Lehrstuhl für Informatik 2
Softwaremodellierung und Verifikation
RWTH AACHEN UNIVERSITY

Exercise Sheet 4

Due date: May 24^{th}. Please hand in your solutions at the start of the exercise class.

Task 1: Tarski–Kantorovich Principle (20 points)

Prove or disprove: Let $(D, \sqsubseteq)$ be a CCPO and let $F\colon D \to D$ be continuous. Moreover, let $d \in D$, such that $d \sqsubseteq F(d)$.

Then F has at least one fixpoint larger than d and the least of those fixpoints is given by

$$\bigsqcup \{F^n(d) \mid n \in \mathbb{N}\} \ .$$

Does this also hold if $d \not\sqsubseteq F(d)$? Justify your answer.

Task 2: Denotational Semantics (80 points)

Consider the following recursive program for $n \in \mathbb{Z}$:

$$\text{fac}(n) := \textsf{if } (n = 0) \textsf{ then } \{1\} \textsf{ else } \{\text{fac}(n-1) * n\};$$

1. Determine the functional $\Phi : (\mathbb{Z} \dashrightarrow \mathbb{Z}) \to (\mathbb{Z} \dashrightarrow \mathbb{Z})$ for $\text{fac}(n)$, as in the lecture.
2. Show that Φ is monotonic and continuous.
3. Show that the partial order $(\mathbb{Z} \dashrightarrow \mathbb{Z}, \sqsubseteq)$ is chain complete.
4. Let $\mathfrak{C}[\![\text{fac}(n)]\!]$ be defined by $\text{fix}(\Phi)$. Compute the denotational semantics of $\text{fac}(3)$.
5. Prove that the program fac calculates the factorial, i.e. $\text{fix}(\Phi)(n) = n!$ for any $n \in \mathbb{Z}$.