Semantics and Verification of Software

Summer Semester 2019

Lecture 1: Introduction

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https://moves.rwth-aachen.de/teaching/ss-19/sv-sw/

Software Modeling and Verification Chair

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Staff

- Lectures: Thomas Noll
  - Lehrstuhl Informatik 2, Room 4211
  - E-mail noll@cs.rwth-aachen.de
- Exercise classes:
  - Christoph Matheja (matheja@cs.rwth-aachen.de)
  - Kevin Batz (later)
- Student assistant: Wanted!!!
  - Evaluation of exercises
  - Organisational support
  - 12 hrs/week contract
  - Previous experience with theory of programming not a prerequisite (but of course helpful)

Target Audience

- MSc Informatik:
  - Theoretische Informatik
- MSc Software Systems Engineering:
  - Theoretical Foundations of SSE
- In general:
  - interest in formal models for programming languages
  - application of mathematical reasoning methods
- Expected: basic knowledge in
  - essential concepts of imperative programming languages
  - formal languages and automata theory
  - mathematical logic

Organisation

- Schedule:
  - Lecture Mon 14:30–16:00 AH 6 (starting 15 April)
  - Lecture Thu 10:30–12:00 5056 (starting 4 April)
  - Exercise class Fri 10:30–12:00 5056 (starting 26 April)
- Irregular lecture dates – checkout web page!
  - in particular, 2nd lecture on 12 April at 5056
- 1st assignment sheet: 18 April on web page
  - submission by 26 April before exercise class
  - presentation on 26 April
- Work on assignments in groups of three
- Examination (6 ECTS credits):
  - oral or written (depending on number of participants)
  - date to be fixed
- Admission requires at least 50% of the points in the exercises
- Written material in English, lecture and exercise classes “on demand”, rest up to you

Aspects of Programming Languages

Syntax: "How does a program look like?"

- hierarchical composition of programs from structural components

$\Rightarrow$ *Compiler Construction*

Semantics: "What does this program mean?"

- output/behaviour/... in dependence of input/environment/...

$\Rightarrow$ this course

Pragmatics: "Is the programming language practically usable?"

- length and understandability of programs
- learnability of programming language
- appropriateness for specific applications, ...

$\Rightarrow$ *Software Engineering*

Historic development:

- Formal syntax since 1960s (scanners, LL/LR parsers); semantics defined by compiler/interpreter
- Formal semantics since 1970s (operational/denotational/axiomatic)

Why Semantics?

Idea: ultimate semantics = compiler!

- Compiler gives each individual program a semantics
  (= “behaviour” of generated machine code)

But:

- Compilers are highly complicated software systems
  - code optimisations
  - memory management
  - interaction with runtime system
  - ...

  $\Rightarrow$ inappropriate level of abstraction
- Most languages have more than one compiler (with different outputs)
- Most compilers have bugs

$\Rightarrow$ Does not help with formal reasoning about programming language or individual programs

The Semantics of "Semantics"

Originally: study of meaning of symbols (linguistics)

Semantics of a program: meaning of a concrete program

- mapping input $\rightarrow$ output values
- interaction behaviour (shared variables, communication, synchronisation, ...)
- ...

Semantics of a programming language: mapping of each (syntactically correct) program of a programming language to its meaning

Semantics of software: various techniques for defining the semantics of diverse programming languages

- operational
- denotational
- axiomatic
- ...

Motivation for Rigorous Formal Treatment I

Example 1.1

1. How often will the following loop be traversed?

```
for i := 2 to 1 do ...
```

   FORTRAN IV: once
   PASCAL: never

2. What if `p = nil` in the following program?

```
while p <> nil and p^.key < val do ...
```

   Pascal: strict boolean operations ↯
   Modula: non-strict boolean operations ✓

3. Are the following assignments to `b` equivalent?

```
boolean f(int x, int y){return (x == 0) && (y == 0);}
boolean b = f(1, 2/0);
boolean b = (1 == 0) && (2/0 == 0);
```

Motivation for Rigorous Formal Treatment II

- Support for development of
  - new programming languages: missing details, ambiguities and inconsistencies can be recognised
  - compilers: automatic compiler generation from appropriately defined semantics
  - programs: exact understanding of semantics avoids uncertainties in the implementation of algorithms
- Support for correctness proofs of
  - programs: comparison of program semantics with expected behaviour (e.g., termination properties, absence of deadlocks, ...)
  - compilers:

$$\begin{array}{ccc} \text{programming language} & \xrightarrow{\text{compiler}} & \text{machine code} \\ \downarrow \text{semantics} & & \downarrow \text{(simple) semantics} \\ \text{meaning} & \overset{?}{=} & \text{meaning} \end{array}$$

  - source-code transformations:

$$\begin{array}{ccc} \text{code} & \xrightarrow{\text{transform}} & \text{code} \\ \downarrow \text{semantics} & & \downarrow \text{semantics} \\ \text{meaning} & \overset{?}{=} & \text{meaning} \end{array}$$

Transformational vs. Reactive Systems

Transformational systems

- "Classical" model for (sequential) software systems

$$\mathit{Program} : \mathit{Input} \to \mathit{Output}$$

- Ignores aspect of interaction between concurrent activities (processes, environment, ...)
- Finite behaviour – non-terminating execution considered as error case
- The approach we will follow here

Reactive systems

- System maintains ongoing interaction with environment and/or among system components
- Infinite behaviour – terminating execution considered as error case
- Examples:
  - operating systems
  - embedded systems controlling mechanical or electrical devices (planes, cars, home appliances, ...)
  - power plants, production lines, ...

$\Rightarrow$ *Concurrency Theory*

(Complementary) Kinds of Formal Semantics

Operational semantics: describes computation of the program on some (very) abstract machine (G. Plotkin)

- example: $\text{(seq)}\dfrac{\langle c_1, \sigma\rangle \to \sigma' \quad \langle c_2, \sigma'\rangle \to \sigma''}{\langle c_1 ; c_2, \sigma\rangle \to \sigma''}$
- application: implementation of programming languages (compilers, interpreters, ...)

Denotational semantics: mathematical definition of input/output relation of the program by induction on its syntactic structure (D. Scott, C. Strachey)

- example: $\mathfrak{C}[\![.]\!] : Cmd \to (\Sigma \dashrightarrow \Sigma)$
  $\mathfrak{C}[\![c_1 ; c_2]\!] := \mathfrak{C}[\![c_2]\!] \circ \mathfrak{C}[\![c_1]\!]$
- application: program analysis

Axiomatic semantics: formalisation of special properties of programs by logical formulae (assertions/proof rules; R. Floyd, T. Hoare)

- example: $\text{(seq)}\dfrac{\{A\}\, c_1\, \{C\} \quad \{C\}\, c_2\, \{B\}}{\{A\}\, c_1 ; c_2\, \{B\}}$
- application: program verification

Overview of the Course

1. The imperative model language WHILE
2. Operational semantics of WHILE
3. Denotational semantics of WHILE
4. Equivalence of operational and denotational semantics
5. Axiomatic semantics of WHILE
6. Applications: compiler correctness etc.
7. Extensions: procedures etc.

Literature

- Formal semantics
  - G. Winskel: *The Formal Semantics of Programming Languages*, The MIT Press, 1996
- Compiler correctness
  - H.R. Nielson, F. Nielson: *Semantics with Applications: An Appetizer*, Springer Undergraduate Topics in Computer Science, 2007

Syntactic Categories

WHILE: simple imperative programming language without procedures or advanced data structures

Syntactic categories:

Category	Domain	Meta variable
Numbers	$\mathbb{Z} = \{0, 1, -1, \ldots\}$	z
Truth values	$\mathbb{B} = \{\text{true}, \text{false}\}$	t
Variables	$\mathit{Var} = \{\mathrm{x}, \mathrm{y}, \ldots\}$	x
Arithmetic expressions	*AExp* (next slide)	a
Boolean expressions	*BExp* (next slide)	b
Commands (statements)	*Cmd* (next slide)	c

Syntax of WHILE Programs

Definition 1.2 (Syntax of WHILE)

The syntax of WHILE Programs is defined by the following context-free grammar:

$a ::= z \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \in \mathit{AExp}$
$b ::= t \mid a_1 = a_2 \mid a_1 > a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in \mathit{BExp}$
$c ::= \texttt{skip} \mid x := a \mid c_1 \,;\, c_2 \mid \texttt{if } b \texttt{ then } c_1 \texttt{ else } c_2 \texttt{ end} \mid \texttt{while } b \texttt{ do } c \texttt{ end} \in \mathit{Cmd}$

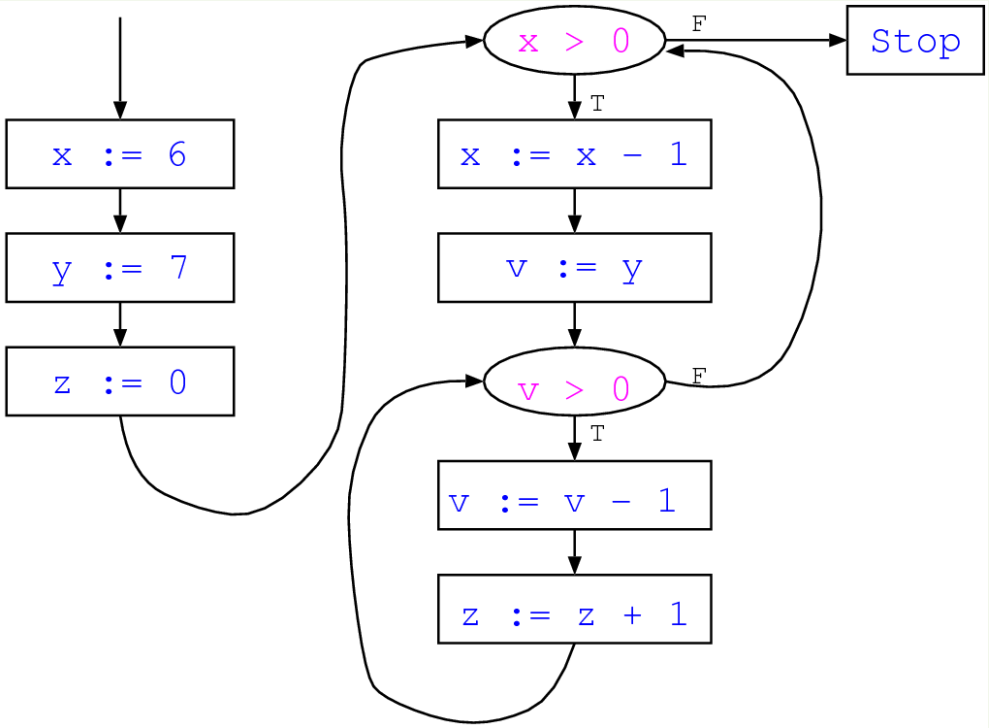
Remarks: we assume that

- the syntax of numbers, truth values and variables is predefined (i.e., no “lexical analysis”)
- the syntactic interpretation of ambiguous constructs (expressions) is uniquely determined (by brackets or priorities)

A WHILE Program and Its Control-Flow Diagram

Example 1.3

```
x := 6;
y := 7;
z := 0;
while x > 0 do
  x := x - 1;
  v := y;
  while v > 0 do
    v := v - 1;
    z := z + 1
  end
end
```



Effect: `z := x * y = 42`