General Remarks

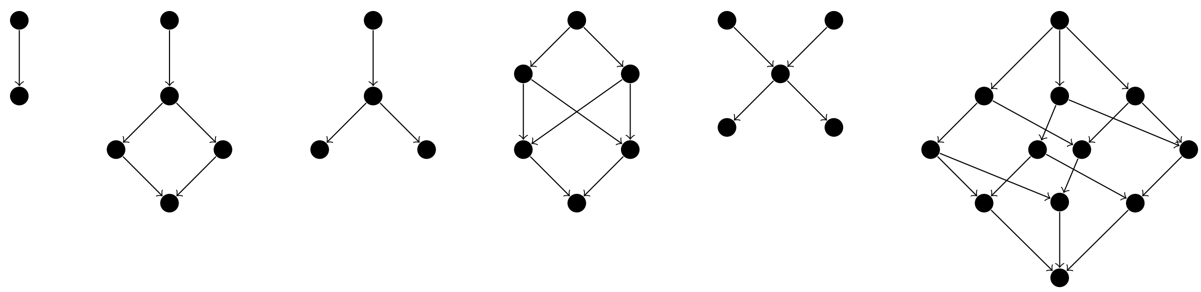
- If you have questions regarding the exercises and/or lecture, feel free to write me an email (`matheja@cs.rwth-aachen.de`) or visit me at the chair (E1, room 4206).
- Please hand in your solutions in groups of three. If you are still looking for a group or your group has less than three members, please use the L2P or contact me after the exercise class.
- Solutions to programming exercises have to be handed in via L2P. Please submit the whole code framework as a single zip file.
- You can hand in your solutions of theoretical exercises online via L2P or at the beginning of the exercise class.
- Please do *not* use the L2P to hand in large high resolution photos/scans of handwritten solutions.
- If you cannot access the L2P due to registration issues please contact me as soon as possible.
- Solutions to all exercises will be published in L2P. All other material, such as slides and exercise sheets, are distributed on our webpage.

Exercise 1 (Complete Lattices): (10 Points)

One can depict a finite partial order as a graph, where the elements are represented by the nodes and the order relation is given as (the transitive closure of) directed edges (we omit the trivial self-loops at every node). An edge between two nodes indicates, that the corresponding elements are comparable where the direction of the edge is determined by the order and there is no element that lies “in between”.

a) Depict the complete lattice of Live Variables Analysis with $Var_c := \{x, y, z\}$ where $\top$ is the upper node.

b) Consider the following graphs and decide for each if it represents a lattice. Justify you answer.



c) Modify the partial order $(\mathbb{Z}_{\leq 0}, \leq)$ such that it forms a complete lattice satisfying ACC. Justify your answer.

Exercise 2 (Safety Properties): (20 Points)

Let $(D, \sqsubseteq)$ be a partial order with a largest element $\top$.
We call a set $M \subseteq D$ *glb-closed* if $\top \in M$ and if $X \subseteq M$ then $\sqcap X$ exists in D and $\sqcap X \in M$.

a) Show that $(M, \sqsubseteq)$ is a complete lattice if $M \subseteq D$ is *glb-closed*.

b) Let Σ be a finite set and $\Sigma^{\leq\omega}$ be the set of all finite and infinite sequences $\sigma = \sigma_1\sigma_2\ldots$ over Σ, where $\sigma_1, \sigma_2, \ldots \in \Sigma$. A set $S \subseteq \Sigma^{\leq\omega}$ is called a *safety property* if

$$\forall\sigma \in \Sigma^{\leq\omega} : (\sigma \notin S) \Leftrightarrow (\exists k \geq 1 : \sigma \downarrow k \notin S) ,$$

where $\sigma \downarrow k := \sigma_1\sigma_2\ldots\sigma_{min\{k,|\sigma|\}}$. Let $Safe(\Sigma^{\leq\omega})$ denote the set of all all safety property over Σ.

Show that $(\mathcal{P}(Safe(\Sigma^{\leq\omega})), \subseteq)$ is a complete lattice, where $\mathcal{P}(S)$ denotes the powerset of set S.

Hint: You may assume that $(\mathcal{P}(\Sigma^{\leq\omega}), \subseteq)$ is a partial order.

Exercise 3 (Fixed Point Theory): (20 Points)

Let $(D, \sqsubseteq)$ be a complete lattice. Moreover, let $F : D \to D$ be a function.
We call F *continuous* if and only if $\forall\emptyset \neq X \subseteq D : F(\sqcup X) = \sqcup\{F(x) \mid x \in X\}$.

a) Show that if F is continuous then

$$\mathrm{fix}(F) = \bigsqcup \{F^k(\bot) \mid k \in \mathbb{N}\} .$$

is a fixed point of F.

b) Give an example of a complete lattice $(D, \sqsubseteq)$ and a continuous function F such that there exists no natural number $k \in \mathbb{N}$ such that

$$F^k(\bot) = F^{k+1}(\bot) .$$

c) Does there exist an example as in (b) if $(D, \sqsubseteq)$ additionally satisfies ACC? Justify your answer.

d) Assume that F is continuous and let $d \in D$. Prove or disprove:

$$F(d) \sqsubseteq d \text{ implies } \mathrm{fix}(F) \sqsubseteq d .$$

Exercise 4 (Sign Analysis): (30 Points)

a) Develop a (non-trivial) *sign analysis* that determines for each program variable whether it is positive, zero, or negative. The analysis should be an instance of the data flow analysis framework from the lecture. Furthermore, it should be based on an abstraction mapping all negative numbers to the symbol $-$, zero to the symbol 0, and all positive numbers to $+$.

For example, the set $\{0, 2, 3, 5, 7, 11, 13, 17, \ldots\}$ is abstracted to $\{0, +\}$.

b) Show that the domain defined in your solution to *a)* satisfies `ACC`.

c) Show that the transfer functions defined in your solution to *a)* are indeed monotonic.

Exercise 5 (Implementation Task): (20 Points)

Please find an update to the code framework used in the first exercise in L2P (`code-framework-02.zip`).

a) Implement the sign analysis designed in exercise 2.3. This analysis should be executed if the `-sgn` option is set. As in the previous exercise, our code framework already contains a skeleton for implementing this analysis in the package `de.rwth.i2.spa.signAnalysis`.

This time, however, you have to implement all components of a data flow system instead of transfer functions only. More precisely, class `SignDomain` captures elements of the complete lattice, class `SignAnalysis` specifies the data flow system and class `SignTransferFunctions` specifies all transfer functions.

Please implement all methods that are marked with a `// TODO` comment. Do not change any method signatures. Of course you can add other classes, methods and variables.

Again, we provide a few example programs to test your implementation. For example, you can run your analysis on the file `examples/Example01.java` as follows:

```
java -jar target/spa-0.2-jar-with-dependencies.jar -cp examples -cn Example01 -m f -sgn
```

b) Write a Java program such that your implemented sign analysis does *not* yield the precise result. Please add the output of your analysis to the solution and mark the lines where the result is not precise.