Static Program Analysis

Lecture 16: Abstract Interpretation VII (Limits & Improvements of CEGAR)

Summer Semester 2018

Thomas Noll
Software Modeling and Verification Group
RWTH Aachen University

https://moves.rwth-aachen.de/teaching/ss-18/spa/

Software Modeling and Verification Chair

RWTH AACHEN UNIVERSITY

Schedule of Forthcoming Lectures

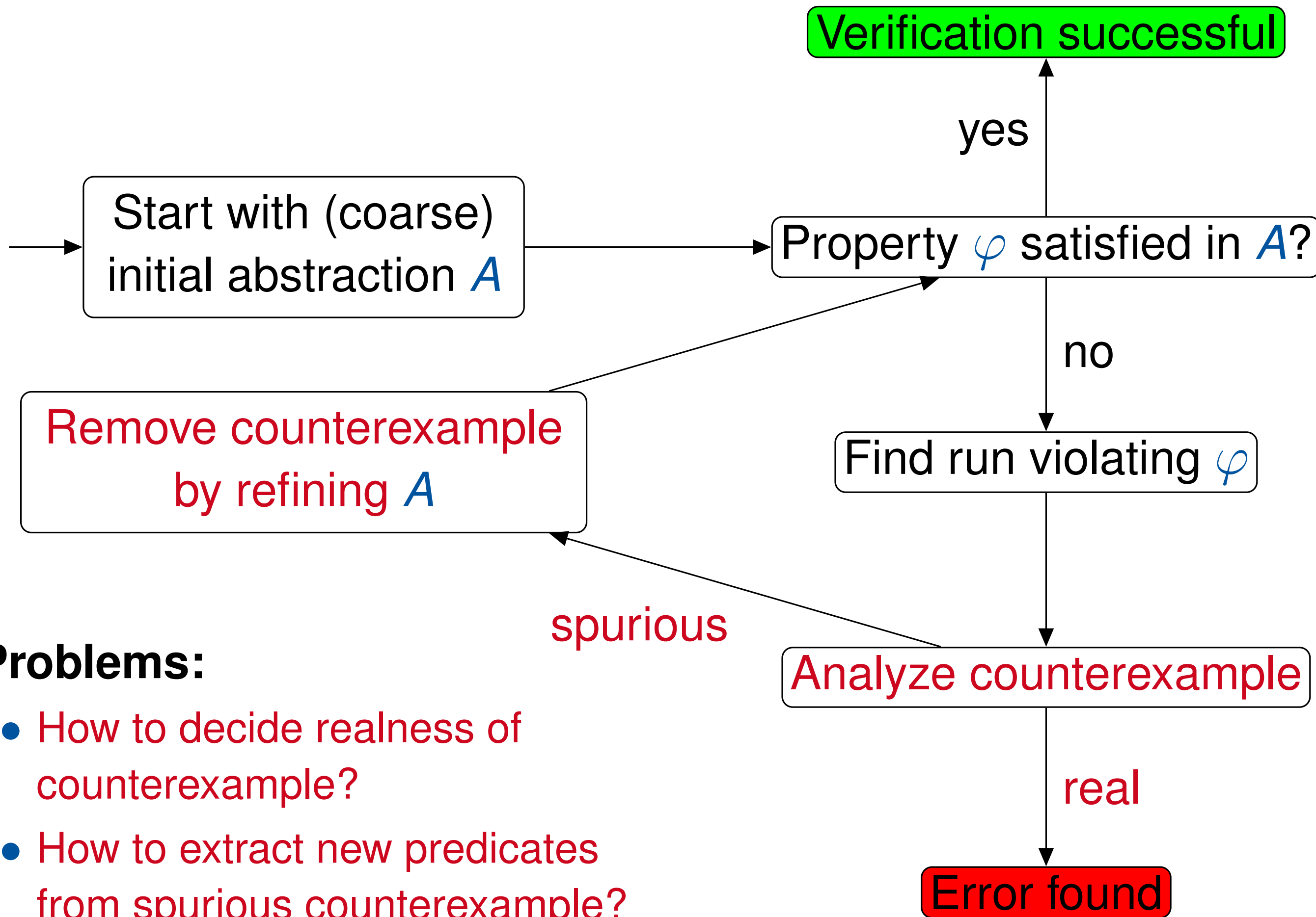
June 25/26: Interprocedural dataflow analysis

July 02/03: Pointer and shape analysis

July 09(/10): Wrap-Up

Recap: Counterexample-Guided Abstraction Refinement (CEGAR)

Reminder: CEGAR



Problems:

- How to decide realness of counterexample?
- How to extract new predicates from spurious counterexample?

Recap: Counterexample-Guided Abstraction Refinement (CEGAR)

Properties of Interest

- A certain program location is not reachable (dead code)
- Division by zero is excluded
- The value of `x` never becomes negative
- After program termination, the value of `y` is even

$\Rightarrow$ All representable as (non-)reachability of "bad locations"

$\Rightarrow$ Counterexample = path to bad locations

Definition (Counterexample)

- A counterexample is a sequence of $k \geq 1$ abstract transitions of the form

$$\langle c_0, Q_0 \rangle \Rightarrow \langle c_1, Q_1 \rangle \Rightarrow \ldots \Rightarrow \langle c_k, Q_k \rangle$$

  where
  - $c_0, \ldots, c_k \in Cmd$ (or $c_k = \downarrow$)
  - $Q_0, \ldots, Q_k \in Abs(P)$ with $Q_0 \equiv \mathsf{true}$ and $Q_k \not\equiv \mathsf{false}$
- It is called real if there exist concrete states $\sigma_0, \ldots, \sigma_k \in \Sigma$ such that

$$\forall i \in \{1, \ldots, k\} : \sigma_i \models Q_i \quad \text{and} \quad \langle c_{i-1}, \sigma_{i-1} \rangle \rightarrow \langle c_i, \sigma_i \rangle$$

- Otherwise it is called spurious.

Elimination of Spurious Counterexamples

Lemma

If $\langle c_0, \text{true}\rangle \Rightarrow \langle c_1, Q_1\rangle \Rightarrow \ldots \Rightarrow \langle c_k, Q_k\rangle$ *is a spurious counterexample, there exist Boolean expressions* $b_0, \ldots, b_k$ *with* $b_0 \equiv \text{true}$, $b_k \equiv \text{false}$, *and*

$$\forall i \in \{1, \ldots, k\}, \sigma, \sigma' \in \Sigma : \sigma \models b_{i-1} \land \langle c_{i-1}, \sigma\rangle \rightarrow \langle c_i, \sigma'\rangle \implies \sigma' \models b_i$$

Proof (idea).

Inductive definition of b_i as strongest postconditions:

1. $b_0 := \text{true}$
2. for $i = 1, \ldots, k$: definition of b_i depending on b_{i-1} and on (axiom) transition rule applied in $\langle c_{i-1}, .\rangle \Rightarrow \langle c_i, .\rangle$:

Atomic commands:

- (skip) $b_i := b_{i-1}$
- (asgn) $b_i := \exists x'.(b_{i-1}[x \mapsto x'] \land x = a[x \mapsto x'])$
  (for "$x := a$"; x' = previous value of x)

Control structures (with guard b):

- (if1) $b_i := b_{i-1} \land b$
- (if2) $b_i := b_{i-1} \land \neg b$
- (wh1) $b_i := b_{i-1} \land b$
- (wh2) $b_i := b_{i-1} \land \neg b$

(yields $b_k \equiv \text{false}$; by induction on k) □

Recap: Counterexample-Guided Abstraction Refinement (CEGAR)

Abstraction Refinement

- Using $b_1, \ldots, b_{k-1}$ as computed before, let $P' := P \cup \{p_1, \ldots, p_n\}$ where $p_1, \ldots, p_n$ are the atomic conjuncts occurring in $b_1, \ldots, b_k$
- Refine $Abs(P)$ to $Abs(P')$

Lemma

After refinement, the spurious counterexample

$$\langle c_0, \text{true}\rangle \Rightarrow \langle c_1, Q_1\rangle \Rightarrow \ldots \Rightarrow \langle c_k, Q_k\rangle$$

with $Q_k \not\equiv \text{false}$ *does not exist any more.*

Proof.

omitted □

Where CEGAR Fails

Example 16.1

- c :=

```
[x := a]^0;
[y := b]^1;
while [¬(x = 0)]^2 do
  [x := x - 1]^3;
  [y := y - 1]^4
end;
if [a = b ∧ ¬(y = 0)]^5 then
  [skip]^6
else
  [skip]^7
end
```

- Interesting property: label 6 unreachable

- Initial abstraction: $P = \emptyset$ $(\implies Abs(P) = \{\text{true}, \text{false}\})$
- Abstraction refinement: on the board
- Observation: iteration yields predicates of the form

$$\texttt{x = a-}k \quad \text{and} \quad \texttt{y = b-}k$$

for all $k \in \mathbb{N}$

- Actually required: loop invariant

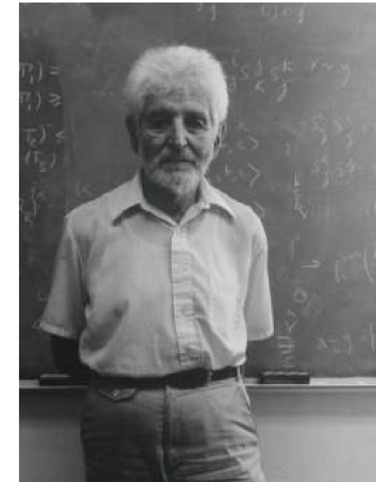
$$\texttt{a = b} \implies \texttt{x = y}$$

but predicate x = y not generated in CEGAR loop

Craig Interpolation

- **Problem:** predicates often unnecessarily complex and involving "irrelevant" variables
- **Idea:** consider only variables that are relevant for previous and future part of execution

William Craig (1918–2016)



Definition 16.2 (Craig interpolant)

Let $b_1, b_2 \in BExp$ where $b_1 \models b_2$. A Craig interpolant of b_1 and b_2 is a formula $b_3 \in BExp$ with $b_1 \models b_3$, $b_3 \models b_2$, and $Var_{b_3} \subseteq Var_{b_1} \cap Var_{b_2}$.

Using Craig Interpolants I

1. Begin with spurious counterexample $\langle c_0, \text{true}\rangle \Rightarrow \langle c_1, Q_1\rangle \Rightarrow \ldots \Rightarrow \langle c_k, Q_k\rangle$ (according to Definition 15.1)
2. Construct strongest postconditions $s_0, \ldots, s_k$ with $s_0 \equiv \text{true}$, $s_k \equiv \text{false}$ (according to Lemma 15.2)
3. Analogously it is possible to construct weakest preconditions $w_0, \ldots, w_k$ with $w_0 \equiv \text{true}$, $w_k \equiv \text{false}$ starting from w_k
   i. $w_k := \text{false}$
   ii. for $i = k-1, \ldots, 0$: definition of w_i depending on w_{i+1} and on (axiom) transition rule applied in $\langle c_i, .\rangle \Rightarrow \langle c_{i+1}, .\rangle$:
      - (skip) $w_i := w_{i+1}$
      - (asgn) $w_i := w_{i+1}[x \mapsto a]$
      - (if1) $w_i := (w_{i+1} \wedge b) \vee \neg b \equiv w_{i+1} \vee \neg b$
      - (if2) $w_i := w_{i+1} \vee b$
      - (wh1) $w_i := w_{i+1} \vee \neg b$
      - (wh2) $w_i := w_{i+1} \vee b$
4. Possible to show: $s_i \models w_i$ for each $i \in \{0, \ldots, k\}$
5. For each $i \in \{0, \ldots, k\}$, choose Craig interpolant b_i of s_i and w_i
6. Refine abstraction by atomic conjuncts occurring in $b_1, \ldots, b_{k-1}$

Remark: Craig interpolants always exist for first-order formulae (but are not necessarily unique)

Using Craig Interpolants II

Example 16.3 (cf. Example 15.3)

Let $c_0 :=$ `[x := z]`^{0}`;[z := z + 1]`^{1}`;[y := z]`^{2}`;`
`if [x = y]`^{3} `then [skip]`^{4} `else [skip]`^{5} `end`

1. Spurious counterexample: $\langle 0, \text{true}\rangle \Rightarrow \langle 1, \text{true}\rangle \Rightarrow \langle 2, \text{true}\rangle \Rightarrow \langle 3, \text{true}\rangle \Rightarrow \langle 4, \text{true}\rangle$
2. Strongest postconditions (cf. Example 15.3): $s_0 = \text{true}$
$s_1 = (\mathtt{x} = \mathtt{z})$
$s_2 = (\mathtt{z} = \mathtt{x} + 1)$
$s_3 = (\mathtt{z} = \mathtt{x} + 1 \wedge \mathtt{y} = \mathtt{z})$
$s_4 = \text{false}$
3. Weakest preconditions w_i: on the board
   - (skip) $w_i := w_{i+1}$
   - (asgn) $w_i := w_{i+1}[x \mapsto a]$
   - (if1) $w_i := (w_{i+1} \wedge b) \vee \neg b \equiv w_{i+1} \vee \neg b$
   - (if2) $w_i := w_{i+1} \vee b$
   - (wh1) $w_i := w_{i+1} \vee \neg b$
   - (wh2) $w_i := w_{i+1} \vee b$
4. Craig interpolants b_i with $s_i \models b_i$, $b_i \models w_i$, $Var_{b_i} \subseteq Var_{s_i} \cap Var_{w_i}$: on the board

CEGAR in Practice

SLAM Tool

- was: “Software, Languages, Analysis, and Modeling”

  “SLAM originally was an acronym but we found it too cumbersome to explain. We now prefer to think of ‘slamming’ the bugs in a program.”
  (T. Ball, B. Cook, V. Levin, S.K. Rajamani: SLAM and Static Driver Verifier: Technology Transfer of Formal Methods inside Microsoft, IFM 2004)
- First implementation of CEGAR for C programs
- Checks behavioural requirements of software interfaces
  - e.g., “a thread may not acquire a lock it has already acquired, or release a lock it does not hold”
- Supports recursive procedures, pointers, and memory allocation
- Sub-tools:
  - `C2bp`: C program $\times$ Predicates $\longrightarrow$ Boolean program (Boolean variables = predicates)
  - BEPOP: symbolic (BDD-based) model checker for (recursive) Boolean programs
  - `newton`: abstraction refinement
- Developed into commercial product (Static Driver Verifier – SDV; part of Windows Driver Foundation development kit)
  - T. Ball, V. Levin, S.K. Rajamani: *A Decade of Software Model Checking with SLAM*. Comm. ACM 54(7), 2011, 68–76
- WWW: `http://research.microsoft.com/en-us/projects/slam/`

CEGAR in Practice

CPAchecker Tool

- CPA: “Configurable Program Analysis”
- Java re-implementation of Berkeley Lazy Abstraction Software Verification Tool (BLAST)
- Software model checker for C programs
- Uses CEGAR with Craig interpolation and lazy abstraction
  - abstraction is constructed on-the-fly
  - model locally refined on demand
  - enables use of different predicates at different program points

  $\Rightarrow$ “abstract reachability tree”
- Sucessfully applied to C programs with $>$ 130,000 LOC
  - D. Beyer, M.E. Keremoglu: *CPAchecker: A Tool for Configurable Software Verification*. Proc. CAV, 2011, 184–190
- WWW: `http://cpachecker.sosy-lab.org/`

CEGAR in Practice

Practical Experiences

(cf. V. D'Silva, D. Kroening, G. Weissenbacher: *A Survey of Automated Techniques for Formal Software Verification*, IEEE Trans. on CAD of Integrated Circuits and Systems 27(7), 2008, 1165–1178)

- Predicate abstraction & CEGAR suitable for checking control-flow-related safety properties
  - predicates good for representation of control flow
  - safety ("Nothing bad is going to happen.") goes well with over-approximation
  - liveness ("Eventually something good will happen.") requires under-approximation
- Does not work well with complex heap-based data structures or arrays
  ($\Rightarrow$ Pointer/Shape Analysis)
- (Real) counterexamples often more useful than correctness proof
- Abstraction refinement cycle may not terminate
- Main application field: safety properties of device drivers and systems code up to 50 kLOCs