Exercise Sheet 7

Due date: June 21$^\text{th}$. Please hand in your solutions at the start of the exercise class.

Task 1: Interpretation of Hoare Triples (25 Points)

Match each of the following Hoare triples

(a) $\{P\}\, c\, \{\mathsf{true}\}$

(b) $\{P\}\, c\, \{\Downarrow \mathsf{true}\}$

(c) $\{\mathsf{true}\}\, c\, \{\mathsf{false}\}$

(d) $\{\mathsf{true}\}\, c\, \{\Downarrow \mathsf{false}\}$

(e) $\{\mathsf{false}\}\, c\, \{\Downarrow Q\}$

with their corresponding interpretation from 1–6. (There may be more than one Hoare triple with the same interpretation in 1–6.) Assume that P and Q contain no logical variable.

1. Program c diverges for all initial states.

2. The program never finishes in a final state satisfying Q.

3. The triple does not say anything about program c; it is logically equivalent to true.

4. The triple is logically equivalent to false.

5. None of the above. Provide yourself the interpretation of the triple.

6. Program c always terminate whenever executed in an initial state that satisfies P.

Task 2: Derived Rules (25 Points)

Which of the following rules hold true? For valid rules, provide a justification (no formal proof required). For rules that do not hold, provide a counterexample.

(a) $\dfrac{\{P\}\, c\, \{Q_1\} \qquad \{P\}\, c\, \{Q_2\}}{\{P\}\, c\, \{Q_1 \wedge Q_2\}}$

(b) $\dfrac{\{P\}\, c\, \{Q_1\} \qquad \{P\}\, c\, \{Q_2\}}{\{P\}\, c\, \{Q_1 \vee Q_2\}}$

(c) $\dfrac{\{P_1\}\, c\, \{Q\} \qquad \{P_2\}\, c\, \{Q\}}{\{P_1 \wedge P_2\}\, c\, \{Q\}}$

(d) $\dfrac{\{P_1\}\, c\, \{Q\} \qquad \{P_2\}\, c\, \{Q\}}{\{P_1 \vee P_2\}\, c\, \{Q\}}$

(e) $\dfrac{\{P\}\, c\, \{Q\}}{\{P \wedge R\}\, c\, \{Q \wedge R\}}$

(f) $\dfrac{\{P\}\, c\, \{Q\}}{\{P \wedge R\}\, c\, \{Q \vee R\}}$

Task 3: Strongest Postconditions (50 Points)

Intuitively, the *strongest liberal postcondition* $\mathtt{slp}[\![c]\!]\,(A)$ of a program c and a precondition A is the strongest postcondition B that holds when running c on a state satisfying A. In contrast to weakest liberal preconditions, we thus apply forward reasoning.

(a) Formalize the intuitive definition of strongest liberal postconditions from above, i.e. give an exact definition of the set of states described by $\mathtt{slp}[\![c]\!]\,(A)$.

(b) Express the validity of a Hoare triple $\{A\}c\{B\}$ in terms of strongest liberal postconditions (cf. Lecture 11, Corollary 11.5.1).

(c) Determine $\mathtt{slp}[\![\textsf{while true do skip end}]\!]\,(\textsf{true})$.

(d) Define $\mathtt{slp}[\![c]\!]\,(A)$ by induction on the structure of c.

(e) Apply your definition from (d) to compute the following strongest liberal postcondition:

$$\mathtt{slp}[\![x := 2 * x; y := x + 2; z := y + x]\!]\,(x = 1)$$

(f) Prove or disprove: For every program $c \in \textsf{Cmd}$, we have

$$\models \{\mathtt{wlp}[\![c]\!]\,(\mathtt{slp}[\![c]\!]\,(\mathit{false}))\}c\{\mathtt{slp}[\![c]\!]\,(\mathtt{wlp}[\![c]\!]\,(\mathit{false}))\}\ .$$