Semantics and Verification of Software

Summer Semester 2019

Lecture 14: Extension by Blocks and Procedures I (Operational Semantics)

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Outline of Lecture 14

Blocks and Procedures

- Extension of WHILE by nested blocks with local variables and recursive procedures

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- Important: scope of variable and procedure identifiers

  static scoping: scope of identifier = declaration environment
  (also: “lexical” scoping; here)

  dynamic scoping: scope of identifier = calling environment
  (old Algol/Lisp dialects)

Extension by Blocks and Procedures

Static and Dynamic Scoping

Example 14.1

```
begin
  var x;
  var y;
  proc P is
    y := x
  end;
  x := 1;
  begin
    var x;
    x := 2;
    call P
  end
end
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dynamic scoping $\Rightarrow$ y = 2

Outline of Lecture 14

Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Procedure identifiers	$PVar = \{\mathtt{P}, \mathtt{Q}, \ldots\}$	P
Procedure declarations	$PDec$	p
Variable declarations	$VDec$	v
Commands (statements)	Cmd	c

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Context-free grammar:

$$p ::= \mathtt{proc}\ P\ \mathtt{is}\ c\ \mathtt{end};p \mid \varepsilon \in PDec$$
$$v ::= \mathtt{var}\ x;v \mid \varepsilon \in VDec$$
$$c ::= \mathtt{skip} \mid x := a \mid c_1;c_2 \mid \mathtt{if}\ b\ \mathtt{then}\ c_1\ \mathtt{else}\ c_2\ \mathtt{end} \mid \mathtt{while}\ b\ \mathtt{do}\ c\ \mathtt{end} \mid \mathtt{call}\ P \mid \mathtt{begin}\ v\ p\ c\ \mathtt{end} \in Cmd$$

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- All used variable/procedure identifiers have to be declared
- Identifiers declared within a block must be distinct

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- Now: explicit control over all (nested) instances of a variable:
  - variable environments

    $$\mathit{VEnv} := \{\rho \mid \rho : \mathit{Var} \dashrightarrow \mathit{Loc}\}$$

    (partial function to maintain declaredness information)
  - locations

    $$\mathit{Loc} := \mathbb{N}$$

  - stores

    $$\mathit{Sto} := \{\sigma \mid \sigma : \mathit{Loc} \dashrightarrow \mathbb{Z}\}$$

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$\Rightarrow$ Two-level access to a variable $x \in \mathit{Var}$:

1. determine current memory location of x:

   $$l := \rho(x)$$

2. reading/writing access to σ at location l

- Thus: previous state information represented as $\sigma \circ \rho : \mathit{Var} \dashrightarrow \mathbb{Z}$

Procedure Environments and Declarations

- Effect of procedure call determined by its body and variable and procedure environment of its declaration:

$$PEnv := \{\pi \mid \pi : PVar \dashrightarrow Cmd \times VEnv \times PEnv\}$$

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- Effect of declaration: update of environment (and store)

$$\begin{aligned} \mathrm{upd}_v[\![.]\!] &: VDec \times VEnv \times Sto \to VEnv \times Sto \\ \mathrm{upd}_v[\![\texttt{var } x\,; v]\!](\rho,\sigma) &:= \mathrm{upd}_v[\![v]\!](\rho[x \mapsto l_x], \sigma[l_x \mapsto 0]) \\ \mathrm{upd}_v[\![\varepsilon]\!](\rho,\sigma) &:= (\rho,\sigma) \\ \mathrm{upd}_p[\![.]\!] &: PDec \times VEnv \times PEnv \to PEnv \\ \mathrm{upd}_p[\![\texttt{proc } P \texttt{ is } c \texttt{ end}\,; p]\!](\rho,\pi) &:= \mathrm{upd}_p[\![p]\!](\rho, \pi[P \mapsto (c,\rho,\pi)]) \\ \mathrm{upd}_p[\![\varepsilon]\!](\rho,\pi) &:= \pi \end{aligned}$$

  where $l_x := \min\{l \in Loc \mid \sigma(l) = \bot\}$

Outline of Lecture 14

Execution Relation I

Definition 14.2 (Execution relation)

For $c \in \mathit{Cmd}$, $\sigma, \sigma' \in \mathit{Sto}$, $\rho \in \mathit{VEnv}$, and $\pi \in \mathit{PEnv}$, the execution relation $(\rho, \pi) \vdash \langle c, \sigma \rangle \to \sigma'$ ("in environment (ρ, π), statement c transforms store σ into σ'") is defined by the following rules:

$$\text{(skip)}\ \frac{}{(\rho, \pi) \vdash \langle \texttt{skip}, \sigma \rangle \to \sigma}$$

$$\text{(asgn)}\ \frac{\langle a, \sigma \circ \rho \rangle \to z}{(\rho, \pi) \vdash \langle x := a, \sigma \rangle \to \sigma[\rho(x) \mapsto z]}$$

$$\text{(seq)}\ \frac{(\rho, \pi) \vdash \langle c_1, \sigma \rangle \to \sigma' \qquad (\rho, \pi) \vdash \langle c_2, \sigma' \rangle \to \sigma''}{(\rho, \pi) \vdash \langle c_1 ; c_2, \sigma \rangle \to \sigma''}$$

$$\text{(if-t)}\ \frac{\langle b, \sigma \circ \rho \rangle \to \text{true} \qquad (\rho, \pi) \vdash \langle c_1, \sigma \rangle \to \sigma'}{(\rho, \pi) \vdash \langle \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2\ \texttt{end}, \sigma \rangle \to \sigma'}$$

Execution Relation II

Definition 14.2 (Execution relation; continued)

$$\text{(if-f)}\ \frac{\langle b, \sigma \circ \rho\rangle \to \text{false} \qquad (\rho, \pi) \vdash \langle c_2, \sigma\rangle \to \sigma'}{(\rho, \pi) \vdash \langle \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2\ \texttt{end}, \sigma\rangle \to \sigma'}$$

$$\text{(wh-f)}\ \frac{\langle b, \sigma \circ \rho\rangle \to \text{false}}{(\rho, \pi) \vdash \langle \texttt{while}\ b\ \texttt{do}\ c\ \texttt{end}, \sigma\rangle \to \sigma}$$

$$\text{(wh-t)}\ \frac{\langle b, \sigma \circ \rho\rangle \to \text{true} \quad (\rho, \pi) \vdash \langle c, \sigma\rangle \to \sigma' \quad (\rho, \pi) \vdash \langle \texttt{while}\ b\ \texttt{do}\ c\ \texttt{end}, \sigma'\rangle \to \sigma''}{(\rho, \pi) \vdash \langle \texttt{while}\ b\ \texttt{do}\ c\ \texttt{end}, \sigma\rangle \to \sigma''}$$

$$\text{(call)}\ \frac{(\rho', \pi'[P \mapsto (c, \rho', \pi')]) \vdash \langle c, \sigma\rangle \to \sigma'}{(\rho, \pi) \vdash \langle \texttt{call}\ P, \sigma\rangle \to \sigma'} \qquad \text{if } \pi(P) = (c, \rho', \pi')$$

$$\text{(block)}\ \frac{\text{upd}_v[\![v]\!](\rho, \sigma) = (\rho', \sigma') \qquad \text{upd}_p[\![p]\!](\rho', \pi) = \pi' \qquad (\rho', \pi') \vdash \langle c, \sigma'\rangle \to \sigma''}{(\rho, \pi) \vdash \langle \texttt{begin}\ v\ p\ c\ \texttt{end}, \sigma\rangle \to \sigma''}$$

Execution Relation III

Remarks:

- Evaluation of (arithmetic and Boolean) expressions can now fail due to undeclared variables
- In rules for composite statements, the execution of sub-statements can have an effect on the environments (due to nested environments), but this effect is transient
- Static scoping is modelled in (call) by using the environments ρ' and π' (as determined in (block)) from the declaration site of procedure P (and not ρ and π from the calling site)
- In (call), the procedure environment associated with procedure P, π', is extended by a P-entry to handle recursive calls of P:

$$\pi'[P \mapsto (c, \rho', \pi')]$$

Execution Relation IV

Example 14.3

```
begin
  var x; var y;                                              } v
  proc F is
    begin
      var z;
      z:=x;
      if z=1 then skip
             else x:=x-1; call F; y:=z*y end                 } c2   } c1   } cF   } p   } c
    end
  end;
  x:=2; y:=1; call F                                         } c0
end
```

Let $\sigma_\emptyset(l) = \rho_\emptyset(x) = \pi_\emptyset(P) = \bot$ for all $l \in Loc, x \in Var, P \in PVar$

Notation: $\sigma_{ijkl} \Leftrightarrow \sigma(0) = i, \sigma(1) = j, \sigma(2) = k, \sigma(3) = l$

Derivation tree for $(\rho_\emptyset, \pi_\emptyset) \vdash \langle c, \sigma_\emptyset \rangle \to \sigma_{1221}$: on the board