Exercise Sheet 2

Due date: May 3rd. Please hand in your solutions at the start of the exercise class.

Task 1: Big-Step Operational Semantics (10 points)

Extend the rule system defining the big-step execution relation $\rightarrow$ from the lecture (cf. Definition 3.2) to incorporate a statement $\mathsf{repeat}\,\{c\}\,\mathsf{until}\,(b)$. Your rules may *not* depend on the existence of the while statements.

Task 2: Termination (15 points)

Show that $\langle \mathsf{while}\ (b)\ \mathsf{do}\ \{c\}, \sigma\rangle \rightarrow \sigma'$ implies that $\langle b, \sigma'\rangle \rightarrow \mathsf{false}$.

Task 3: Decomposition Lemma for AM programs (20 points)

The following statement is known as the *decomposition lemma* for AM programs:

Let $c_1, c_2 \in \mathsf{Cmd}$ and $pc \in \{0, ..., |\mathfrak{T}_c[\![c_1]\!]| - 1\}$. If

$$\mathfrak{T}_c[\![c_1]\!]; \mathfrak{T}_c[\![c_2]\!] \vdash \langle pc, e, \sigma\rangle \rhd^k \langle |\mathfrak{T}_c[\![c_1]\!]; \mathfrak{T}_c[\![c_2]\!]|, e'', \sigma''\rangle,$$

then there exists a configuration $\langle pc', e', \sigma'\rangle$ and $k_1, k_2 \in \mathbb{N}$ with $k = k_1 + k_2$ such that

$$\mathfrak{T}_c[\![c_1]\!] \vdash \langle pc, e, \sigma\rangle \rhd^{k_1} \langle |\mathfrak{T}_c[\![c_1]\!]|, e', \sigma'\rangle$$

and

$$\mathfrak{T}_c[\![c_1]\!]; \mathfrak{T}_c[\![c_2]\!] \vdash \langle |\mathfrak{T}_c[\![c_1]\!]|, e', \sigma'\rangle \rhd^{k_2} \langle |\mathfrak{T}_c[\![c_1]\!]; \mathfrak{T}_c[\![c_2]\!]|, e'', \sigma''\rangle.$$

Prove that the decomposition lemma is correct. You may use the following proposition without giving an explicit proof by structural induction:

$$\forall j \in \mathbb{N} : \mathfrak{T}_c[\![c]\!] \vdash \langle pc, e, \sigma\rangle \rhd^j \langle pc', e', \sigma'\rangle \text{ implies } pc' \in \{0, 1, ..., |\mathfrak{T}_c[\![c]\!]|\},$$

where $pc \in \{0, ..., |\mathfrak{T}_c[\![c_1]\!]| - 1\}$.

Task 4: Soundness of Command Translation (30 points)

Prove Lemma 5.12 from the lecture. That is, show that, for every $c \in \mathsf{Cmd}$, $\sigma, \sigma' \in \Sigma$, and $e \in \mathit{Stk}$, we have

$$\mathfrak{T}_c[\![c]\!] \vdash \langle 0, \varepsilon, \sigma\rangle \rhd^* \langle |\mathfrak{T}_c[\![c]\!]|, e, \sigma'\rangle \qquad \text{implies} \qquad \langle c, \sigma\rangle \rightarrow \sigma' \text{ and } e = \varepsilon.$$

Task 5: AM Semantics (25 points)

a) Extend the translation function $\mathfrak{T}_c$ such that programs containing statements of the form $\textsf{repeat}\ \{c\}\ \textsf{until}\ (b)$ can be translated directly into `AM` code.

b) Prove that for all for all $\sigma, \sigma' \in \Sigma$,

$$\begin{array}{ll} & \langle \textsf{repeat}\ \{c\}\ \textsf{until}\ (b), \sigma \rangle \to \sigma' \\ \text{implies} & \mathfrak{T}_{\textsf{repeat}\ \{c\}\ \textsf{until}\ (b)}[\![c]\!] \vdash \langle 0, \varepsilon, \sigma \rangle \ \rhd^* \ \langle |\mathfrak{T}_c[\![\textsf{repeat}\ \{c\}\ \textsf{until}\ (b)]\!]|, e, \sigma' \rangle \end{array}$$