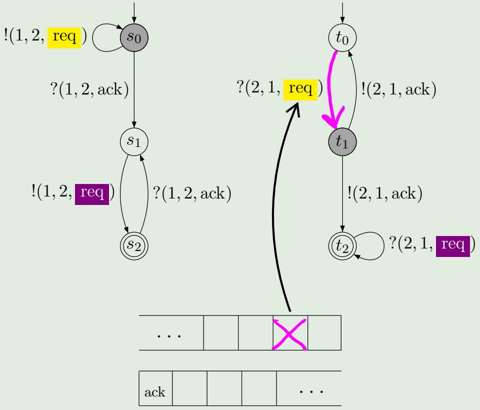
Example



!(1, 2, req) !(1, 2, req) ?(2, 1, req) !(2, 1, ack)

Communicating finite-state machines

Example



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Communicating finite-state machines

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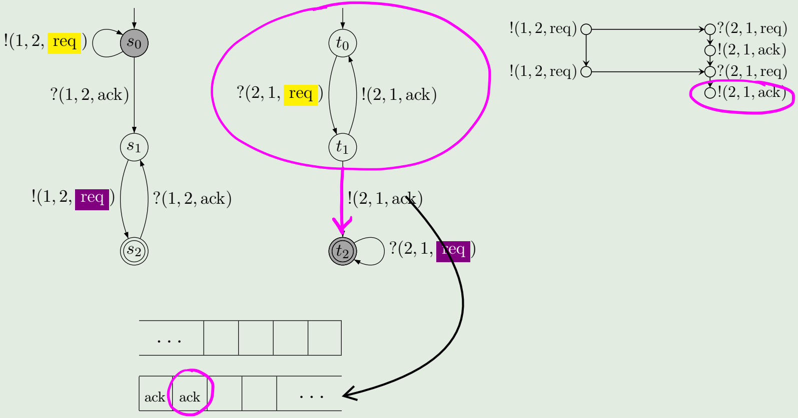

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Communicating finite-state machines

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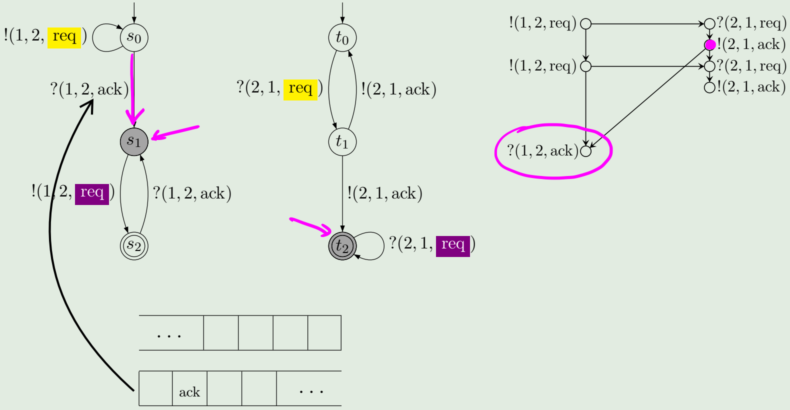

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Communicating finite-state machines

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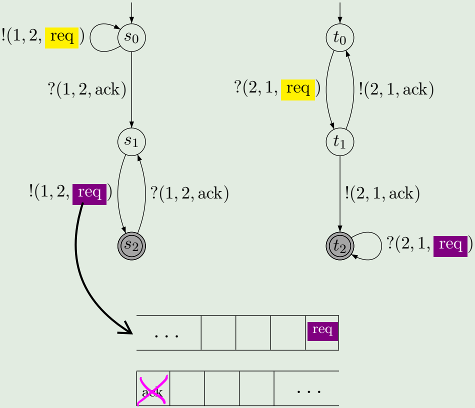

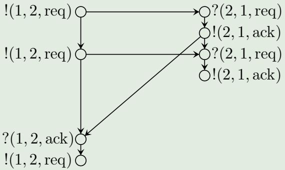

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Communicating finite-state machines

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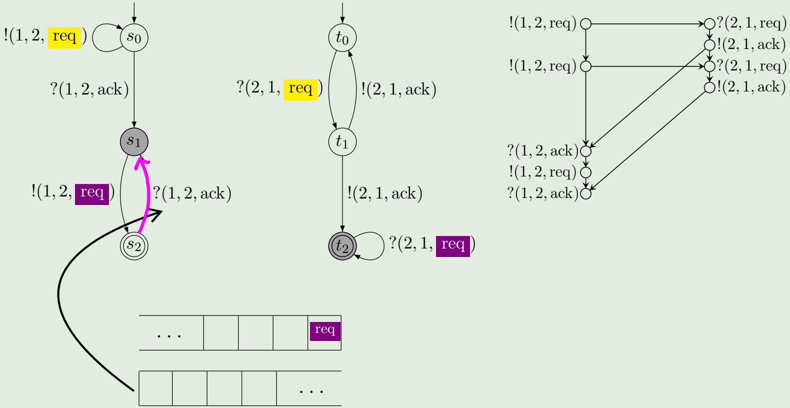

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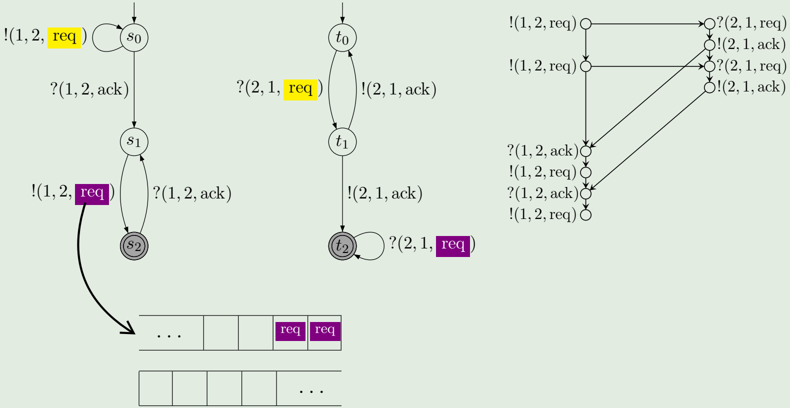

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Communicating finite-state machines

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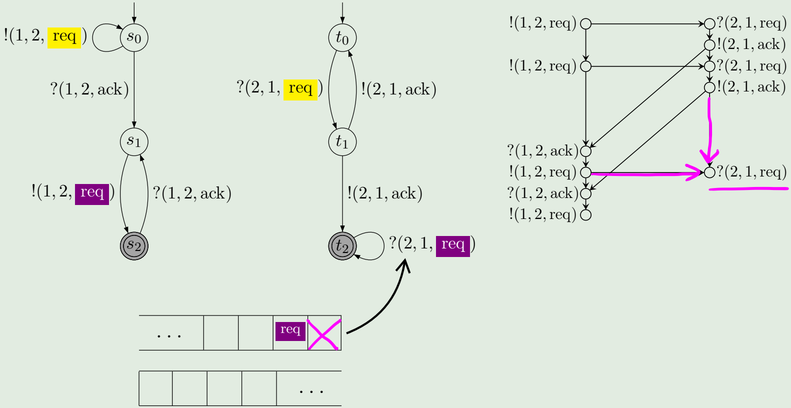

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Communicating finite-state machines

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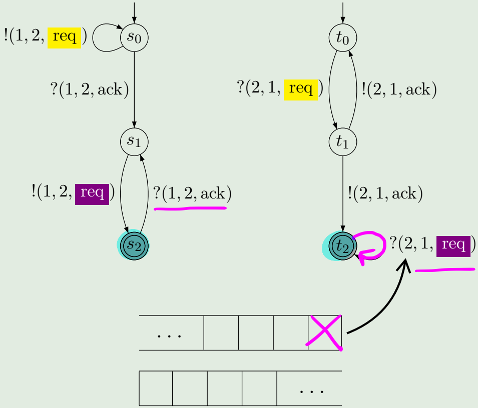

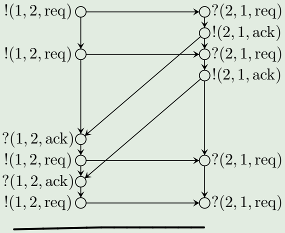

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Communicating finite-state machines

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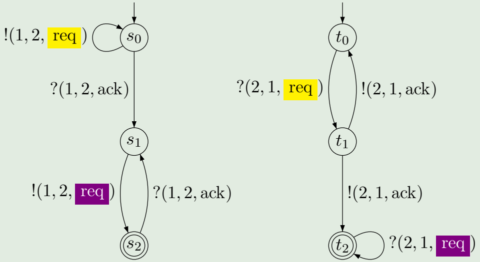

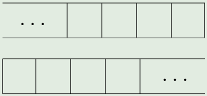

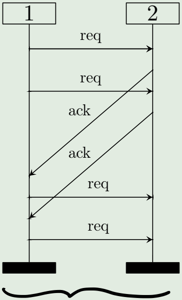

MSC M

M is accepted by this CFM.

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Overview

Formal semantics of CFMs

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition (configurations)

Configurations of $\mathcal{A}$: $Conf_{\mathcal{A}} := S_{\mathcal{A}} \times \underbrace{\{\eta \mid \eta : Ch \rightarrow (\mathcal{C} \times \mathbb{D})^*\}}$

global state (= a local state for every process p)

"the content of all channels of the CFM"

$\eta : Ch \rightarrow (C \times D)^*$

$\eta((p,q)) = \varepsilon$

$\eta((p,q)) = (a, \,), (b, \,)$

Formal semantics of CFMs

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Definition (configurations)

Configurations of $\mathcal{A}$: $Conf_{\mathcal{A}} := S_{\mathcal{A}} \times \{\eta \mid \eta : Ch \to (\mathcal{C} \times \mathbb{D})^*\}$

Definition (global step)

$\Longrightarrow_{\mathcal{A}} \subseteq Conf_{\mathcal{A}} \times Act \times \mathbb{D} \times Conf_{\mathcal{A}}$ is defined as follows:

$|P| = k$

$((s_1, \ldots, s_i, \ldots, s_k), \eta) \xrightarrow{!(s_i, s_j, a),\, m} ((s_1, \ldots, s_i', \ldots, s_k), \eta')$

$Ch \to (C \times D)^*$

$?(s_i, s_j, a), m$

Formal semantics of CFMs

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition (configurations)

Configurations of $\mathcal{A}$: $Conf_{\mathcal{A}} := S_{\mathcal{A}} \times \{\eta \mid \eta : Ch \to (\mathcal{C} \times \mathbb{D})^*\}$

Definition (global step)

$\Longrightarrow_{\mathcal{A}} \subseteq Conf_{\mathcal{A}} \times Act \times \mathbb{D} \times Conf_{\mathcal{A}}$ is defined as follows: $|P| = k$

- sending a message: $((\overline{s}, \eta), \underbrace{!(p,q,a), m}, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - ✓ $(\overline{s}[p], !(p,q,a), m, \overline{s}'[p]) \in \Delta_p$
  - ✓ $\eta' = \eta[(p,q) := (a,m) \cdot \eta((p,q))]$
  - ✓ $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

$\overline{s}[p] = s_p$ local state at process p

$s_p \xrightarrow{!(p,q,a),m} s'_p$

(p,q): $(a,m) \times \times \times \times$ $\underbrace{\quad}_{\eta}$

$(\overline{s} = (s_1, \ldots s_p \ldots, s_k), \eta)$

$\downarrow \; !(p,q,a), m$

$(\overline{s}' = \underbrace{(s_1, \ldots s'_p, \ldots s_k)}, \eta')$

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  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$
- receipt of a message: $((\overline{s}, \eta), ?(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], ?(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta((q, p)) = w \cdot (a, m) \neq \epsilon$ and $\eta' = \eta[(q, p) := w]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

<u>receipt of a message:</u> $((\bar{s},\eta), ?(p,q,a), m, (\bar{s}',\eta')) \in \Longrightarrow_{\mathcal{A}}$ if

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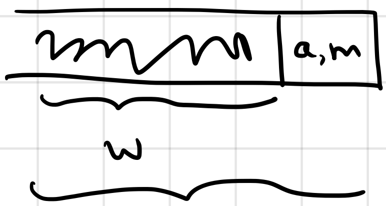
- ✓ $\bar{s}[r] = \bar{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

$$\begin{array}{c} ((\underbrace{s_1,\ldots\ldots}, s_p, \underbrace{\ldots\ldots, s_k}), \eta) \\ \Big\downarrow \; ?(p,q,a),\, m \\ ((\underbrace{s_1,\ldots\ldots}, s_p', \underbrace{\ldots\ldots, s_k}), \eta') \end{array}$$

current configuration

$q \rightarrow p$



$\neq \varepsilon$

$\eta'((q,p)) = w$

for all other channels c

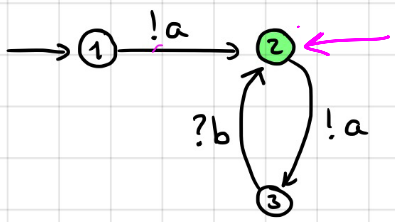
$\eta'(c) = \eta(c)$

$$s_p \xrightarrow{?(p,q,a),\, m} s_p'$$

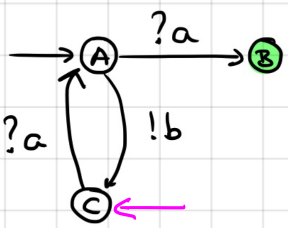
Example

Example 2

process p



process q



global initial state = $(1, A)$

global final states = $\{(2, B)\}$

$\eta((p,q)) = \varepsilon$

$\eta((q,p)) = \varepsilon$

starting configuration: $((1,A),(\varepsilon,\varepsilon)) = \gamma_0$

$$\Downarrow \text{empty}$$

$$((1,C),(\varepsilon,b)) = \gamma_1$$

$$\Downarrow \text{empty}$$

$$((2,C),(a,b)) = \gamma_2$$

$$\Downarrow \text{--}$$

$\eta((p,q)) = aa$

$\eta((q,p)) = b$

$$\ldots \underbrace{((2,C),(aa,\varepsilon))}_{=\gamma_4} \Leftarrow ((3,C),(aa,b)) = \gamma_3$$

Linearizations of a CFM

$!(p,q,a)$, $?(q,p,b)$

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition (accepting runs)

A run ρ of CFM $\mathcal{A}$ on word $w = \sigma_1 \ldots \sigma_n \in Act^*$ is an alternating sequence $\rho = \gamma_0\, m_1\, \gamma_1 \ldots \gamma_{n-1}\, m_n\, \gamma_n$ such that

1. $\gamma_0 = (s_{init}, \eta_\varepsilon)$ with η_ε mapping any channel to ε (empty content)
2. $\gamma_{i-1} \xRightarrow{\sigma_i, m_i}_{\mathcal{A}} \gamma_i$ for any $i \in \{1, \ldots, n\}$

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The run ρ is accepting if $\underline{\gamma_n} \in \underline{F} \times \underline{\{\eta_\varepsilon\}}$.

γ_n = global final state + all channels are empty.

Linearizations of a CFM

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

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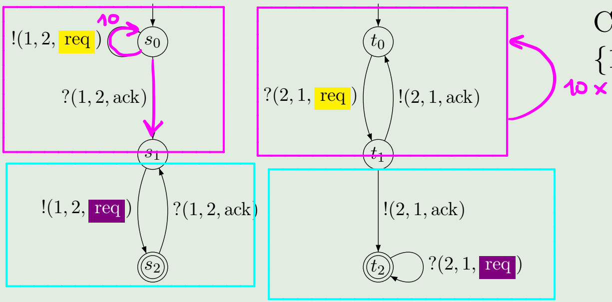
The run ρ is accepting if $\gamma_n \in F \times \{\eta_\varepsilon\}$.

Definition (linearization of a CFM)

The (word) language of CFM $\mathcal{A}$ is defined by:

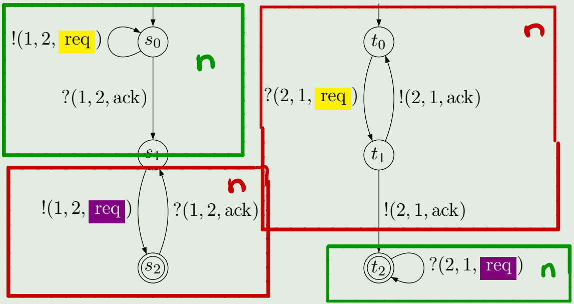
$Lin(\mathcal{A}) := \{\underline{w \in Act^*} \mid \underline{\text{there is an accepting run of } \mathcal{A} \text{ on } w}\}$

Example



CFM $\mathcal{A}$ over $\{1,2\}$ and $\{\text{req},\text{ack}\}$

Example



CFM $\mathcal{A}$ over $\{1, 2\}$ and $\{\text{req}, \text{ack}\}$

w↾1 = the sequence of actions in w that occur at process 1

w↾2 = ... for process 2..

$$Lin(\mathcal{A}) = \Big\{ w \in Act^* \mid \text{there is } n \geqslant 1 \text{ such that:}$$

$$w \restriction 1 = \,!(1,2,\text{req}))^n \; (?(1,2,\text{ack}) \; !(1,2,\text{req}))^n \qquad (*)$$

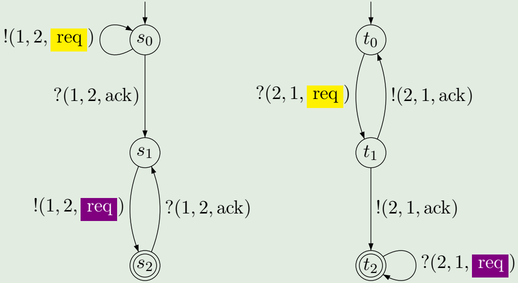
$$w \restriction 2 = (?(2,1,\text{req}) \; !(2,1,\text{ack}))^n \; (?(2,1,\text{req}))^n \qquad (**)$$

for any $u \in Pref(w)$ and $(p, q) \in Ch$:

$$\sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \geqslant 0 \Big\} \qquad (***)$$

Linearizations of an example CFM

Example



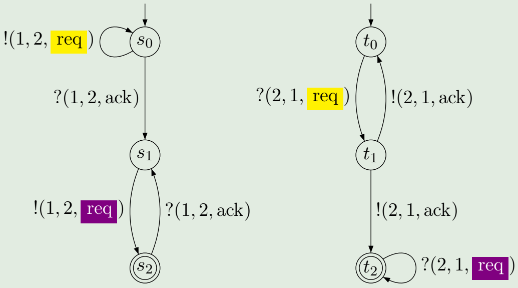
CFM $\mathcal{A}$ over $\{1, 2\}$ and $\{\text{req}, \text{ack}\}$

- $!(1, 2, \text{req})$ and $!(2, 1, \text{ack})$ are always independent.
- $!(1, 2, \text{req})$ and $?(1, 2, \text{ack})$ are always dependent.
- $!(1, 2, \text{req})$ and $?(2, 1, \text{req})$ are sometimes independent.

$\leadsto$ non-regular (word) languages → more expressive than finite-state automata!

Linearizations and MSCs of an example CFM

Example



CFM $\mathcal{A}$ over $\{1,2\}$ and $\{\text{req}, \text{ack}\}$

$Lin(\mathcal{A}) = \Big\{ w \in Act^* \mid$ there is $n \geqslant 1$ such that:

$$w \upharpoonright 1 = (!(1,2,\text{req}))^n \; (?(1,2,\text{ack}) \; !(1,2,\text{req}))^n$$

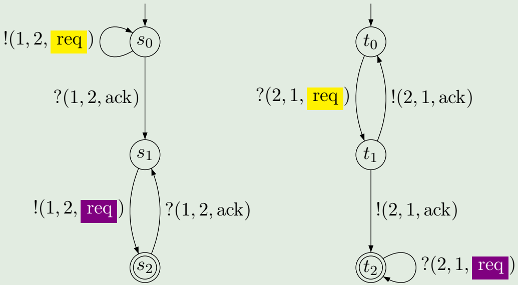
$$w \upharpoonright 2 = (?(2,1,\text{req}) \; !(2,1,\text{ack}))^n \; (?(2,1,\text{req}))^n$$

for any $u \in Pref(w)$ and $(p,q) \in Ch$:

$$\sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \geqslant 0 \Big\}$$

Linearizations and MSCs of an example CFM

Example



CFM $\mathcal{A}$ over $\{1, 2\}$ and $\{\text{req}, \text{ack}\}$

$\mathcal{L}(\mathcal{A}) = \Big\{ M \in \mathbb{M} \mid$ there is $n \geq 1$ such that:

$$M \upharpoonright 1 = \overbrace{(!(1,2,\text{req}))^n}^{s_0} \; \overbrace{(?(1,2,\text{ack}) \; !(1,2,\text{req}))^n}^{s_1 s_2}$$

$$M \upharpoonright 2 = \underbrace{(?(2,1,\text{req}) \; !(2,1,\text{ack}))^n}_{t_0 t_1} \; \underbrace{(?(2,1,\text{req}))^n}_{t_2} \Big\}$$

set of MSCs accepted by CFM A.

Overview

CFMs are more expressive than finite-state automata

does a CFM accept at least one word? undecidable ☹

Elementary questions are undecidable for CFMs

Emptiness of CFMs is undecidable [Brand & Zafiropulo 1983]

The following problem is undecidable (even if $\mathcal{C}$ is a singleton):

INPUT:	CFM $\mathcal{A}$ over processes $\mathcal{P}$ and message contents $\mathcal{C}$
QUESTION:	Is $\mathcal{L}(\mathcal{A})$ empty?

e.g. $C = \{a\}$

the set of MSCs accepted by CFM A

$\updownarrow$

the set of linearisations accepted by CFM A.

Lin(A)

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QUESTION: Is $\mathcal{L}(\mathcal{A})$ empty?

Proof (sketch)

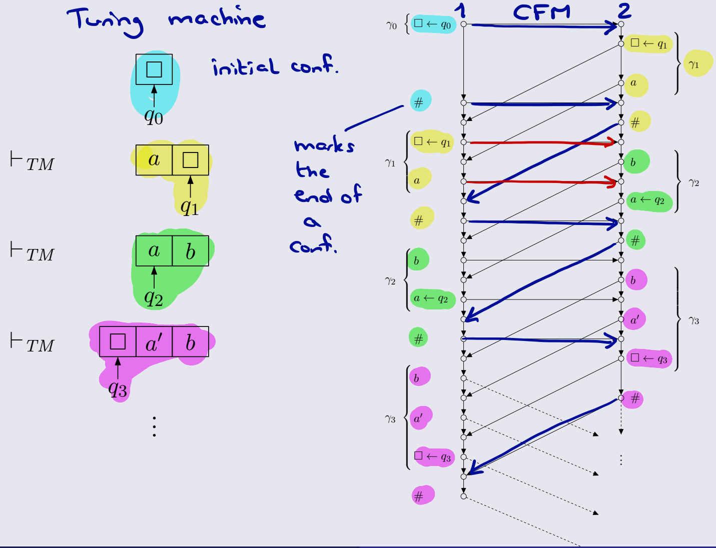
Reduction from the halting problem for Turing machine $TM = (Q, \Sigma, \Delta, \Box, q_0, q_f)$ to emptiness for a CFM with two processes.
Build CFM $\mathcal{A} = ((\mathcal{A}_1, \mathcal{A}_2), \mathbb{D}, s_{init}, F)$ over $\{1, 2\}$ and some singleton set $\mathcal{C}$ such that $\boxed{\mathcal{L}(\mathcal{A}) \neq \emptyset \text{ iff } TM \text{ can reach } q_f\text{, i.e., } TM \text{ accepts.}}$

- Process 1 sends current configurations to process 2 ✓
- Process 2 chooses successor configurations and sends them to 1 ✓
- $\mathbb{D} = \Big((\Sigma \cup \{\Box\}) \times (Q \cup \{\_\})\Big) \cup \{\#\}$

of the TM

A CFM simulating a Turing machine

Proof (contd.)



A CFM simulating a Turing machine

Proof (contd.)

- Left or standstill transition: Process 2 may just wait for a symbol containing a state of TM and to alter it correspondingly. In the example, the left-moving transition (q_2, a, a', L, q_3) is applied so that process 2
  - sends b unchanged back to process 1
  - detects (receives) $a \leftarrow q_2$
  - sends a' to process 1 entering a state indicating that the symbol to be sent next has to be equipped with q_3
  - receives $\#$ so that the symbol $\Box \leftarrow q_3$ has to be inserted before returning $\#$

A CFM simulating a Turing machine

Proof (contd.)

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  - receives $\#$ so that the symbol $\Box \leftarrow q_3$ has to be inserted before returning $\#$
- Right transition: Process 2 has to guess what the position right before the head is. For example, provided process 2 decided in favor of (q_2, a, a', R, q_3) while reading b, it would have to
  - send $b \leftarrow q_3$ instead of just b, entering some state $t(a \leftarrow q_2)$
  - receive $a \leftarrow q_2$ (no other symbol can be received in state $t(a \leftarrow q_2)$)
  - send a' back to process 1

Communicating Finite-state Machines

"realisation" of system

operational model of an implementation

(c) MSG = "requirements"

all scenarios a system should exhibit

A CFM simulating a Turing machine

Proof (contd.)

- Introduce local final states s_f and t_f, one for process 1 and one for process 2, respectively (i.e., $F = \{(s_f, t_f)\}$ and $\mathcal{A}$ is locally accepting).
- At any time, process 1 may switch into s_f, in which arbitrary and arbitrarily many messages can be received to empty channel $(2, 1)$.
- Process 2 is allowed to move into t_f and to empty the channel $(1, 2)$ as soon as it receives a letter $c \leftarrow q_f$ for some c.
- As process 2 modifies a configuration of TM locally, finitely many states are sufficient in $\mathcal{A}$. $\square$