

---

## Exercise Sheet 6

**Due date:** June 7<sup>th</sup>. Please hand in your solutions at the start of the exercise class.

### Task 1: Partial Correctness Properties (30 Points)

Goldbach's conjecture states that every even natural number  $n \in \mathbb{N}$  can be written as the sum of two primes  $p, q \in \mathbb{N}$ . Such a pair  $(p, q)$  is called a *Goldbach partition* of  $n$ .

- (a) Show that there exists a partial correctness property  $\{A\}c\{B\}$  of a program  $c$  that computes a Goldbach partition of any given natural number  $n$ . (You do not have to search for such a program  $c$ , it suffices to find suitable assertions  $A, B$ .)
- (b) Does the existence of a program  $c$  satisfying the partial correctness property from (a) prove Goldbach's conjecture? Justify your answer.

### Task 2: Expressivity (30 Points)

Prove that the language of Boolean expressions  $\text{BExp}$  (i.e. assertions without logical variables) is *not* expressive. That is, provide a program  $c \in \text{Cmd}$  and a Boolean expression  $B \in \text{BExp}$  such that there is no  $A \in \text{BExp}$  with  $A = \text{wlp}[[c]](B)$ .

*Hint:* You may assume that there is no Boolean expression  $b$  such that for every state  $\sigma$

$$\mathfrak{B}[[b]]\sigma = \text{true} \quad \text{iff} \quad \sigma(x) \text{ is even .}$$

### Task 3: Total Correctness (40 Points)

Consider the following program  $c$ :

```
while  $x \neq 0$  do  
   $x := x - 1$   
end
```

Prove or disprove each of the following partial and total correctness properties. In case the property holds, prove it in Hoare logic. Otherwise, provide an initial state witnessing the violation of the property.

- (a)  $\{\text{true}\}c\{\text{true}\}$
- (b)  $\{x > 0\}c\{\downarrow \text{true}\}$
- (c)  $\{\text{true}\}c\{\downarrow \text{true}\}$