Theoretical Foundations of the UML - SS 2020
— Exercise Sheet 3 —

Hand in until May 11, 09:00 am via RWTHmoodle

General Remarks

- The exercises should be solved in groups of *three* students.
- *Only one student per group* is supposed to upload a solution sheet as a PDF file, where the names and matriculation numbers of all the group members have to be explicitly indicated.
- Questions regarding lectures and exercises, if any, are expected in the Q&A sessions via Zoom (instead of e-mails), with the next on Thursday 7 May, at 16:00. Zoom ID: 369 366 100, Password: FUML-QA

Exercise 1 (Definedness of CMSC-Concatenation) (3 Points)

Prove or disprove: There exists a CMSC M_1 with process set $\mathcal{P}_1 = \{p_1, p_2\}$, such that for all CMSC M_2 which satisfy the following side conditions, it holds that $M_1 \bullet M_2$ violates the FIFO property.

The side conditions are:

- For the process set $\mathcal{P}_2$ of M_2 it holds that $\mathcal{P}_2 = \mathcal{P}_1$, and
- M_2 contains an unmatched receive event of the form "p_2 receives message content a from p_1".

Exercise 2 (Non-Associativity of CMSC-Concat.) (3 Points)

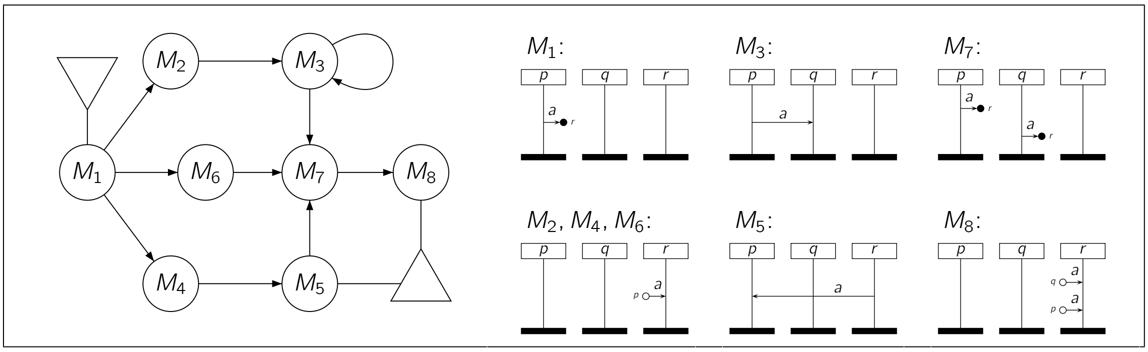
Give three CMSCs M_1, M_2, and M_3, such that

$$(M_1 \bullet M_2) \bullet M_3 \;\neq\; M_1 \bullet (M_2 \bullet M_3)\ ,$$

even though both $(M_1 \bullet M_2) \bullet M_3$ and $M_1 \bullet (M_2 \bullet M_3)$ are defined, i.e., they are valid CMSCs which satisfy the FIFO property.

Exercise 3 (Locally Safe CMSGs) (4 Points)

1. Given the definition of *locally safe* (cf. Definition 1): is the following CMSG locally safe? Justify your answer in detail by calculating the sets $V^1, V^2, \prod$ and arguing about $M(\pi)$, $\pi \in \prod$.



2. Prove the following statement:

   If a CMSG G is locally safe then G is *finitely generated* (cf. Definition 2) and there exists an MSG G', such that $L(G') = L(G)$.

Definition 1: Let the following sets be given for a CMSG $G = \{V, \rightarrow, v_0, F, \lambda\}$,

- $V^1 := \{v_0\} \cup \{v \in V \mid |Succ_G(v)| > 1\} \cup \{v \in F \mid |Succ_G(v)| > 0\}$
- $V^2 := Pred_G(V^1 \setminus \{v_0\}) \cup F$
- $\prod := \{\pi \text{ is path in } G \mid \pi = v_1, \ldots, v_n, n \in \mathbb{N}, v_1 \in V^1, v_n \in V^2 \setminus V^1, v_i \notin V^2 \text{ for } 1 < i < n\} \cup \{v \mid v \in V^1 \cap V^2\}$

A compositional Message Sequence Graph G is called *locally safe* if for every path $\pi \in \prod$: $M(\pi) \in \mathbb{M}$ (i.e., $M(\pi)$ is an MSC).

For a graph $G = (V, \rightarrow)$ and a node $v \in V$:

- $Succ_{\mathcal{G}}(v) := \{(v, w) \in \rightarrow \mid w \in V\}$
- $Pred_{\mathcal{G}}(v) := \{(w, v) \in \rightarrow \mid w \in V\}$ and $Pred_{\mathcal{G}}(V) = \bigcup_{v \in V} Pred_{\mathcal{G}}(v)$

Definition 2: A CMSG G is called *finitely generated* if there is a finite set $\mathcal{B} = \{M_1, \ldots, M_k\}$ of MSCs such that for any $M \in L(G)$ there are $n \in \mathbb{N}$ and indices $i_1, \ldots, i_n \in \{1, \ldots, k\}$ such that $M = \prod_{j=1}^{n} M_{i_j}$.