Theoretical Foundations of the UML - SS 2020
— Exercise Sheet 2 —

Hand in until Monday May 4, 09:00 am via RWTHmoodle

General Remarks

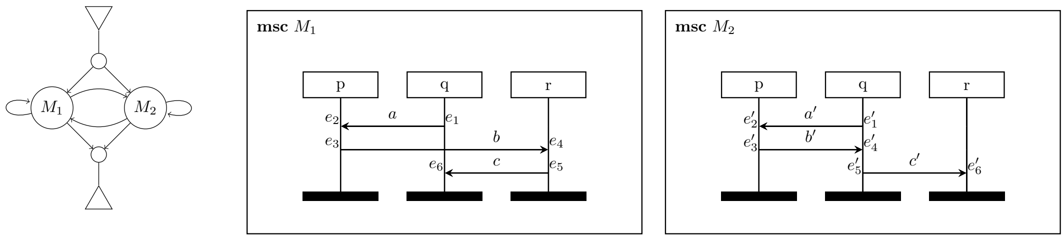
- The exercises should be solved in groups of *three* students.
- *Only one student per group* is supposed to upload a solution sheet as a PDF file, where the names and matriculation numbers of all the group members have to be explicitly indicated.
- Questions regarding the lectures and exercises, if any, are expected in the Q&A session via Zoom (instead of emails), with the next on Thursday 30 April, at 16:00. Zoom ID: 369 366 110, Password: FUML-QA

Exercise 1 (Warm Up) (1 Points)

Prove or disprove: An MSC with the set of processes $\mathcal{P}$ has a race only if $|\mathcal{P}| \geq 3$.

Exercise 2 (Paths, Language and Races in MSGs) (1+2+2 Points)

Consider the MSG $\mathcal{G}$:



1) Give 3 different accepting paths of the MSG $\mathcal{G}$.
2) Determine the MSC language $L(\mathcal{G})$.
3) Determine whether $\mathcal{G}$ contains a race, and if so give all its races. Justify your answer.

Exercise 3 (Strong Concatenation) (4+2+2 Points)

As presented in the second lecture, the (weak) concatenation of two MSCs M_1 and M_2 (denoted by $M_1 \bullet_w M_2$), intuitively is constructed by gluing the process lines together such that M_1 is situated on top of M_2 (cf. Figure 2.1).

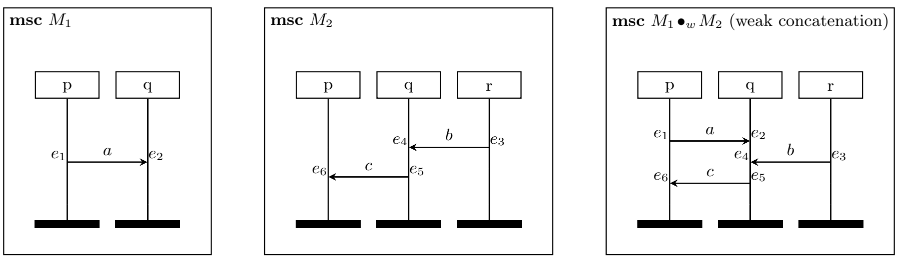
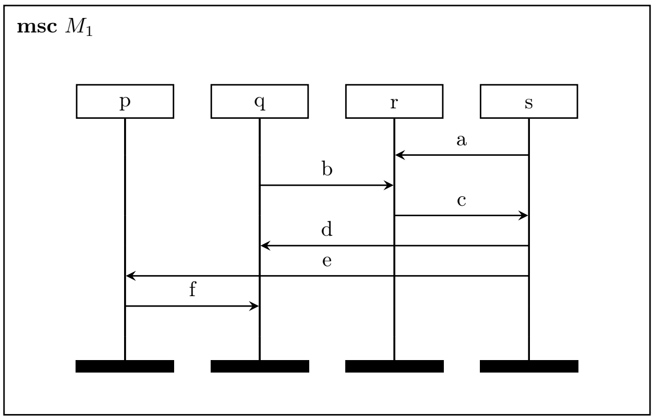


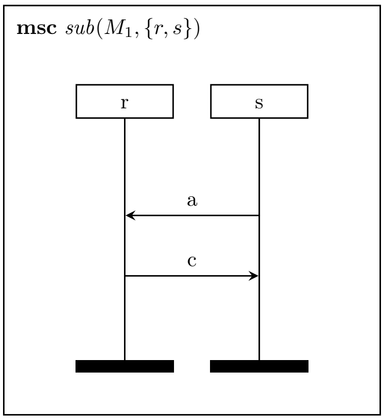
Figure 2.1: Two MSCs and their weak concatenation

1) Define the variant operator called *strong concatenation* $\bullet_s$ of two MSCs M_1 and M_2, namely, all events of MSC M_1 have to be executed before the first event of M_2. For this purpose determine a structure $M = M_1 \bullet_s M_2 := (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$, for $M_i = (\mathcal{P}_i, E_i, \mathcal{C}_i, l_i, m_i, \preceq_i)$ for $i \in \{1, 2\}$, that results from concatenating the two MSCs strongly.
2) For M_1 and M_2 in Figure 2.1, draw the Hasse diagrams of $M_1 \bullet_w M_2$ and $M_1 \bullet_s M_2$, respectively.
3) Prove or disprove: (M_1 is race-free $\wedge$ M_2 is race-free) $\implies$ $M_1 \bullet_s M_2$ is race-free.

Exercise 4 (Sub-MSCs) (2+2+2 Points)

For an MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$ and a subset of processes $\mathcal{P}' \subseteq \mathcal{P}$ we consider the sub-MSC $sub(M, \mathcal{P}')$ which arises from M by erasing all processes in $\mathcal{P} \setminus \mathcal{P}'$ as well as the incoming and outgoing messages of these processes. For example, we depict an MSC M_1 and the sub-MSC $sub(M_1, \{r, s\})$ below.





1) Provide the formal definition of the MSC $sub(M, \mathcal{P}')$.
2) Prove or disprove: M is race-free $\implies$ for any $\mathcal{P}' \subseteq \mathcal{P}$, $sub(M, \mathcal{P}')$ is race-free.
3) Prove or disprove: $sub(M, \mathcal{P}')$ is race-free for some $\mathcal{P}' \subseteq \mathcal{P}$ $\implies$ M is race-free.

Exercise 5 (Race-Freeness) (6 Points)

It was shown in the lecture that (weak) concatenation of MSCs does not preserve race-freeness:

$$(M_1 \text{ is race-free} \wedge M_2 \text{ is race-free}) \;\not\Longrightarrow\; M_1 \bullet M_2 \text{ is race-free.}$$

We will now have a look at the other direction:

Proposition 2.1:

$$(M_1 \textit{ has a race} \vee M_2 \textit{ has a race}) \implies M_1 \bullet M_2 \textit{ has a race.}$$

Prove or disprove Proposition 2.1.