Theoretical Foundations of the UML

Lecture 5+6: Compositional Message Sequence Graphs

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Outline

two decision poblems

undecidable

decidable

Compositional MSCs

[Gunter, Muscholl, Peled 2001]

Solution: drop restriction that e and $m(e)$ belong to the same MSC (= allow for incomplete message transfer)

Definition (Compositional MSC)

$M = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$ is a compositional MSC (CMSC, for short) where $\mathcal{P}, E, \mathcal{C}$ and l are defined as before, and

- $m : E_! \rightarrow E_?$ is a partial, injective function such that (as before):

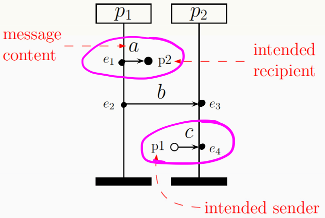
$$m(e) = e' \wedge l(e) = !(p, q, a) \quad \text{implies} \quad l(e') = ?(q, p, a)$$

- $\preceq \; = \Big(\bigcup_{p \in \mathcal{P}} <_p \quad \cup \quad \underbrace{\{(e, m(e)) \mid e \in \underbrace{dom(m)}_{\text{domain of } m} \}}_{\text{"}m(e) \text{ is defined"}} \Big)^*$

Note:

An MSC is a CMSC where m is total and bijective.

CMSC example



- $m(e_2) = e_3$ ✓
- $e_1 \notin dom(m)$
- $e_4 \notin rng(m)$ ✓

m(e_1) is not defined

$E_! = \{e_1, e_2\}$

$E_? = \{e_3, e_4\}$

$\neg \exists e.\ m(e) = e_4$

Paths

Let $G = (V, \to, v_0, F, \lambda)$ be a CMSG.

$\lambda : V \longrightarrow CM$

Definition (Path in a CMSG)

A path π of G is a finite sequence

$$\pi = u_0\ u_1\ \ldots\ u_n \text{ with } u_i \in V\ \ (0 \leq i \leq n) \text{ and } u_i \to u_{i+1}\ \ (0 \leq i < n)$$

Definition (Accepting path of a CMSG)

Path $\pi = u_0\ \ldots\ u_n$ is accepting if: $u_0 = v_0$ and $u_n \in F$.

Definition (CMSC of a path)

The CMSC of a path $\pi = u_0\ \ldots\ u_n$ is:

$$M(\pi) \;=\; (\ldots(\lambda(u_0) \bullet \lambda(u_1)) \bullet \lambda(u_2) \ldots) \bullet \lambda(u_n)$$

where CMSC concatenation is left associative.

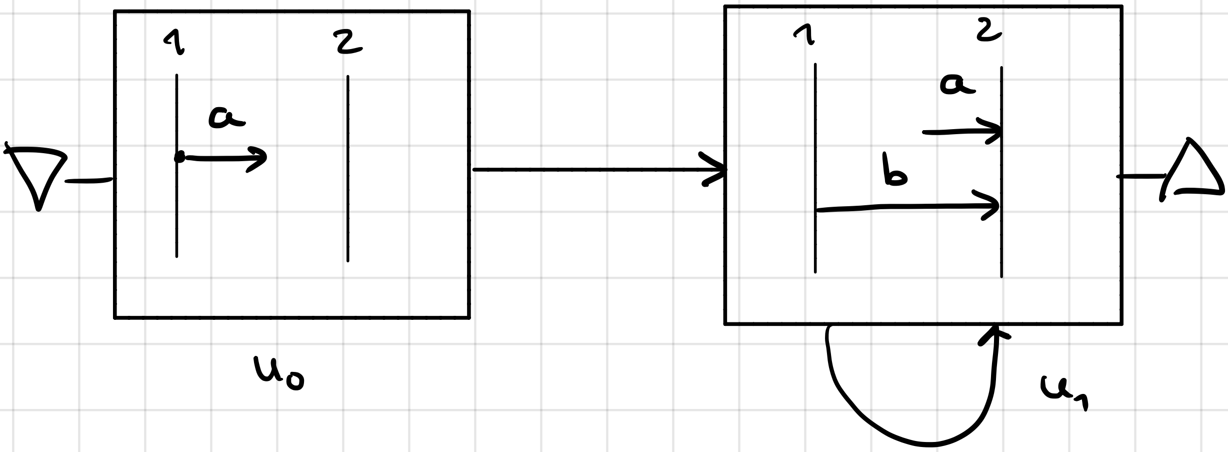
The MSC language of a CMSG

Definition (Language of a CMSG)

The (MSC) language of CMSG G is defined by:

$$L(G) = \{ \underbrace{M(\pi) \in \mathbb{M}}_{\text{only "real" MSCs}} \mid \pi \text{ is an accepting path of } G \}.$$

Note: Accepting paths that give rise to an CMSC (which is not an MSC) are not part of $L(G)$.

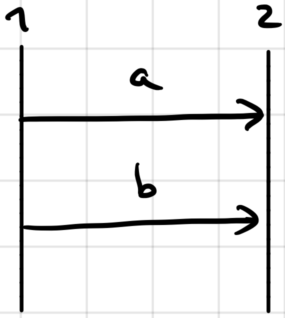


CMSG $\mathcal{G}$

accepting path: $\pi = u_0 u_1$

$M(\pi) \in \mathbb{M}$

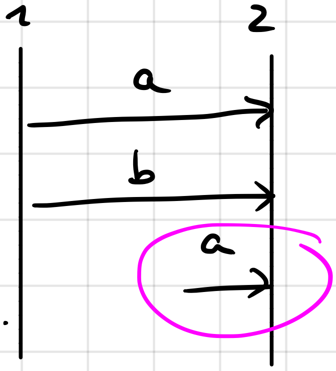
thus $M(\pi) \in L(\mathcal{G})$



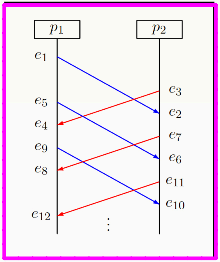
accepting path $\pi' = u_0 u_1 u_1$

$M(\pi') \notin \mathbb{M}$

$M(\pi') \notin L(\mathcal{G})$.



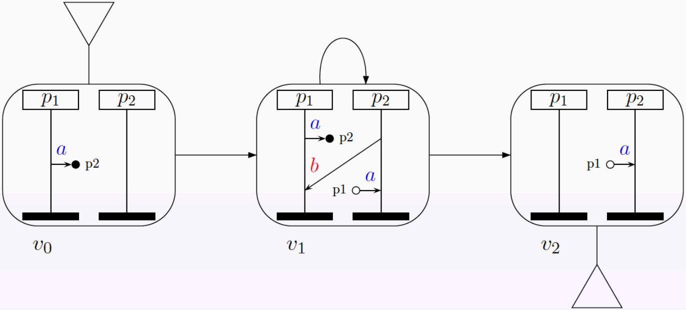
Yannakakis' example as compositional MSG



This MSC cannot be modeled for $n > 1$ by:

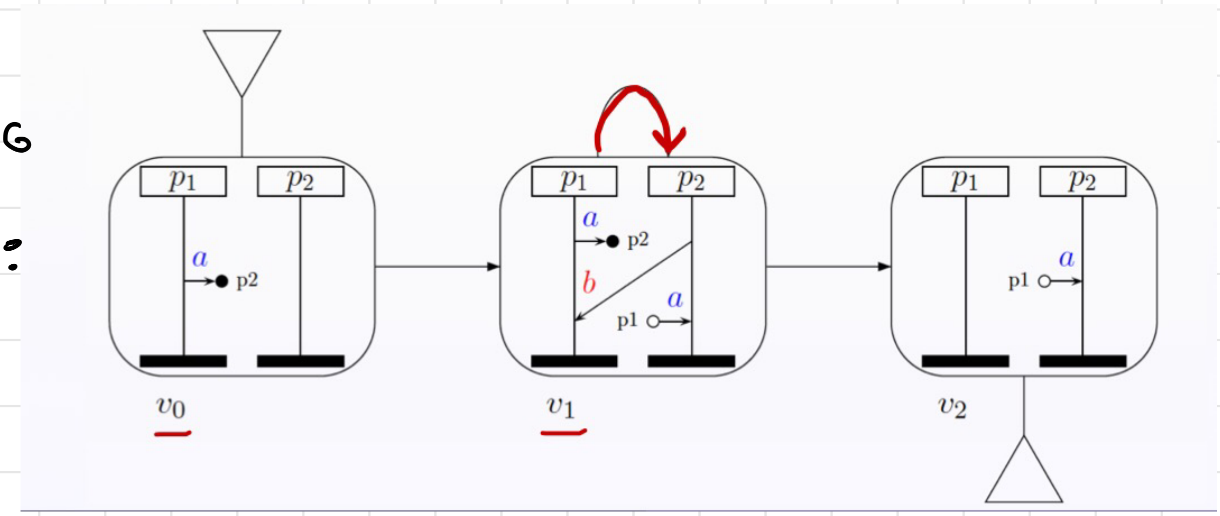
$$M = M_1 \bullet M_2 \bullet \ldots \bullet M_n \quad \text{with} \quad M_i \in \mathbb{M}$$

Thus it cannot be modeled by a MSG.
But it can be modeled as compositional MSG:

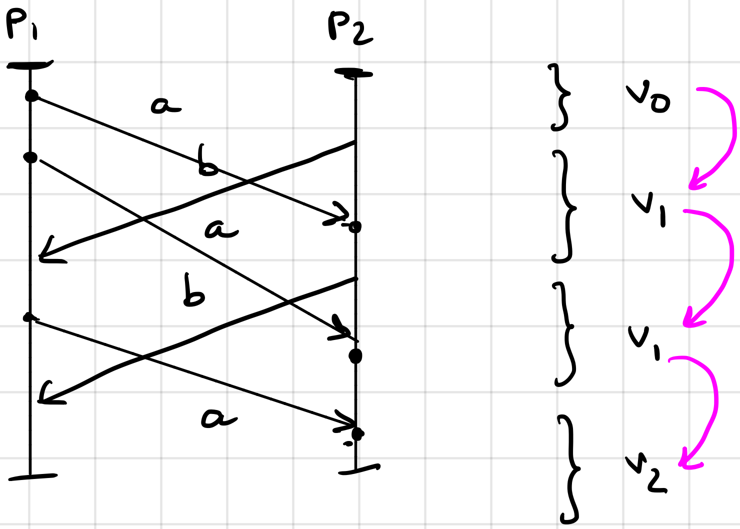


CMSG

$\mathcal{G}$:



MSC $M_1 \in L(\mathcal{G})$



safe

Every accepting path π for G : $\underline{M(\pi) \text{ is an MSC}}$

$\rightarrow M(\pi) \in L(\mathcal{G})$

CMSG $\mathcal{G}$ is called <u>safe</u>

Safe paths and CMSGs

is an MSC

Definition (Safe path)

Path π of CMSG G is safe whenever $\underbrace{M(\pi)}_{\text{CMSC of } \pi} \in \mathbb{M}$.

Existence of a safe accepting path

Theorem: undecidability of existence of a safe path

The decision problem "does CMSG G have at least one safe, accepting path?" is undecidable.

Proof.

By a reduction from Post's Correspondence Problem (PCP).

... black board ... $\square$

The complement decision problem "does CMSG G have no safe, accepting path?" is undecidable too.

Universality of safe accepting paths

Theorem: undecidability of existence of a safe path

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Universality of safe accepting paths

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Theorem: decidability of universality of safe paths

The decision problem "are all accepting paths of CMSG G safe?" is decidable in PTIME.

Universality of safe accepting paths

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The decision problem "does CMSG G have at least one safe, accepting path?" is undecidable.

Theorem: decidability of universality of safe paths

The decision problem "are all accepting paths of CMSG G safe?" is decidable in PTIME.

Proof.

Polynomial reduction to reachability problem in (non-deterministic) pushdown automata.

... see details on the next slides ... □

Pushdown automata

Definition (Pushdown automaton)

A pushdown automaton (PDA, for short) $K = \underline{(Q, q_0, \Gamma, \Sigma, \Delta)}$ with

- Q, a finite set of control states
- $q_0 \in Q$, the initial state
- Γ, a finite stack alphabet — which symbols can be put on the stack
- Σ, a finite input alphabet — a, b, c
- $\Delta \subseteq Q \times \Sigma \times \Gamma \times Q \times \Gamma^*$, the transition relation.

current state

next input symbol that is to be read

top symbol of the stack

next state

next stack content (at the top)

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Transition relation

$(q, a, \gamma, q', \text{pop}) \in \Delta$ means: in state q, on reading input symbol a and top of stack is symbol γ, change to q' and pop γ from the stack.

$L = \{ 0^n 1^n \mid n > 0 \}$

$01 \in L$

$0011 \in L$

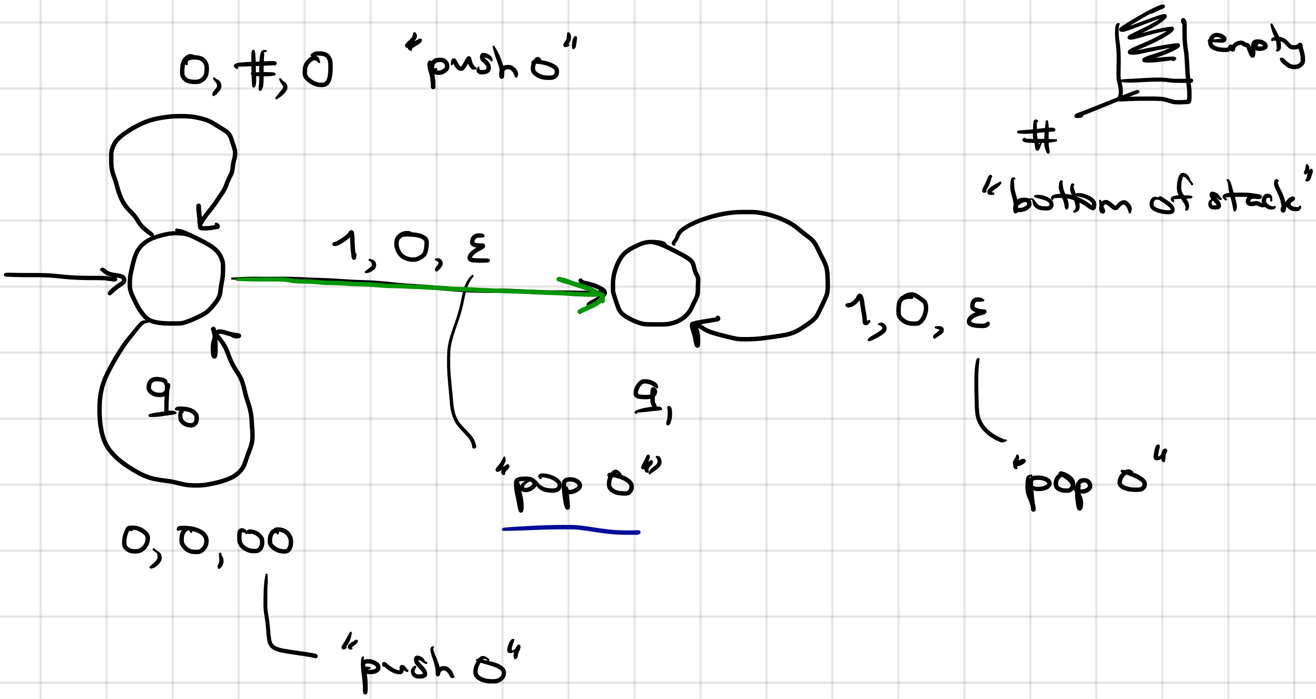
$011 \notin L$

$010 \notin L$

Construct a PDA K such that K accepts the language L

<u>Intuition</u>

- PDA K starts in initial control state q_0
- if input word $w = \varepsilon$ or if w start with a "1" : reject
- otherwise, "scan" all 0s and push them on the stack
- on reading the first "1", move to control state q_1 + pop 0 from the stack
- in q_1, on reading a "1", we pop a "0" from the stack
- in q_1 : reject if a "0" is read, or if $\underbrace{\text{input word is } \varepsilon \text{ but the stack is not}}_{\text{nr. of 0s} > 1\text{s}}$
- in q_1; accept if input word and the stack are both empty.



$Q = \{q_0, q_1\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, \#\}$

$q_0 = q_0$

transitions:

$\underline{(q_0, 1, 0, q_1, \varepsilon)} \in \Delta$

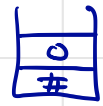
$(q_0, 0, 0, q_0, 00) \in \Delta$

$(q_1, 1, 0, q_1, \varepsilon) \in \Delta$

$(q_0, 0, \#, q_0, 0) \in \Delta$

Example configurations:

$(q_0, \underbrace{0011}_{w}, \underbrace{\#}_{z})$, $(q_1, \underbrace{1}_{w}, \underbrace{0}_{z})$

$z \{$ 

Change of configuration

$(q_0, 0011, \#) \vdash (q_0, 011, 0) \vdash (q_0, 11, 00) \vdash$

$(q_1, 1, 0) \vdash (q_1, \varepsilon, \varepsilon)$

$$\boxed{\text{Is } (q_1, \varepsilon, \varepsilon) \text{ reachable from } (q_0, 0011, \#)\,?}$$

Definition

A configuration c is a triple (state q, stack content Z, rest input w).

control

Reachability in pushdown automata

Definition

A configuration c is a triple (state q, stack content Z, rest input w).

Definition

Given a transition in Δ, a (direct) successor configuration c' of c is obtained: $c \vdash c'$.

Reachability in pushdown automata

Definition

A configuration c is a triple (state q, stack content Z, rest input w).

Definition

Given a transition in Δ, a (direct) successor configuration c' of c is obtained: $c \vdash c'$.

Reachability problem

For configuration c, and initial configuration c_0: $c_0 \vdash^* c$?

Reachability in pushdown automata

Definition

A configuration c is a triple (state q, stack content Z, rest input w).

Definition

Given a transition in Δ, a (direct) successor configuration c' of c is obtained: $c \vdash c'$.

Reachability problem

For configuration c, and initial configuration c_0: $c_0 \vdash^* c$?

Theorem: [Esparza et al. 2000]

The reachability problem for PDA is decidable in PTIME.

Remark: Dyck language

$\Sigma = \{ [,] \}$ square brackets

Dyck language

"send" "receive"

$\{ u \in \Sigma^* \mid$ all prefixes of u contain no more "]" than "[", and the number of "[" equals the number of "]" in u $\}$

linearization

Exercise construct a PDA that accepts the Dyck language.

$$(q_0, \underbrace{00111}_{\notin L}, \#) \vdash (q_0, 0011, 0)$$

$$\vdash (q_0, 011, 00)$$

$$\vdash (q_0, 11, 000)$$

$$\vdash (q_1, 1, 00)$$

$$\vdash (q_1, \varepsilon, 0) \qquad \text{"reject"}$$

Sometimes "reject" is modeled explicitly by a separate control state q_{err}; similarly "accept" by a control state q_F

In our example this means that there are transitions:

$$(q_0, 1, \#, q_{err}, \#)$$

$$(q_1, 0, \#, q_{err}, \#)$$

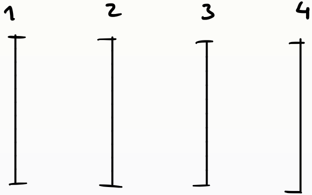
$$(q_1, 0, 0, q_{err}, \#)$$

etcetera.

Checking whether a CMSG is safe is decidable

- Consider any ordered pair (p_i, p_j) of processes in CMSG G

is every accepting path of G safe?



(1,2)	(1,3)	(1,4)
$\boxed{(2,1)}$	(3,1)	(4,1)
(2,3)	(3,2)	$\boxed{(3,4)}$
(2,4)	(4,2)	(4,3)

Checking whether a CMSG is safe is decidable

- Consider any ordered pair (p_i, p_j) of processes in CMSG G
- Proof idea: construct a PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ such that

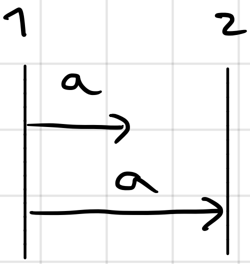
$$\boxed{\text{CMSG } G \text{ is } \underbrace{\text{not safe}}_{\text{in fact "not left-closed"}} \text{ wrt. } (p_i, p_j) \quad \text{iff} \quad \text{PDA } K_{i,j} \text{ accepts}}$$

$K_{i,j}$ is going check for possible violations of being safe

Definition (left-closed CMSC)

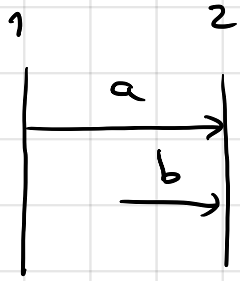
A CMSC is <u>left-closed</u> if it does not contain unmatched receive events, or any send events that are not yet matched and precede other matched send events (of the same type).

Examples



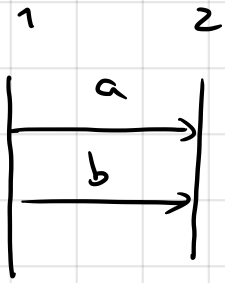
is not left-closed (unsafe)

accept



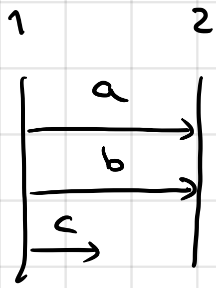
not left-closed (unsafe)

accept



left-closed (and safe)

reject



left-closed (not-safe)

reject

Checking whether a CMSG is safe is decidable

- Consider any ordered pair (p_i, p_j) of processes in CMSG G
- Proof idea: construct a PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ such that

$$\boxed{\text{CMSG } G \text{ is not lc wrt. } (p_i, p_j) \quad \text{iff} \quad \text{PDA } K_{i,j} \text{ accepts}}$$

- For accepting path $u_0 \ldots u_k$ in G, feed $K_{i,j}$ with all words

$$\rho_0 \ldots \rho_k \quad \text{where} \quad \rho_i \in Lin(\lambda(u_i))$$

such that unmatched sends (of some type) precede all unmatched receipts (of the same type)

↳ not a restriction, because such linearisations do always exist

+ assume that matched events are indicated explicitly

!?(p,q,a)

Checking whether a CMSG is safe is decidable

- Consider any ordered pair (p_i, p_j) of processes in CMSG G

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such that unmatched sends (of some type) precede all unmatched receipts (of the same type)

a $i \to j$

b at j from i

- Possible violations that $K_{i,j}$ may encounter:
  1. nr. of unmatched $!(p_i, p_j, \cdot)$ $>$ nr. of unmatched $?(p_j, p_i, \cdot)$
  2. type of k-th unmatched send $\neq$ type of k-th unmatched receive
  3. non-FIFO communication

The nondeterministic PDA $K_{i,j}$

Let $\underbrace{\{a_1, \ldots, a_k\}}$ be the message contents in CMSG G for (p_i, p_j).

all message in CMSG $\mathcal{G}$
send from i to j, or
received at j from i.

The nondeterministic PDA $K_{i,j}$

Let $\{a_1, \ldots, a_k\}$ be the message contents in CMSG G for (p_i, p_j).

Nondeterministic PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ where:

- Control states $Q = \{q_0, \underbrace{q_{a_1}, \ldots, q_{a_k}}, q_{err}, q_F\}$

initial   reject   accept

The nondeterministic PDA $K_{i,j}$

Let $\{a_1, \ldots, a_k\}$ be the message contents in CMSG G for (p_i, p_j).

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- Control states $Q = \{q_0, q_{a_1}, \ldots, q_{a_k}, q_{err}, q_F\}$
- Stack alphabet $\Gamma = \{1, \#\}$

  1 counts nr. of unmatched $!(p_i, p_j, a_m)$, and $\#$ is bottom of stack

The nondeterministic PDA $K_{i,j}$

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- Stack alphabet $\Gamma = \{1, \#\}$

  1 counts nr. of unmatched $!(p_i, p_j, a_m)$, and $\#$ is bottom of stack

  possible symbols in the linearisations
- Input alphabet

$$\Sigma = \begin{cases} \text{unmatched action } !(p_i, p_j, a_m) & \checkmark \\ \text{unmatched action } ?(p_j, p_i, a_m) & \checkmark \\ \text{matched actions } !?(p_i, p_j, a_m) & \checkmark \end{cases}$$

The nondeterministic PDA $K_{i,j}$

Let $\{a_1, \ldots, a_k\}$ be the message contents in CMSG G for (p_i, p_j).

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- Stack alphabet $\Gamma = \{1, \#\}$

  1 counts nr. of unmatched $!(p_i, p_j, a_m)$, and # is bottom of stack
- Input alphabet

$$\Sigma = \begin{cases} \text{unmatched action } !(p_i, p_j, a_m) \\ \text{unmatched action } ?(p_j, p_i, a_m) \\ \text{matched action } !?(p_i, p_j, a_m) \end{cases}$$

- Transition function Δ is described on next slide

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \#, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \#, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G

- On reading $!(p_i, p_j, a_m)$ in q_0, push 1 on stack
  - nondeterministically move to state q_{a_m} or stay in q_0

"seen an unmatched send from p_i to p_j with content a_m"

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \#, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G

- On reading $!(p_i, p_j, a_m)$ in q_0, push 1 on stack
  - nondeterministically move to state q_{a_m} or stay in q_0

pending unmatched send $p_i \to p_j$

- On reading $?(p_j, p_i, a_m)$ in $\underline{q_0}$, proceed as follows:
  - if 1 is on stack, pop it
  - otherwise, i.e., if stack is empty, accept (i.e., move to q_F)

not left-closed

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  - if 1 is on stack, pop it
  - otherwise, i.e., if stack is empty, accept (i.e., move to q_F)
- On reading matched send $!?(p_i, p_j, a_k)$ in q_0
  - stack empty (i.e., equal to $\#$)? ignore input; otherwise, accept

not left-closed

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \#, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G
- On reading $!(p_i, p_j, a_m)$ in q_0, push 1 on stack
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- On reading matched send $!?(p_i, p_j, a_k)$ in q_0
  - stack empty (i.e., equal to $\#$)? ignore input; otherwise, accept
- Ignore the following inputs in state q_0:
  - matched send events $!?(p_j, p_i, a_k)$, and
  - unmatched sends or receipts not related to p_i and p_j

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \#, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G
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  - nondeterministically move to state q_{a_m} or stay in q_0
- On reading $?(p_j, p_i, a_m)$ in q_0, proceed as follows:
  - if 1 is on stack, pop it
  - otherwise, i.e., if stack is empty, accept (i.e., move to q_F)

left-closed.

X
- On reading matched send $!?(p_i, p_j, a_k)$ in q_0
  - stack empty (i.e., equal to $\#$)? ignore input; otherwise, accept
- Ignore the following inputs in state q_0:
  - matched send events $!?(p_j, p_i, a_k)$, and
  - unmatched sends or receipts not related to p_i and p_j

still pending sends/receives

- Remaining input w empty? Accept, if stack non-empty; else reject

Safeness of CMSGs (3)

The behaviour in state q_{a_m} for $0 < m \leqslant k$:

- Ignore all actions except $?(p_j, p_i, a_\ell)$ for all $0 < \ell \leqslant k$

Safeness of CMSGs (3)

The behaviour in state q_{a_m} for $0 < m \leqslant k$:

- Ignore all actions except $?(p_j, p_i, a_\ell)$ for all $0 < \ell \leqslant k$
- On reading $?(p_j, p_i, a_\ell)$ (for some $0 < \ell \leqslant k$) in state q_{a_m} do:
  - if 1 is on top of stack, pop it

Safeness of CMSGs (3)

The behaviour in state q_{a_m} for $0 < m \leqslant k$:

- Ignore all actions except $?(p_j, p_i, a_\ell)$ for all $0 < \ell \leqslant k$
- On reading $?(p_j, p_i, a_\ell)$ (for some $0 < \ell \leqslant k$) in state q_{a_m} do:
  - if 1 is on top of stack, pop it

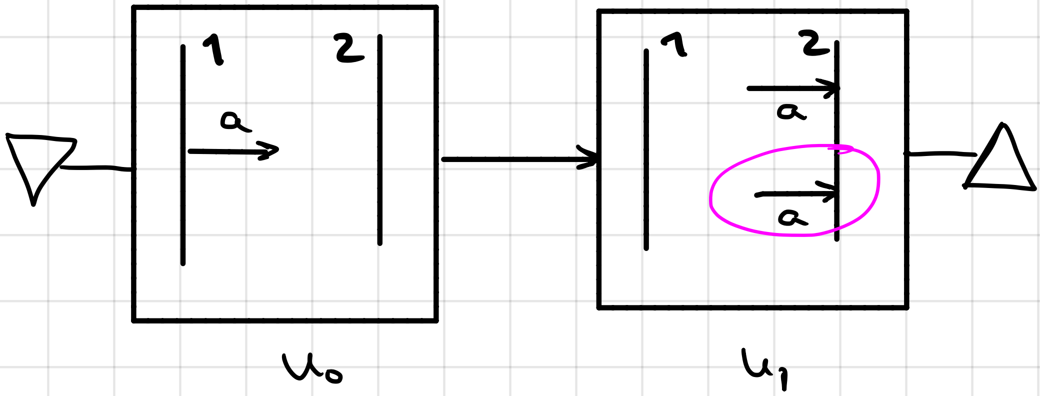
not left-closed

- If stack is empty:
  - if last receive differs from a_m, accept
  - otherwise reject, while ignoring the rest (if any) of the input

left-closed

Example 1.

CMSG $\mathcal{G}_1$



$$\underbrace{(q_0, \#, !a?a?a)}_{\text{initial configuration}} \vdash (q_a, 1, ?a?a) \vdash (q_a, \#, ?a)$$

reject

$$(q_0, \#, !a?a?a) \vdash (q_0, 1, ?a?a) \vdash (q_0, \#, ?a)$$

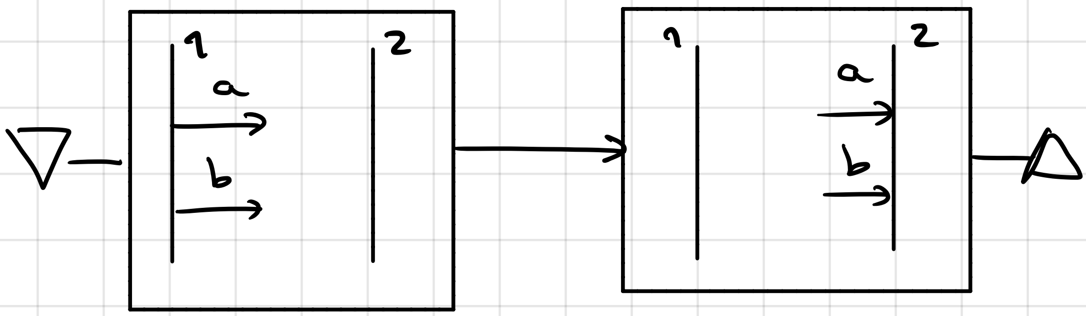
accept

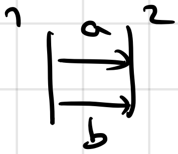
Thus the PDA $K_{1,2}$ accepts the input word $\Longrightarrow$ $\mathcal{G}_1$ is not left-closed

Example 2

CMSG $\mathcal{G}_2$:

safe.



$L(\mathcal{G}) =$ 

PDA $\mathcal{K}_{1,2}$

$(q_0, \#, !a!b?a?b) \vdash (q_a, 1, !b?a?b)$

$(q_0, \#, !a!b?a?b) \vdash (q_0, 1, !b?a?b)$

$(q_0, 1, !b?a?b) \vdash (q_0, 11, ?a?b) \vdash (q_0, 1, ?b) \vdash (q_0, \#, \varepsilon)$ reject

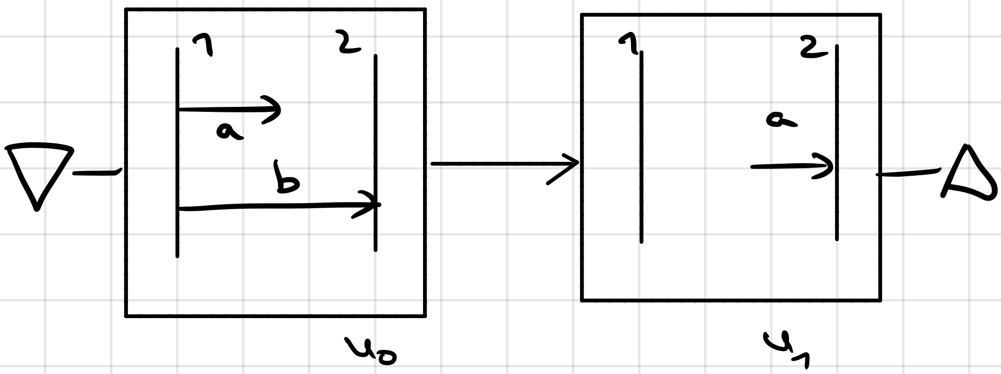
$(q_0, 1, !b?a?b) \vdash (q_b, 11, ?a?b) \vdash (q_b, 1, ?b) \vdash (q_b, \#, \varepsilon)$ reject

$(q_a, 1, !b?a?b) \vdash (q_a, 1, ?a?b) \vdash (q_a, \#, ?b) \vdash (q_a, \#, \varepsilon)$ reject

PDA $\mathcal{K}_{1,2}$ has no accepting run on $!a!b?a?b$ and thus CMSG $\mathcal{G}_2$ is <u>left-closed</u>

Example

CMSG $\mathcal{G}_3$



PDA $K_{1,2}$

$(q_0, \#, \ !a\ !?b\ ?a) \vdash (q_0, 1, \ !?b\ ?a)$

accept

$\top$

$(q_a, 1, \ !?b\ ?a)$

$\longrightarrow$ violation of FIFO property.

$\top$

$(q_a, 1, ?a)$

$\top$

$(q_a, \#, \varepsilon)$

reject

PDA $K_{1,2}$ accepts

$\Longrightarrow$ CMSG $\mathcal{G}_3$ is not left-closed

$\Longrightarrow$ is not safe.

Safeness of CMSGs (4)

It follows: PDA $K_{i,j}$ accepts iff CMSG G is not lc wrt. (p_i, p_j)

$\Longrightarrow$ CMSG G is not lc wrt. (p_i, p_j) iff configuration $\underbrace{(q_F, \cdot, \cdot)}_{\text{accept}}$ is reachable.

Safeness of CMSGs (4)

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$\Longrightarrow$ CMSG G is not lc wrt. (p_i, p_j) iff configuration $(q_F, \cdot, \cdot)$ is reachable.

$\Longrightarrow$ reachability of a configuration in a PDA is in PTIME, hence checking safeness wrt. (p_i, p_j) is in PTIME.

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Time complexity

The worst-case time complexity of checking whether CMSG G is safe is in $\mathcal{O}(k^2 \cdot N^2 \cdot L \cdot |E|^2)$ where $k = |\mathcal{P}|$, $N = |V|$, and $L = |\mathcal{C}|$.

lc

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Lc

Proof.

Checking reachability in PDA $K_{i,j}$ is in $\mathcal{O}(L \cdot |E|^2)$.

Safeness of CMSGs (4)

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$\Longrightarrow$ CMSG G is not safe wrt. (p_i, p_j) iff configuration $(q_F, \cdot, \cdot)$ is reachable.

$\Longrightarrow$ reachability of a configuration in a PDA is in PTIME, hence checking safeness wrt. (p_i, p_j) is in PTIME.

Time complexity

The worst-case time complexity of checking whether CMSG G is $\underbrace{\text{safe}}_{k}$ is in $\mathcal{O}(k^2 \cdot N^2 \cdot L \cdot |E|^2)$ where $k = |\mathcal{P}|$, $N = |V|$, and $L = |\mathcal{C}|$.

Proof.

Checking reachability in PDA $K_{i,j}$ is in $\mathcal{O}(L \cdot |E|^2)$. The number of PDAs is k^2, as we consider ordered pairs in $\mathcal{P}$.

Safeness of CMSGs (4)

It follows: PDA $K_{i,j}$ accepts iff CMSG G is not safe wrt. (p_i, p_j)

$\Longrightarrow$ CMSG G is not safe wrt. (p_i, p_j) iff configuration $(q_F, \cdot, \cdot)$ is reachable.

$\Longrightarrow$ reachability of a configuration in a PDA is in PTIME, hence checking safeness wrt. (p_i, p_j) is in PTIME.

Time complexity

The worst-case time complexity of checking whether CMSG G is $\underbrace{\text{safe}}_{\text{lc}}$ is in $\mathcal{O}(k^2 \cdot N^2 \cdot L \cdot |E|^2)$ where $k = |\mathcal{P}|$, $N = |V|$, and $L = |\mathcal{C}|$.

Proof.

Checking reachability in PDA $K_{i,j}$ is in $\mathcal{O}(L \cdot |E|^2)$. The number of PDAs is k^2, as we consider ordered pairs in $\mathcal{P}$. The number of paths in the CMSG G for each pair that need to be checked is in $\mathcal{O}(N^2)$, as a single traversal for each loop in G suffices. □