Overview

Introduction
Modelling parallel systems
Linear Time Properties
Regular Properties
Linear Temporal Logic (LTL)
Computation Tree Logic
- syntax and semantics of CTL
- expressiveness of CTL and LTL
- CTL model checking
- CTL with fairness ⟵
- counterexamples/witnesses, CTL^+ and CTL^*

Equivalences and Abstraction

Complexity of CTL and LTL model checking CTLFAIR4.4-1

Complexity of CTL and LTL model checking

LTL model checking problem:

PSPACE-complete and solvable in time

$$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(|\varphi|))$$

CTL model checking problem:

solvable in polynomial time

$$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot |\Phi|)$$

LTL model checking problem:

PSPACE-complete and solvable in time

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CTL model checking problem:

solvable in polynomial time (even PTIME-complete)

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Complexity of CTL and LTL model checking

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LTL with fairness: $\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(|\varphi| + |\mathit{fair}|))$

CTL model checking problem:

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CTL model checking problem:

solvable in polynomial time (even PTIME-complete)

$$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot |\Phi|)$$

CTL with fairness: $\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot |\Phi| \cdot |\mathit{fair}|)$

Recall: LTL fairness assumptions

CTLFAIR4.4-2

Recall: LTL fairness assumptions

are conjunctions of **LTL** formulas of the form

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\psi \rightarrow \Box\Diamond\phi$
- weak fairness $\Diamond\Box\psi \rightarrow \Box\Diamond\phi$

where ϕ, ψ are propositional formulas

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Reduction of $\models_{fair}$ to $\models$

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iff for all paths π in $\mathcal{T}$:

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CTL fairness assumptions

CTL fairness assumptions

conjunctions of “formulas” of the type

- unconditional fairness: $\Box\Diamond\Phi$
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where Ψ, Φ are CTL state formulas

CTL fairness assumptions

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note: CTL fairness assumptions

- are <u>not</u> CTL (state or path) formulas
- just a syntactic formalism to specify fairness assumptions

CTL fairness assumptions

conjunctions of "formulas" of the type

- unconditional fairness: $\Box\Diamond\Phi$
- strong fairness: $\Box\Diamond\Psi \rightarrow \Box\Diamond\Phi$
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where Ψ, Φ are CTL state formulas

e.g., a strong CTL fairness assumption has the form:

$$fair \;=\; \bigwedge_{1\leq j\leq k} (\Box\Diamond\Psi_j \rightarrow \Box\Diamond\Phi_j)$$

where Ψ_j, Φ_j are CTL state formulas

Satisfaction relation for CTL with fairness

$s \models_{fair} \mathit{true}$

$s \models_{fair} a$ iff $a \in L(s)$

$s \models_{fair} \neg\Phi$ iff $s \not\models_{fair} \Phi$

$s \models_{fair} \Phi_1 \wedge \Phi_2$ iff $s \models_{fair} \Phi_1$ and $s \models_{fair} \Phi_2$

Satisfaction relation for CTL with fairness

$s \models_{fair} true$		
$s \models_{fair} a$	iff	$a \in L(s)$
$s \models_{fair} \neg\Phi$	iff	$s \not\models_{fair} \Phi$
$s \models_{fair} \Phi_1 \wedge \Phi_2$	iff	$s \models_{fair} \Phi_1$ and $s \models_{fair} \Phi_2$
$s \models_{fair} \exists\varphi$	iff	there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$

Satisfaction relation for CTL with fairness

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$s \models_{fair} \neg\Phi$	iff	$s \not\models_{fair} \Phi$
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$s \models_{fair} \exists\varphi$	iff	there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$
$s \models_{fair} \forall\varphi$	iff	for all $\pi \in Paths(s)$: $\pi \models fair$ implies $\pi \models_{fair} \varphi$

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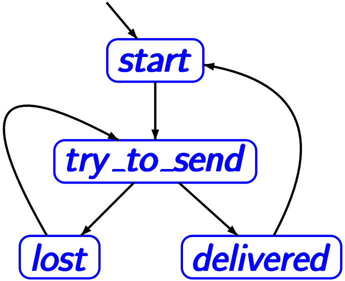
$s \models_{fair} \exists\varphi$	iff	there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$
$s \models_{fair} \forall\varphi$	iff	for all $\pi \in Paths(s)$: $\pi \models fair$ implies $\pi \models_{fair} \varphi$

e.g., $s_0\, s_1\, s_2 \ldots \models \Box\Diamond\Phi$ iff $\overset{\infty}{\exists}\, i \geq 0$ s.t. $s_i \models \Phi$

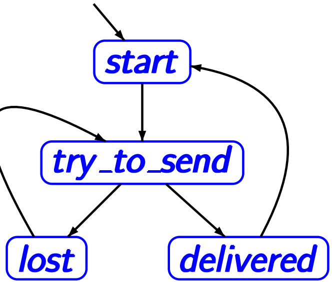
Simple communication protocol



CTL formula

$\Phi = \forall\Box\forall\Diamond start$

Simple communication protocol

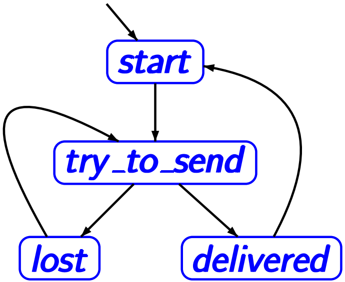


CTL formula

$\Phi = \forall\Box\forall\Diamond \mathit{start}$

$\mathcal{T} \not\models \Phi$

Simple communication protocol



CTL formula

$\Phi = \forall\Box\forall\Diamond start$

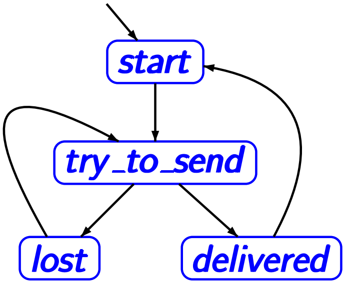
$\mathcal{T} \not\models \Phi$

$\mathcal{T} \models_{ufair} \Phi$

unconditional CTL fairness assumption:

$ufair = \Box\Diamond delivered$

Simple communication protocol



CTL formula

$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \not\models \Phi$

$\mathcal{T} \models_{ufair} \Phi$

$\mathcal{T} \models_{sfair} \Phi$

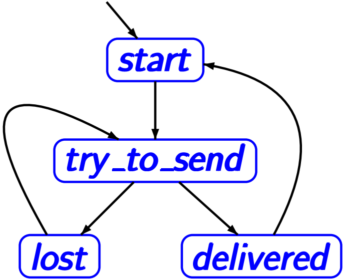
unconditional CTL fairness assumption:

$ufair = \Box\Diamond delivered$

strong CTL fairness assumption:

$sfair = \Box\Diamond try\_to\_send \rightarrow \Box\Diamond delivered$

Simple communication protocol

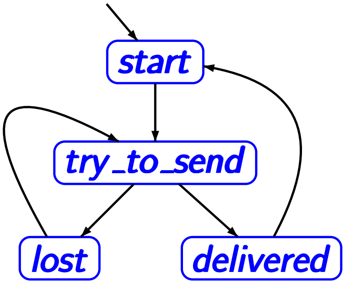


$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$?

unconditional fairness: $ufair = \Box\Diamond\,\exists\bigcirc start$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond start$

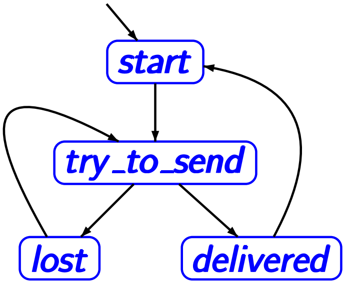
$\mathcal{T} \models_{ufair} \Phi$?

unconditional fairness: $ufair = \Box\Diamond \exists\bigcirc start$

$\uparrow$

$Sat(\exists\bigcirc start) = \{delivered\}$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond \mathit{start}$

$\mathcal{T} \models_{\mathit{ufair}} \Phi$?

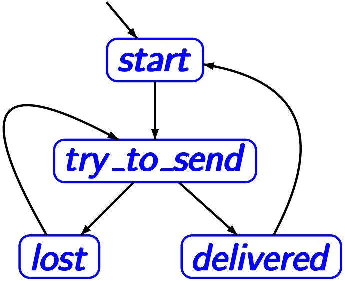
unconditional fairness: $\mathit{ufair} = \Box\Diamond \boxed{\exists\bigcirc \mathit{start}}$

$\uparrow$

$$\mathit{Sat}(\exists\bigcirc \mathit{start}) = \{\mathit{delivered}\}$$

$$\mathit{ufair} \;\widehat{=}\; \Box\Diamond \mathit{delivered}$$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$ ✓

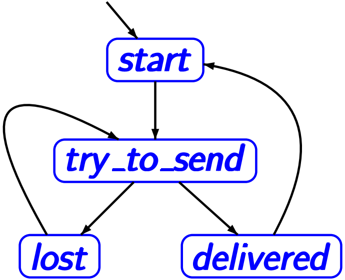
unconditional fairness: $ufair = \Box\Diamond\, \exists\bigcirc start$

↑

$Sat(\exists\bigcirc start) = \{delivered\}$

$ufair \;\widehat{=}\; \Box\Diamond delivered$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond start$

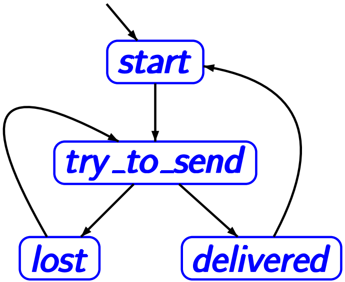
$\mathcal{T} \models_{ufair} \Phi$ ✓

$\mathcal{T} \models_{wfair} \Phi$?

unconditional fairness: $ufair = \Box\Diamond\, \exists\bigcirc start$

weak fairness: $wfair = \Diamond\Box\, \exists\bigcirc delivered \rightarrow \Box\Diamond delivered$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond start$

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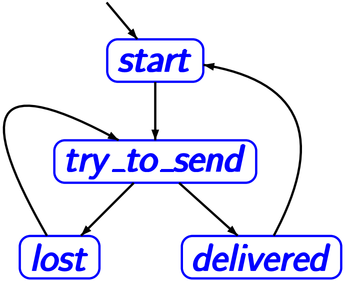
unconditional fairness: $ufair = \Box\Diamond\,\exists\bigcirc start$

weak fairness: $wfair = \Diamond\Box\,\exists\bigcirc delivered \longrightarrow \Box\Diamond delivered$

$Sat(\exists\bigcirc delivered) = \{try\_to\_send\}$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$ ✓

$\mathcal{T} \models_{wfair} \Phi$?

unconditional fairness: $ufair = \Box\Diamond\, \exists\bigcirc start$

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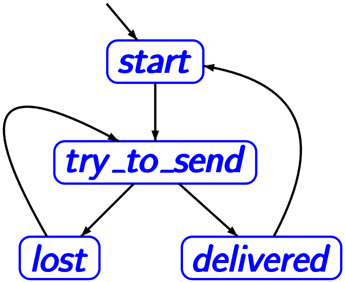
$wfair \mathrel{\widehat{=}} \Diamond\Box try\_to\_send \rightarrow \Box\Diamond delivered$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond \mathit{start}$

$\mathcal{T} \models_{\mathit{ufair}} \Phi$ $\quad\surd$

$\mathcal{T} \models_{\mathit{wfair}} \Phi$ $\quad$ wrong

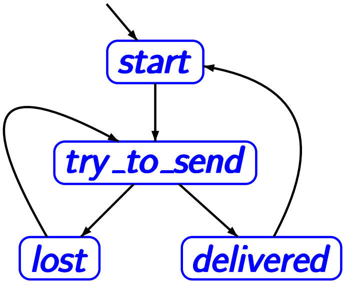
unconditional fairness: $\mathit{ufair} = \Box\Diamond\,\exists\bigcirc \mathit{start}$

weak fairness: $\mathit{wfair} = \Diamond\Box\,\exists\bigcirc \mathit{delivered} \rightarrow \Box\Diamond \mathit{delivered}$

$\mathit{Sat}(\exists\bigcirc \mathit{delivered}) = \{\mathit{try\_to\_send}\}$

$\mathit{wfair} \mathrel{\widehat{=}} \Diamond\Box \mathit{try\_to\_send} \rightarrow \Box\Diamond \mathit{delivered}$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond \mathit{start}$

$\mathcal{T} \models_{\mathit{ufair}} \Phi$ $\checkmark$

$\mathcal{T} \not\models_{\mathit{wfair}} \Phi$

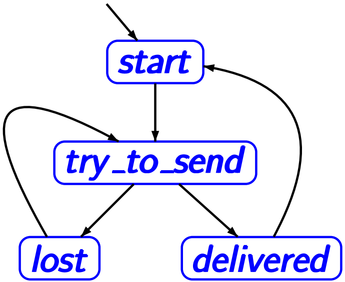
$\mathcal{T} \models_{\mathit{sfair}} \Phi$ **?**

unconditional fairness: $\mathit{ufair} = \Box\Diamond\, \exists\bigcirc \mathit{start}$

weak fairness: $\mathit{wfair} = \Diamond\Box\, \exists\bigcirc \mathit{delivered} \rightarrow \Box\Diamond \mathit{delivered}$

strong fairness: $\mathit{sfair} = \Box\Diamond\, \exists\bigcirc \mathit{delivered} \rightarrow \Box\Diamond \mathit{delivered}$

Simple communication protocol



$\Phi = \forall\Box\forall\Diamond start$

$\mathcal{T} \models_{ufair} \Phi$ ✓

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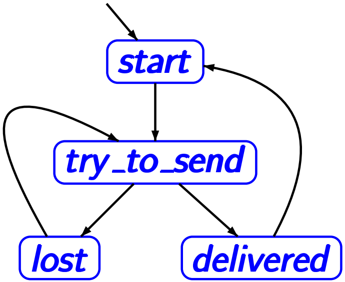
Simple communication protocol



$\Phi = \forall\Box\forall\Diamond\, start$

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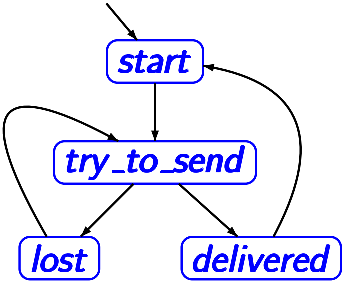

$\Phi = \forall\Box\forall\Diamond\, start$

$\mathcal{T} \models_{ufair} \Phi$ ✓

$\mathcal{T} \not\models_{wfair} \Phi$

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Correct or wrong?

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct.

Correct or wrong?

$$\text{If } s \models \forall\Diamond a \text{ where } a \in AP \text{ then } s \models_{fair} \forall\Diamond a$$

correct. Note that:

$$s \models \forall\varphi \quad \Longrightarrow \quad \text{for all } \pi \in Paths(s)\text{:} \;\; \pi \models \varphi$$

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If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct. Note that:

$$s \models \forall\varphi \quad \Longrightarrow \quad \text{for all } \pi \in Paths(s)\text{: } \pi \models \varphi$$

$$\Longrightarrow \quad \text{for all } \pi \in Paths(s)\text{: } \pi \models fair \text{ implies } \pi \models \varphi$$

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct. Note that:

$$s \models \forall\varphi \implies \text{for all } \pi \in Paths(s):\ \pi \models \varphi$$

$$\implies \text{for all } \pi \in Paths(s):\ \pi \models fair \text{ implies } \pi \models \varphi$$

$$\implies s \models_{fair} \forall\varphi$$

Correct or wrong?

If $s \models \forall\Diamond a$ where $a \in AP$ then $s \models_{fair} \forall\Diamond a$

correct.

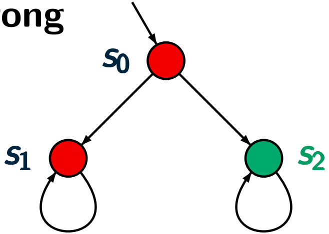
Does the same condition hold if a is replaced with an arbitrary state formula **?**

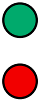
Correct or wrong?

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong



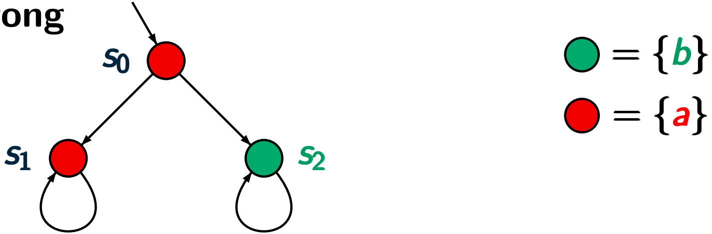


$$Sat(\exists\Box a) = \{s_0, s_1\}$$

$$Sat(\forall\Diamond\exists\Box a) = \{s_0, s_1\}$$

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong



$fair = \Box\Diamond b$

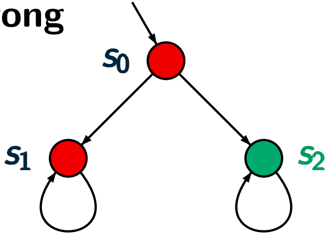
$Sat(\exists\Box a) = \{s_0, s_1\}$

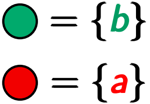
$Sat(\forall\Diamond\exists\Box a) = \{s_0, s_1\}$

Correct or wrong?

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong





$fair = \Box\Diamond b$

$Sat(\exists\Box a) = \{s_0, s_1\}$

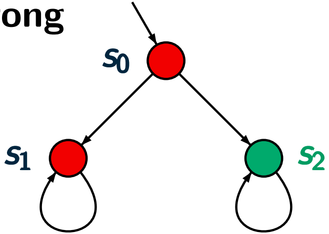
$Sat_{fair}(\exists\Box a) = \varnothing$

$Sat(\forall\Diamond\exists\Box a) = \{s_0, s_1\}$

Correct or wrong?

If $s \models \forall\Diamond\exists\Box a$ then $s \models_{fair} \forall\Diamond\exists\Box a$

wrong



$\bullet = \{b\}$ (green)

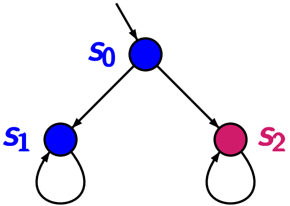
$\bullet = \{a\}$ (red)

$fair = \Box\Diamond b$

$Sat(\exists\Box a) = \{s_0, s_1\}$ $\qquad$ $Sat_{fair}(\exists\Box a) = \varnothing$

$Sat(\forall\Diamond\exists\Box a) = \{s_0, s_1\}$ $\qquad$ $Sat_{fair}(\forall\Diamond\exists\Box a) = \varnothing$

$Sat_{fair}(\exists\Box true) = ?$



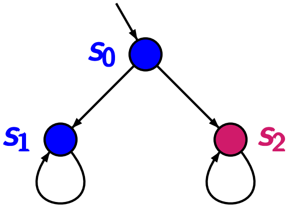
$\bullet = \{a\}$ (pink)

$\bullet = \varnothing$ (blue)

$fair = \Box\Diamond a$

$Sat_{fair}(\exists\Box true) = \{s_0, s_2\}$

$Sat_{fair}(\exists\Box true) = ?$



$\bullet = \{a\}$ (red)

$\bullet = \varnothing$ (blue)

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$= \{s : \exists\pi \in Paths(s) \text{ s.t. } \pi \models fair\}$

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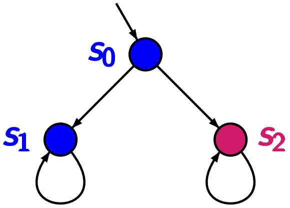

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fair is realizable iff

$Sat_{fair}(\exists\Box true) \supseteq$ set of all reachable states

Model checking problem for FairCTL

Model checking problem for FairCTL

given: finite transition system $\mathcal{T}$
CTL formula Φ
CTL fairness assumption ***fair***

question: does $\mathcal{T} \models_{\mathit{fair}} \Phi$ hold **?**

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for simplicity:

we suppose that Φ is in existential normal form,
i.e., a $\forall$-free CTL formula with temporal modalities

$$\exists\bigcirc, \exists \mathsf{U}, \exists\Box$$

Preprocessing of FairCTL model checking

given: finite transition system $\mathcal{T}$
CTL formula Φ in $\exists$-normal form
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Preprocessing of FairCTL model checking

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CTL formula Φ in $\exists$-normal form
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$$fair = \bigwedge_{1 \leq i \leq k} \Box\Diamond b_i \rightarrow \Box\Diamond c_i \text{ with } b_i, c_i \in AP$$

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preprocessing: apply a standard CTL model checker to evaluate the CTL state subformulas of *fair*

- compute $Sat(\Psi_{i,1})$ and $Sat(\Psi_{i,2})$
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Idea of FairCTL model checking

given: finite transition system $\mathcal{T}$
CTL formula Φ in $\exists$-normal form
CTL fairness assumption ***fair***

question: does $\mathcal{T} \models_{\mathit{fair}} \Phi$ hold **?**

1. ... preprocessing ...

Idea of FairCTL model checking

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CTL formula Φ in $\exists$-normal form
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1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
   - ***true***, ***a*** $\in$ ***AP***, $\wedge$, $\neg$: as for standard **CTL**
   - $\exists\bigcirc$, $\exists$**U**: via standard **CTL** model checking
   - $\exists\Box$: via analysis of **SCCs**

CTL model checking with fairness

recursive computation of the fair satisfaction sets:

$$Sat_{fair}(\Psi) = \{ s \in S : s \models_{fair} \Psi \}$$

simple cases: $\Psi = \mathit{true}$ or $\Psi = a \in AP$ or the outer most operator of Ψ is a negation or conjunction:

$$\begin{array}{lcl} Sat_{fair}(\mathit{true}) & = & S \\ Sat_{fair}(a) & = & \{ s \in S : \; a \in L(s) \} \\ Sat_{fair}(\neg\Psi) & = & S \setminus Sat_{fair}(\Psi) \\ Sat_{fair}(\Psi_1 \wedge \Psi_2) & = & Sat_{fair}(\Psi_1) \cap Sat_{fair}(\Psi_2) \end{array}$$

Idea of FairCTL model checking

given: finite transition system $\mathcal{T}$
CTL formula Φ in $\exists$-normal form
CTL fairness assumption ***fair***

question: does $\mathcal{T} \models_{fair} \Phi$ hold **?**

1. ... preprocessing ...
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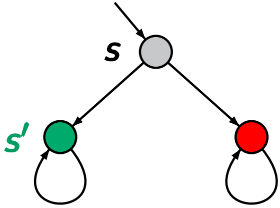
   - $\exists\Box$: via analysis of SCCs

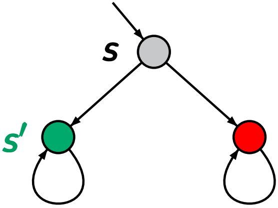
FairCTL model checking: treatment of $\exists\bigcirc$

FairCTL model checking: treatment of $\exists\bigcirc$



$fair = \Box\Diamond red$

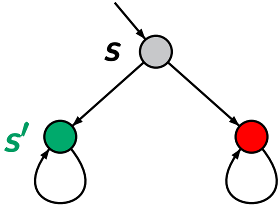
FairCTL model checking: treatment of $\exists\bigcirc$



$fair = \Box\Diamond red$

$s \not\models_{fair} \exists\bigcirc green$

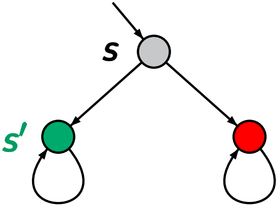
FairCTL model checking: treatment of $\exists\bigcirc$



$fair = \Box\Diamond red$

$s \not\models_{fair} \exists\bigcirc green$

as $s' \not\models_{fair} \exists\Box true$



$fair = \Box\Diamond red$

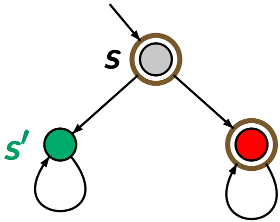
$s \not\models_{fair} \exists\bigcirc green$

as $s' \not\models_{fair} \exists\Box true$

introduce an additional atomic proposition a_{fair} s.t. for all states s:

$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

FairCTL model checking: treatment of $\exists\bigcirc$



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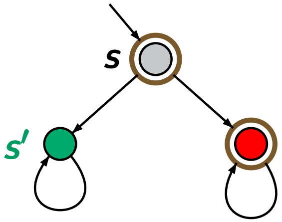

$fair = \Box\Diamond red$

$s \not\models_{fair} \exists\bigcirc green$

as $s' \not\models_{fair} \exists\Box true$

introduce an additional atomic proposition a_{fair} s.t. for all states s:

$$a_{fair} \in L(s) \text{ iff } s \models_{fair} \exists\Box true$$

This yields that for all $b \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \text{ iff } s \models \exists\bigcirc(b \wedge a_{fair})$$

FairCTL model checking: $\exists\bigcirc$ and $\exists U$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

This yields that for all $b, c \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$

$$s \models_{fair} \exists(c \, U \, b) \quad \text{iff} \quad ?$$

FairCTL model checking: $\exists\bigcirc$ and $\exists\mathsf{U}$

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$$s \models_{fair} \exists(c \,\mathsf{U}\, b) \quad \text{iff} \quad s \models \exists(c \,\mathsf{U}(b \wedge a_{fair}))$$

FairCTL model checking: $\exists\bigcirc$ and $\exists U$

introduce an additional atomic proposition a_{fair} s.t.

$$a_{fair} \in L(s) \quad \text{iff} \quad s \models_{fair} \exists\Box true$$

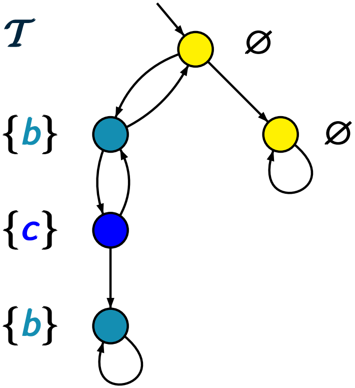
This yields that for all $b, c \in AP$ and all states s:

$$s \models_{fair} \exists\bigcirc b \quad \text{iff} \quad s \models \exists\bigcirc(b \wedge a_{fair})$$
$$s \models_{fair} \exists(c \, U \, b) \quad \text{iff} \quad s \models \exists(c \, U(b \wedge a_{fair}))$$

hence: treatment of $\exists\bigcirc$ and $\exists U$ for FairCTL via

- special methods to compute $Sat_{fair}(\exists\Box true)$
- standard CTL model checking for $\exists\bigcirc$ and $\exists U$

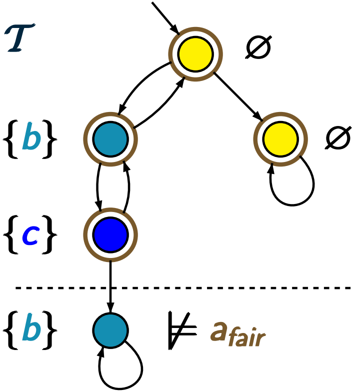
Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

strong fairness assumption: $\mathit{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$

Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

$\downarrow$

$\exists\Diamond(c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

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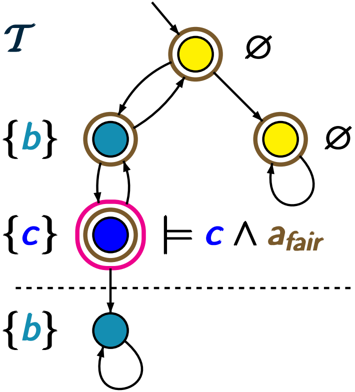

CTL formula $\exists\Diamond c$

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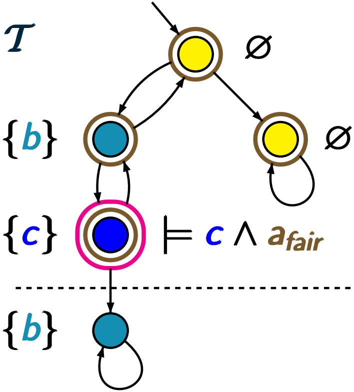


CTL formula $\exists\Diamond c$

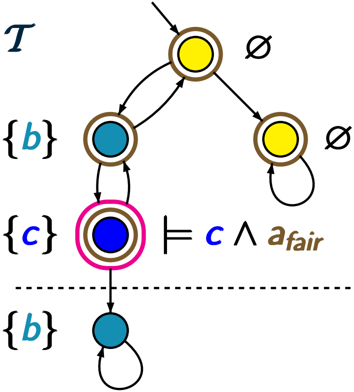
$\downarrow$

$\exists\Diamond (c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists\Diamond(c \wedge a_{fair})$

Example: treatment of $\exists\Diamond$



CTL formula $\exists\Diamond c$

$\downarrow$

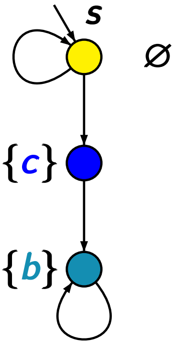
$\exists\Diamond(c \wedge a_{fair})$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$$\mathcal{T} \models \exists\Diamond(c \wedge a_{fair}) \implies \mathcal{T} \models_{fair} \exists\Diamond c$$

Example: treatment of ∃U

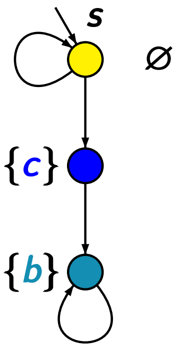
$\mathcal{T}$:



$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of ∃U

$\mathcal{T}$:



strong fairness assumption: $fair = \Box\Diamond b \to \Box\Diamond c$

$$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$$

Example: treatment of ∃U

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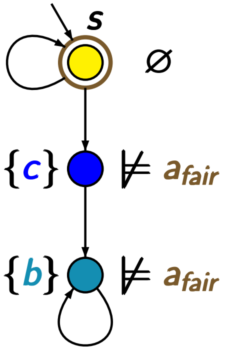

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

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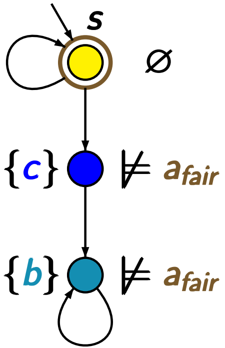
$\mathcal{T}$:



$Sat(c \land a_{fair}) = \varnothing$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists(\neg b \, \mathsf{U} \, c)$

Example: treatment of ∃U

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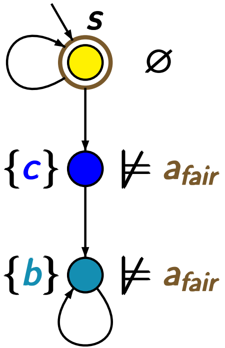

$$s \not\models \exists(\neg b \,\mathsf{U}\, (c \wedge a_{fair}))$$

$$\Uparrow$$

$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

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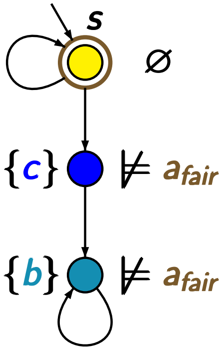

$$s \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

$$\Uparrow$$

$$s \not\models \exists(\neg b \,\mathsf{U}(c \wedge a_{fair}))$$

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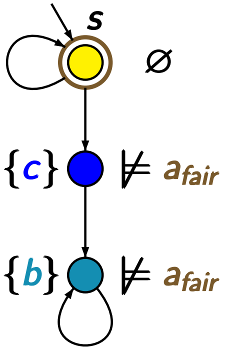


$$s \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$$

$$\Uparrow$$

$$s \not\models \exists(\neg b \,\mathsf{U}\,(c \wedge a_{fair}))$$

$$\Uparrow$$

$$Sat(c \wedge a_{fair}) = \varnothing$$

strong fairness assumption: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

$\mathcal{T} \models \exists(\neg b \,\mathsf{U}\, c)$, but $\mathcal{T} \not\models_{fair} \exists(\neg b \,\mathsf{U}\, c)$

Correct or wrong?

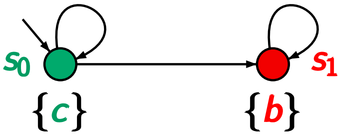
$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.

$fair = \Box\Diamond b$



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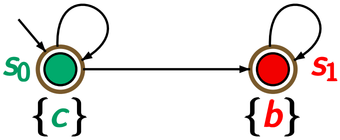


$fair = \Box\Diamond b$

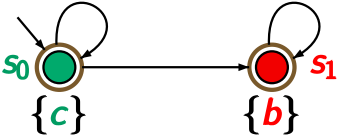
$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



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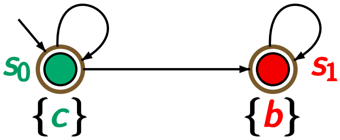

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Correct or wrong?

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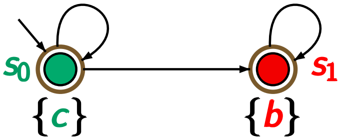


$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

regard state $s = s_0$:

$s \models \exists\Box(c \wedge a_{fair})$,

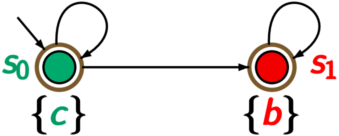
$\uparrow$

path $\pi = s_0\, s_0\, s_0\, s_0 \ldots \models \Box(c \wedge a_{fair})$

Correct or wrong?

$$s \models_{fair} \exists\Box c \quad \text{iff} \quad s \models \exists\Box(c \wedge a_{fair})$$

wrong.



$fair = \Box\Diamond b$

$s_0 \models a_{fair}$

$s_1 \models a_{fair}$

regard state $s = s_0$:

$s \models \exists\Box(c \wedge a_{fair})$, but $s \not\models_{fair} \exists\Box c$

$\uparrow$

path $\pi = s_0\, s_0\, s_0\, s_0 \ldots \models \Box(c \wedge a_{fair})$

Idea of FairCTL model checking

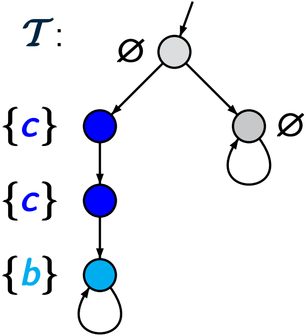
given: finite transition system $\mathcal{T}$
CTL formula Φ in $\exists$-normal form
CTL fairness assumption ***fair***

question: does $\mathcal{T} \models_{fair} \Phi$ hold ?

1. ... preprocessing ...
2. Build the parse tree of Φ and process it in bottom-up-manner. Treatment of:
   - ***true***, $a \in AP$, $\wedge$, $\neg$: as for standard CTL
   - $\exists\bigcirc$, $\exists\mathsf{U}$: via standard CTL model checking
   - $\exists\Box$: via analysis of **SCCs**

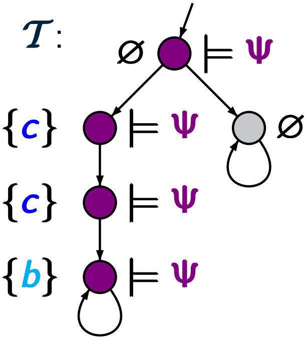
$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{fair} \exists\Box\Psi$?

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ

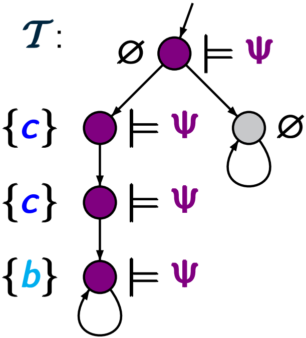


$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$

$\exists\Box\Psi$ under strong fairness

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$, CTL state formula Ψ



$\mathcal{T} \models_{fair} \exists\Box\Psi$?

1. calculate $Sat_{fair}(\Psi)$
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$\exists\Box\Psi$ under strong fairness

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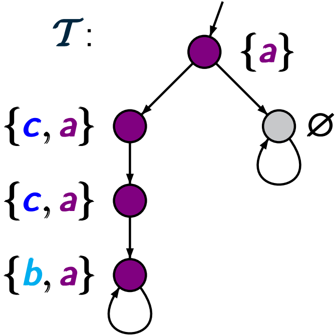

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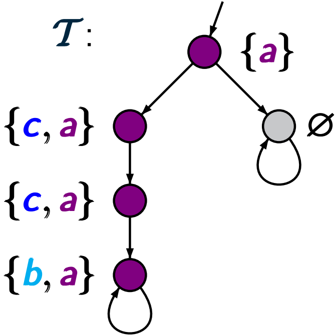

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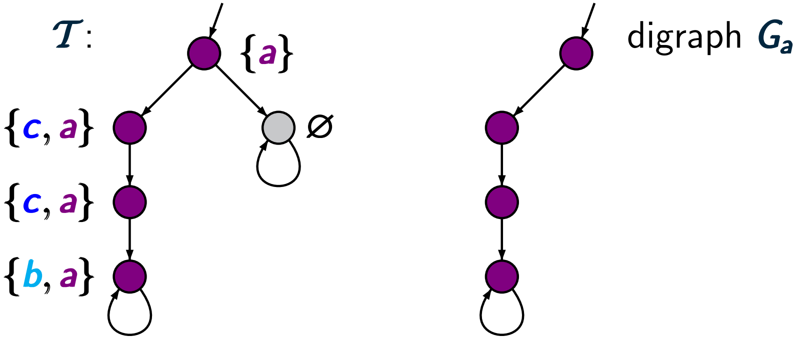

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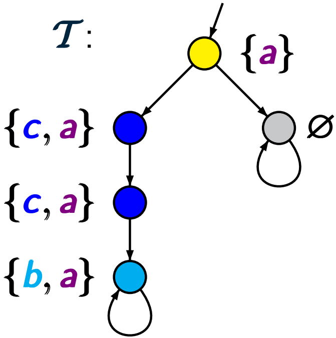

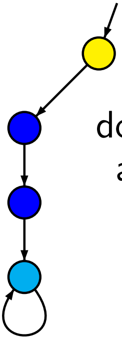

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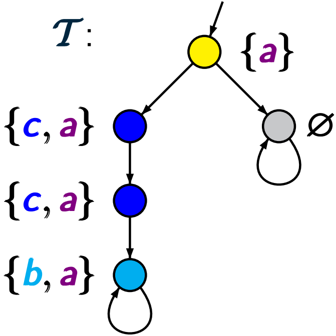

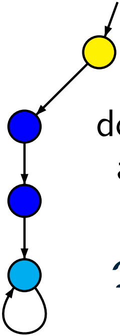

doesn't contain any fair path

$\Downarrow$

$\mathcal{T} \not\models_{fair} \exists\Box\Psi$

1. calculate $Sat_{fair}(\Psi)$
2. replace Ψ with a fresh atomic proposition $a = a_\Psi$
3. calculate $Sat_{fair}(\exists\Box a) = \varnothing$

Treatment of $\exists\Box a$ for FairCTL

given: finite TS $\mathcal{T}$, atomic proposition a
CTL fairness assumption *fair*

goal: compute $Sat_{fair}(\exists\Box a)$

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CTL fairness assumption *fair*

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if all states are labeled by a:

this technique yields a method
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given: finite TS $\mathcal{T}$, atomic proposition a
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here: explanations only for strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

Treatment of $\exists\Box$ under strong fairness

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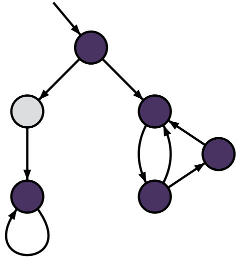
or $$\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(c_i) \neq \varnothing$$

$\exists\Box a$ under strong fairness

does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



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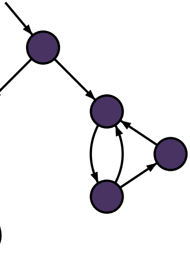

$\bullet \models a \qquad \circ \not\models a$

analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$

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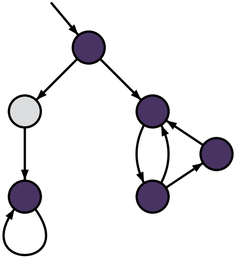
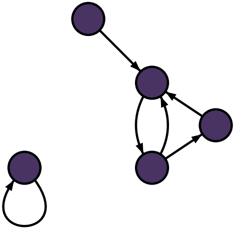


digraph G_a



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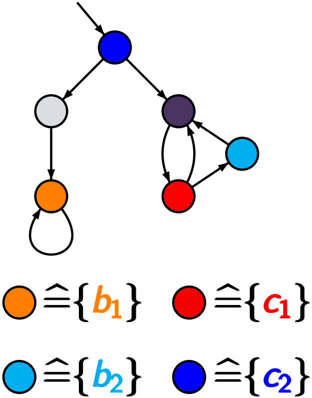

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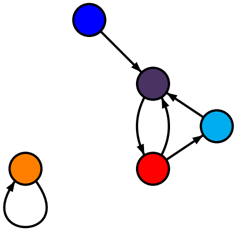

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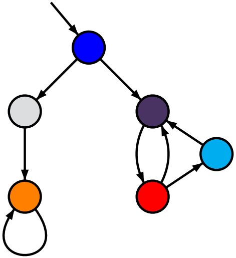

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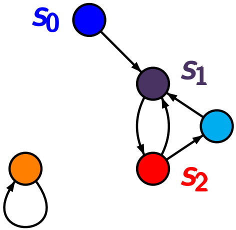

● $\widehat{=}\{b_1\}$ ● $\widehat{=}\{c_1\}$

● $\widehat{=}\{b_2\}$ ● $\widehat{=}\{c_2\}$

$s_0\,(s_1\,s_2)^\omega \models \neg\Box\Diamond b_2 \wedge \Box\Diamond c_1$

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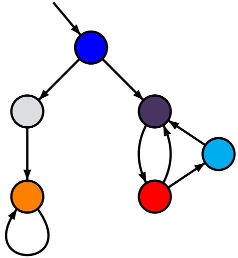

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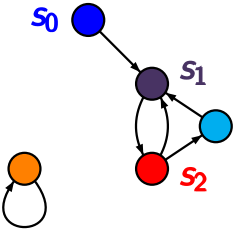

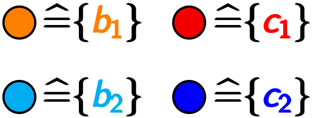

$s_0\,(s_1\,s_2)^\omega \models \neg\Box\Diamond b_2 \wedge \Box\Diamond c_1$

$s_0\,(s_1\,s_2)^\omega \models fair$

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Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \to \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

$$s_0\, s_1 \ldots s_n \ldots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n + r$
- for all $1 \leq i \leq k$: $\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(b_i) = \varnothing$
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$\uparrow$

Thus: $D = \{s_{n+1}, \ldots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a

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↑

Thus: $D = \{s_{n+1}, \ldots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a (possibly not an SCC)

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G_a: digraph that arises from $\mathcal{T}$ by removing all states s' with $s' \not\models a$

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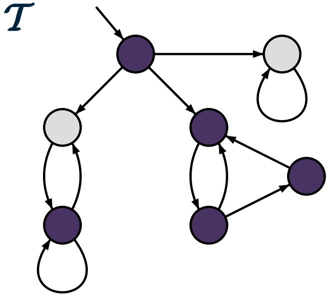
note: if $s \models_{fair} \exists\Box a$ then there might be **no SCC** D where (1) and (2) hold

Example: computation of $Sat_{fair}(\exists\Box a)$



$\models a$ $\quad$ $\not\models a$

computation of $Sat_{fair}(\exists\Box a)$

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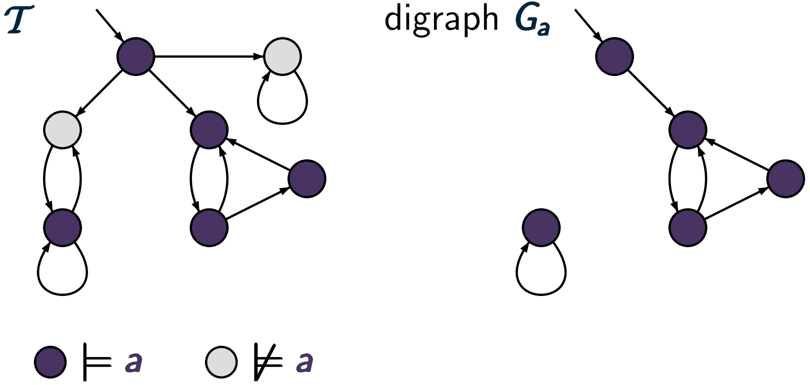
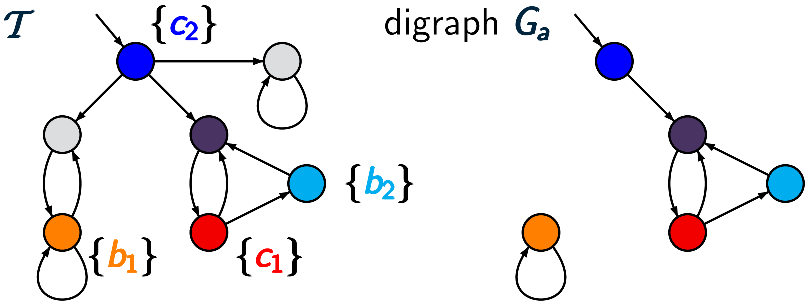


computation of $Sat_{fair}(\exists\Box a)$
by analyzing the digraph G_a

Example: computation of $Sat_{fair}(\exists\Box a)$



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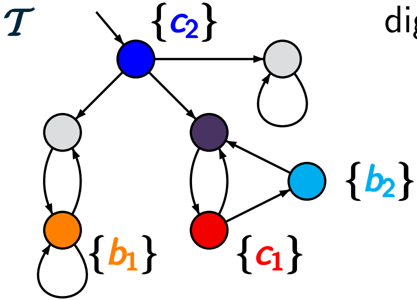

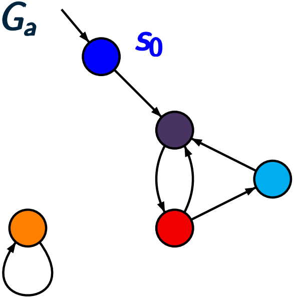

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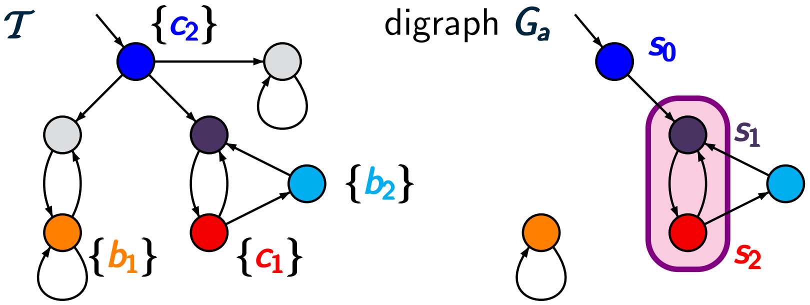

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$s_0 \models_{fair} \exists\Box a$ as $s_0\, s_1\, s_2\, s_1\, s_2 \ldots \models_{LTL} fair$

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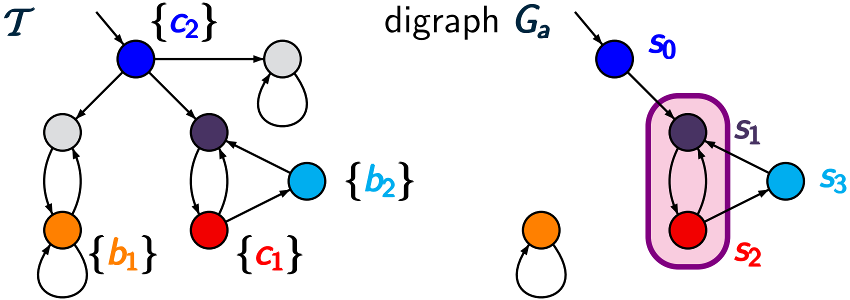

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$s_0 \models_{fair} \exists\Box a$ as $s_0\, s_1\, s_2\, s_1\, s_2 \ldots \models_{LTL} fair$

$Sat_{fair}(\exists\Box a) = \{s_0, s_1, s_2, s_3\}$

CTL model checking with fairness

treatment of $\exists\Box$ for **CTL** with fairness

CTL model checking with fairness

treatment of $\exists\Box$ for **CTL** with fairness

here: explanations only for strong fairness

weak fairness and combinations of weak/strong fairness can be treated in an analogous way

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treatment of $\exists\Box$ for **CTL** with fairness

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case 1: unconditional fairness

case 2: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

case 3: arbitrary strong fairness assumption

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treatment of $\exists\Box$ for **CTL** with fairness

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$\exists\Box a$ under unconditional fairness

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$s \models_{fair} \exists\Box a$ iff **?**

$\exists\Box a$ under unconditional fairness

$$fair = \bigwedge_{1 \leq i \leq k} \Box\Diamond c_i$$

$s \models_{fair} \exists\Box a$	iff	there exists a nontrivial **SCC** C in G_a that is reachable from s and $C \cap Sat(c_i) \neq \varnothing$ for $i = 1, \ldots, k$

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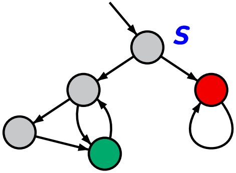
digraph G_a



fairness assumption:

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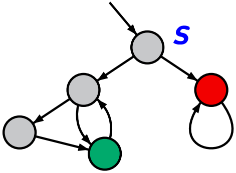

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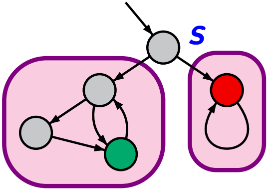

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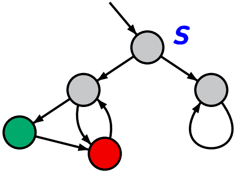

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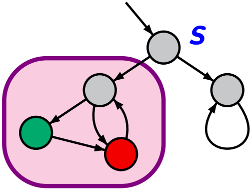
$fair = \Box\Diamond c_1 \wedge \Box\Diamond c_2$

$\exists\Box a$ under unconditional fairness

$$fair = \bigwedge_{1 \leq i \leq k} \Box\Diamond c_i$$

$s \models_{fair} \exists\Box a$ iff there exists a nontrivial **SCC** C in G_a that is reachable from s and $C \cap Sat(c_i) \neq \varnothing$ for $i = 1, \ldots, k$

digraph G_a



fairness assumption:

$fair = \Box\Diamond c_1 \wedge \Box\Diamond c_2$

$s \models_{fair} \exists\Box a$

treatment of $\exists\Box$ for CTL with fairness

here: explanations only for strong fairness

case 1:	unconditional fairness	✓
case 2:	$\mathit{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$	
case 3:	arbitrary strong fairness assumption	

$$\mathit{fair} = \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

treatment of $\exists\Box$ for CTL with fairness

here: explanations only for strong fairness

case 1: unconditional fairness √

case 2: $\mathit{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$

case 3: arbitrary strong fairness assumption

$$\mathit{fair} = \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

Strong fairness: 1 fairness requirement

$$fair = \Box\Diamond b \rightarrow \Box\Diamond c$$

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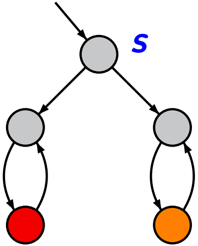
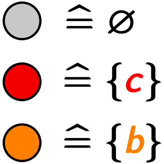
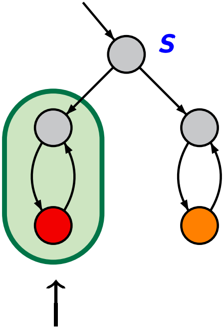
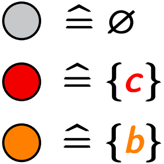




$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a



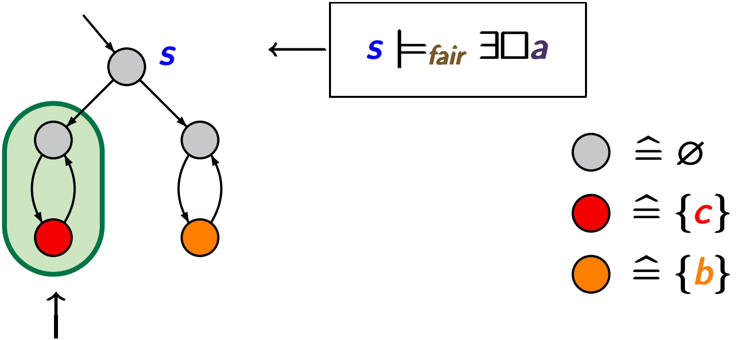


nontrivial **SCC** C of G_a with $C \cap Sat(c) \neq \varnothing$

Strong fairness: 1 fairness requirement

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a



nontrivial SCC C of G_a with $C \cap Sat(c) \neq \varnothing$

Strong fairness: 1 fairness requirement

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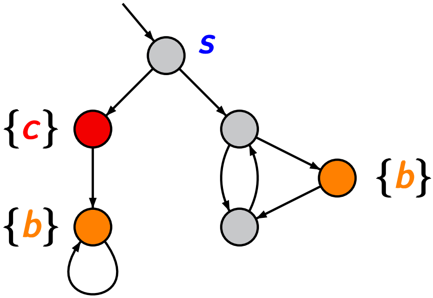
digraph G_a



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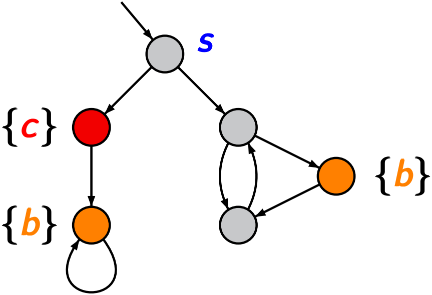
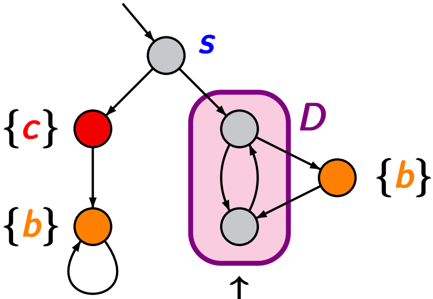


$\longleftarrow$ $s \models_{fair} \exists\Box a$

Strong fairness: 1 fairness requirement

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a



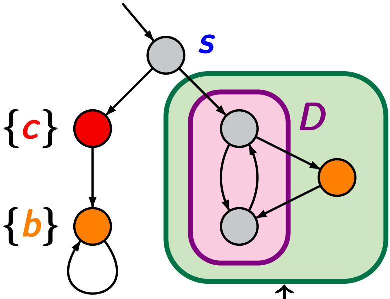
$s \models_{fair} \exists\Box a$

strongly connected node-set D of G_a with
$D \cap Sat(b) = \varnothing$

Strong fairness: 1 fairness requirement

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a



$s \models_{fair} \exists\Box a$

nontrivial **SCC** C of G_a that contains a nontrivial **SCC** D of $G_a|_C \setminus Sat(b)$

treatment of $\exists\Box$ for CTL with fairness

here: explanations only for strong fairness

case 1: unconditional fairness $\surd$

case 2: $\mathit{fair} = \Box\Diamond b \rightarrow \Box\Diamond c$ $\surd$

case 3: arbitrary strong fairness assumption

$$\mathit{fair} = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

CTL model checking with fairness

treatment of $\exists\Box$ for CTL with fairness

here: explanations only for strong fairness

case 1: unconditional fairness √

case 2: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$ √

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Example: 2 strong fairness conditions

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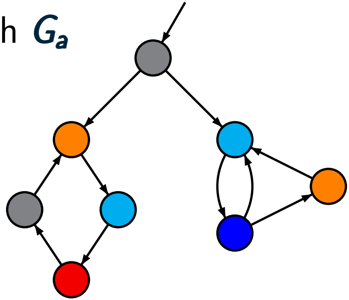

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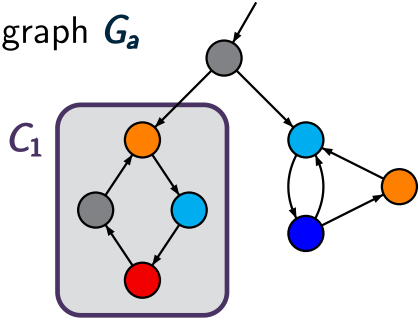

first SCC: $C_1 \cap Sat(c_2) = \varnothing$

Example: 2 strong fairness conditions

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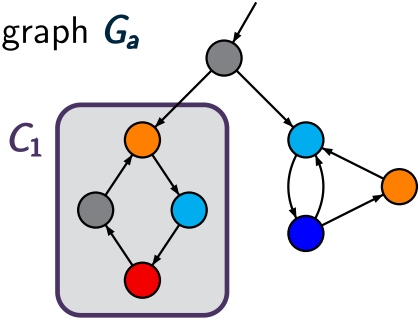
digraph G_a



first SCC: $C_1 \cap Sat(c_2) = \varnothing$

analyze $C_1 \setminus Sat(b_2)$ w.r.t. $\Box\Diamond b_1 \rightarrow \Box\Diamond c_1$

Example: 2 strong fairness conditions

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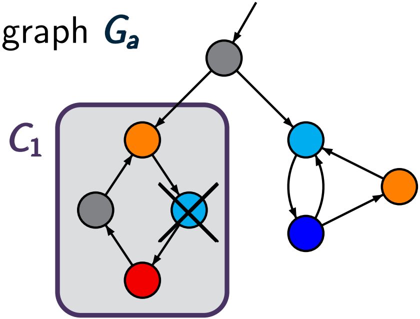

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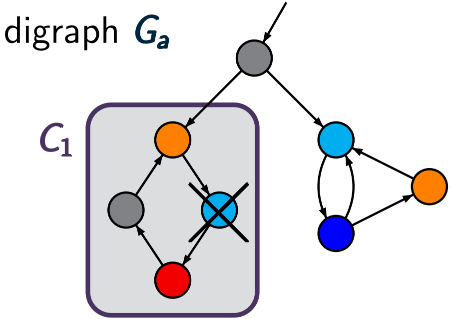

first SCC: $C_1 \cap Sat(c_2) = \varnothing$

analyze $C_1 \setminus Sat(b_2)$ w.r.t. $\Box\Diamond b_1 \rightarrow \Box\Diamond c_1$

$\rightsquigarrow$ there is no cycle

Example: 2 strong fairness conditions

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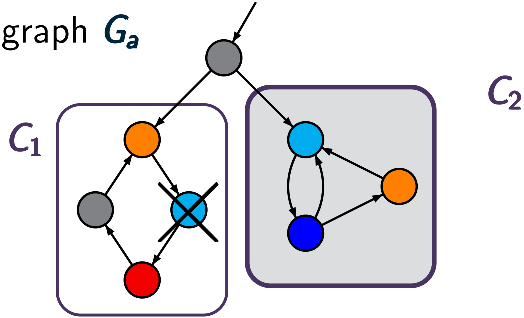
digraph G_a



second SCC:

Example: 2 strong fairness conditions

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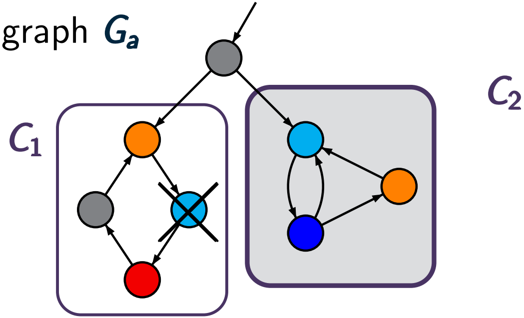

second SCC: $C_2 \cap Sat(c_1) = \varnothing$

Example: 2 strong fairness conditions

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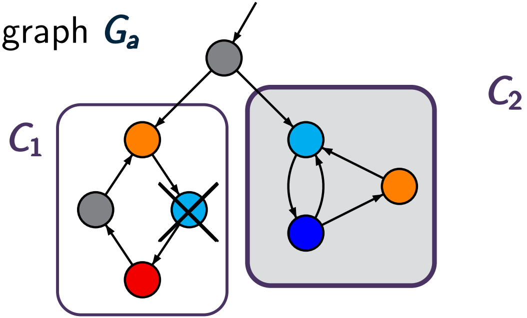

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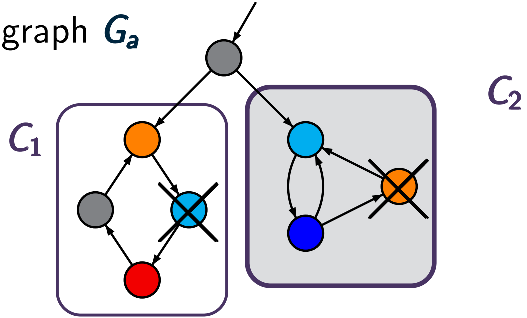

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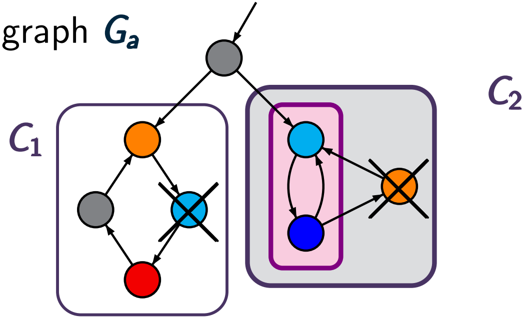

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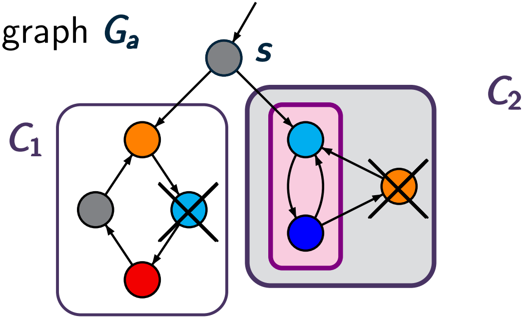

second SCC: $C_2 \cap Sat(c_1) = \varnothing$

analyze $C_2 \setminus Sat(b_1)$ w.r.t. $\Box\Diamond b_2 \rightarrow \Box\Diamond c_2$

hence: $s \models_{fair} \exists\Box a$

Calculation of $Sat_{fair}(\exists\Box a)$

compute the SCCs of the digraph G_a;

$T := \varnothing$;

FOR ALL nontrivial SCCs C of G_a DO

IF $CheckFair(C, \ldots)$ THEN $T := T \cup C$ FI

OD

$$Sat_{fair}(\exists\Box a) := \{ s \in S : Reach_{G_a}(s) \cap T \neq \varnothing \}$$

↑ backward search from T

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time complexity: $\mathcal{O}(size(\mathcal{T}) \cdot |fair|)$

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Recursive algorithm *CheckFair*(...)

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algorithm *CheckFair*$\left(C, k, \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)\right)$

Recursive algorithm *CheckFair*(...)

algorithm *CheckFair*$(C, k, \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i))$ returns

"true" if there exists a cyclic path fragment $s_0\, s_1 \ldots s_n$ in C such that

$$(s_0\, s_1 \ldots s_{n-1})^\omega \models \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

"false" otherwise

Recursive algorithm $\mathit{CheckFair}(\ldots)$

pseudo code for $\mathit{CheckFair}(C, k, \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i))$

```
IF ∀i ∈ {1,...,k}. C ∩ Sat(c_i) ≠ ∅ THEN return "true" FI
choose j ∈ {1,...,k} with C ∩ Sat(c_j) = ∅;
remove all states in Sat(b_j);
IF the resulting graph G is acyclic THEN return "false" FI
FOR ALL nontrivial SCCs D of G DO
   IF CheckFair(D, k−1, ⋀_{i≠j} (□◊b_i → □◊c_i))
   THEN return "true"                                FI
OD
return "false"
```

pseudo code for $\mathit{CheckFair}(C, k, \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i))$

IF $\forall i \in \{1, ..., k\}.\ C \cap \mathit{Sat}(c_i) \neq \varnothing$ THEN return "true" FI
choose $j \in \{1, ..., k\}$ with $C \cap \mathit{Sat}(c_j) = \varnothing$;
remove all states in $\mathit{Sat}(b_j)$;
IF the resulting graph G is acyclic THEN return "false" FI
FOR ALL nontrivial **SCCs** D of G DO
  IF $\mathit{CheckFair}(D, k-1, \bigwedge_{i \neq j} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i))$
  THEN return "true"
OD
return "false"

time complexity:
$\mathcal{O}(\mathit{size}(C) \cdot k)$

Model checking algorithm for FairCTL

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calculate $Sat_{fair}(\exists\Box true)$;

label all states in $Sat_{fair}(\exists\Box true)$ with a_{fair}

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FOR ALL subformulas Ψ of Φ DO

CASE Ψ is:

⋮

$$
\begin{array}{rcll}
\exists\bigcirc a & : & Sat_{fair}(\Psi) & := & Sat(\exists\bigcirc(a \wedge a_{fair})); \\
\exists(a_1 \mathsf{U}\, a_2) & : & Sat_{fair}(\Psi) & := & Sat(\exists(a_1 \mathsf{U}(a_2 \wedge a_{fair}))); \\
\exists\Box a & : & Sat_{fair}(\Psi) & := & \ldots
\end{array}
$$

replace Ψ with a fresh atomic proposition a_Ψ

OD

Model checking algorithm for FairCTL

calculate $Sat_{fair}(\exists\Box true)$;

label all states in $Sat_{fair}(\exists\Box true)$ with a_{fair}

FOR ALL subformulas Ψ of Φ DO

CASE Ψ is:

$\vdots$

$$\begin{array}{rcl} \exists\bigcirc a & : Sat_{fair}(\Psi) & := \; Sat(\exists\bigcirc(a \wedge a_{fair})); \\ \exists(a_1 \, \mathsf{U} \, a_2) & : Sat_{fair}(\Psi) & := \; Sat(\exists(a_1 \, \mathsf{U}(a_2 \wedge a_{fair}))); \\ \exists\Box a & : Sat_{fair}(\Psi) & := \; \ldots \end{array}$$

replace Ψ with a fresh atomic proposition a_Ψ

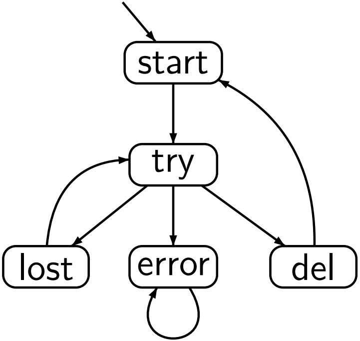
OD

IF $S_0 \subseteq Sat_{fair}(\Phi)$ THEN return "**yes**"

ELSE return "**no**"

FI

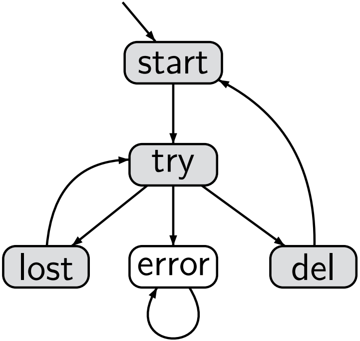
Example: CTL model checking with fairness



$fair = \Box\Diamond\ \exists\Diamond del$

$\Phi = \exists\Diamond\ \forall\bigcirc(lost \vee del)$

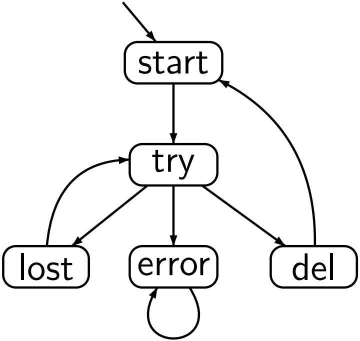
Example: CTL model checking with fairness



$$\Phi = \exists\Diamond\ \forall\bigcirc(lost \vee del)$$

$$fair = \Box\Diamond\ \boxed{\exists\Diamond del} \rightsquigarrow \Box\Diamond c \text{ where } Sat(c) = S \setminus \{error\}$$

Example: CTL model checking with fairness

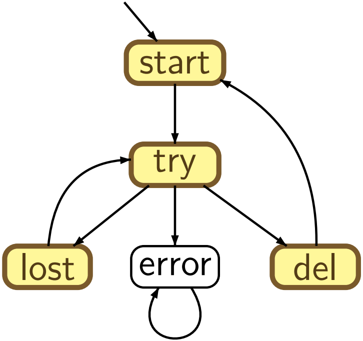


$\Phi = \exists\Diamond\ \forall\bigcirc(lost \vee del)$

$fair = \Box\Diamond\ \exists\Diamond del \ \leadsto \Box\Diamond c$ where $Sat(c) = S \setminus \{error\}$

$Sat_{fair}(\exists\Box true)$

Example: CTL model checking with fairness

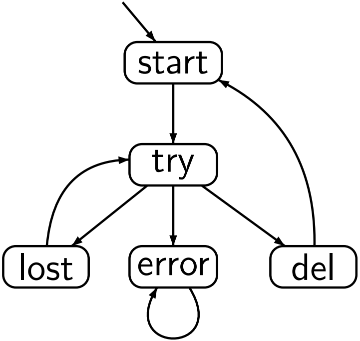


$$\Phi = \exists\Diamond \forall\bigcirc(lost \vee del)$$

$$fair = \Box\Diamond \exists\Diamond del \rightsquigarrow \Box\Diamond c \text{ where } Sat(c) = S \setminus \{error\}$$

$$Sat_{fair}(\exists\Box true) = Sat(a_{fair}) = S \setminus \{error\}$$

Example: CTL model checking with fairness



$$\Phi \;=\; \exists\Diamond\; \forall\bigcirc(\mathit{lost} \vee \mathit{del})$$

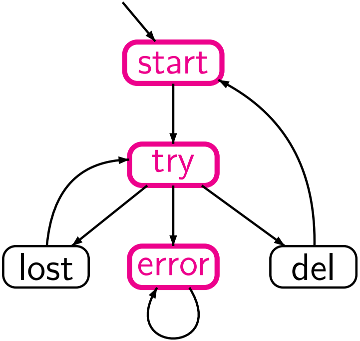
$$\equiv\; \exists\Diamond\; \neg\exists\bigcirc(\neg\mathit{lost} \wedge \neg\mathit{del})$$

existential normal form

$\mathit{fair} = \Box\Diamond\; \exists\Diamond \mathit{del} \;\rightsquigarrow\; \Box\Diamond c$ where $\mathit{Sat}(c) = S \setminus \{\mathit{error}\}$

$\mathit{Sat}_{\mathit{fair}}(\exists\Box \mathit{true}) \;=\; \mathit{Sat}(a_{\mathit{fair}}) = S \setminus \{\mathit{error}\}$

Example: CTL model checking with fairness



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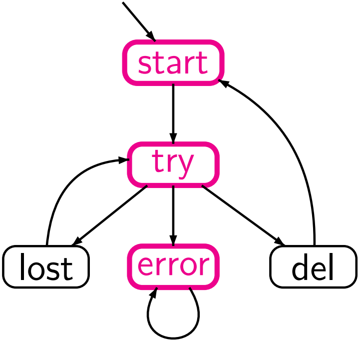

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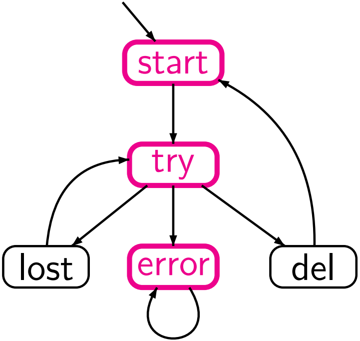
$$\rightsquigarrow \exists\Diamond\neg\exists\bigcirc a$$

$fair = \Box\Diamond\ \exists\Diamond del \ \rightsquigarrow \Box\Diamond c$ where $Sat(c) = S \setminus \{error\}$

$Sat_{fair}(\exists\Box true) \ = Sat(a_{fair}) = S \setminus \{error\}$

Example: CTL model checking with fairness



$$\Phi = \exists\Diamond\ \forall\bigcirc(lost \vee del)$$

$$\equiv\ \exists\Diamond\neg\exists\bigcirc\ (\neg lost \wedge \neg del)$$

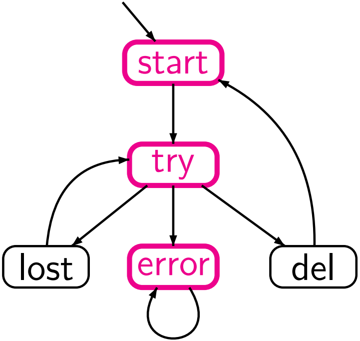
$$\rightsquigarrow\ \exists\Diamond\neg\boxed{\exists\bigcirc a}$$

$fair = \Box\Diamond\ \exists\Diamond del\ \rightsquigarrow \Box\Diamond c$ where $Sat(c) = S \setminus \{error\}$

$Sat_{fair}(\exists\Box true)\ = Sat(a_{fair}) = S \setminus \{error\}$

$Sat_{fair}(\exists\bigcirc a)$

Example: CTL model checking with fairness



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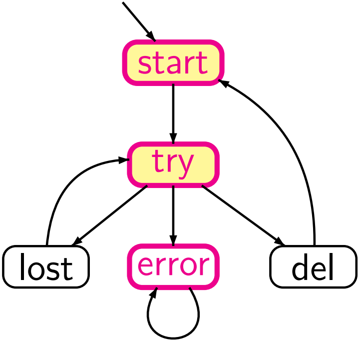
$$Sat_{fair}(\exists\Box true) = Sat(a_{fair}) = S \setminus \{error\}$$

$$Sat_{fair}(\exists\bigcirc a) = Sat(\exists\bigcirc(\ a \wedge a_{fair}\ ))$$

Example: CTL model checking with fairness



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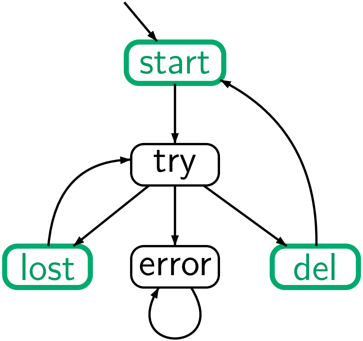

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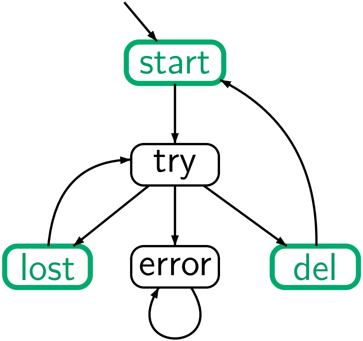

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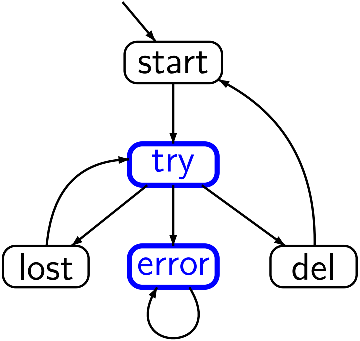

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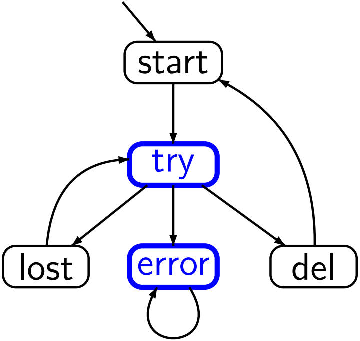

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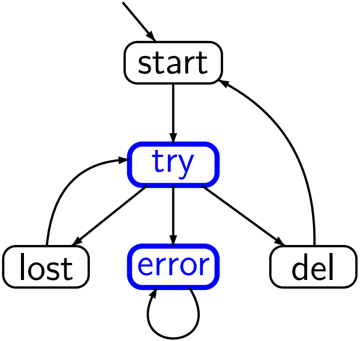

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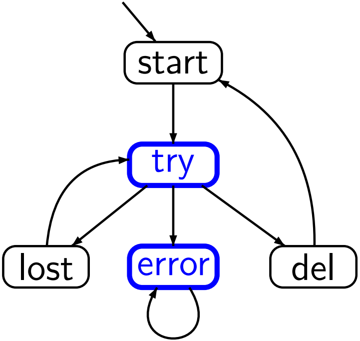

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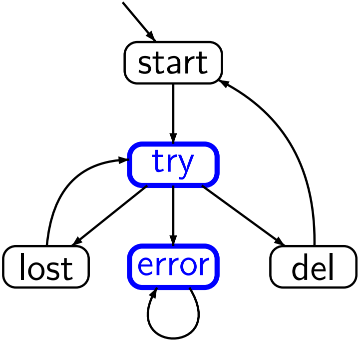

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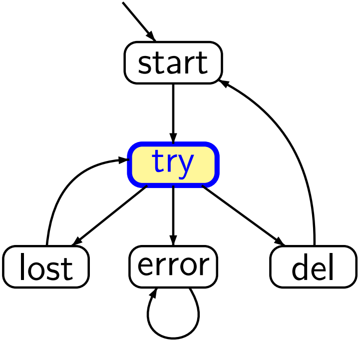

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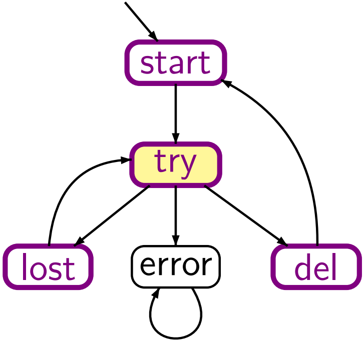

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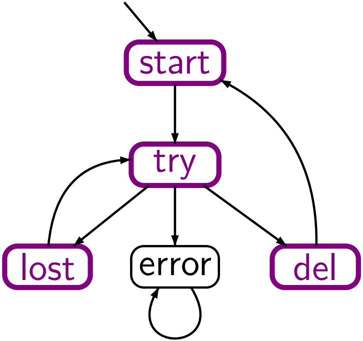

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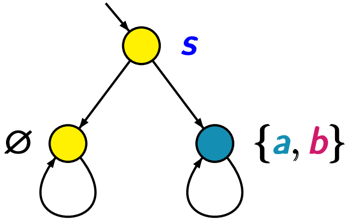

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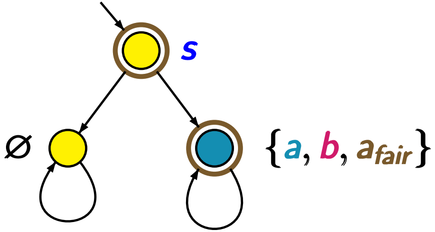

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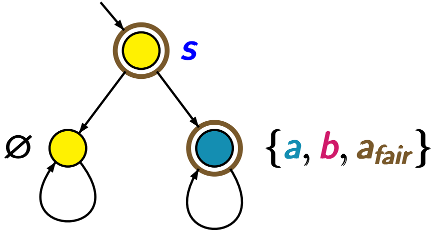

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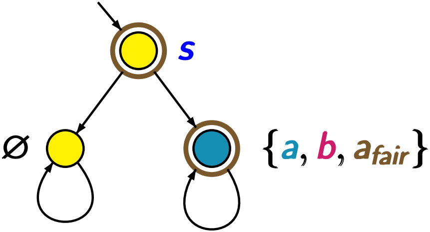

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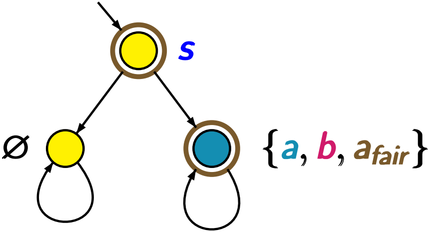

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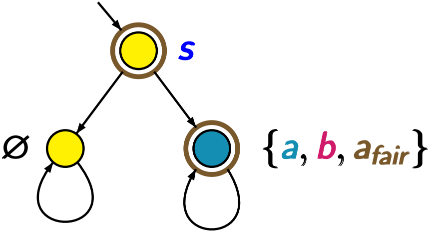

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Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is <u>no</u> state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

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$$
\begin{aligned}
s \models_{fair} \forall\Box a \quad & \text{iff} \quad s \models_{fair} \neg\exists\Diamond\neg a \\
& \text{iff} \quad s \not\models_{fair} \exists\Diamond\neg a \\
& \text{iff} \quad s \not\models \exists\Diamond(\neg a \wedge a_{fair}) \\
& \text{iff} \quad s \models \neg\exists\Diamond(\neg a \wedge a_{fair}) \;\equiv\; \forall\Box(a_{fair} \rightarrow a)
\end{aligned}
$$

Summary

We just saw:

$$s \models_{fair} \forall\bigcirc a \quad \text{iff} \quad s \models \forall\bigcirc(a_{fair} \rightarrow a)$$

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Is the following statement correct **?**

$$s \models_{fair} \forall(b\, \mathsf{U}\, a) \quad \text{iff} \quad s \models \forall\big(b\, \mathsf{U}(a_{fair} \rightarrow a)\big)$$

We just saw:

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Summary: fairness in CTL

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CTL fairness assumptions: formulas similar to **LTL**

e.g., $fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond\Psi_i \rightarrow \Box\Diamond\Phi_i)$

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CTL satisfaction relation with fairness:

$s \models_{fair} \exists\varphi$ iff there exists $\pi \in Paths(s)$ with

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model checking for **CTL** with fairness:

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- complexity: $\mathcal{O}(size(\mathcal{T}) \cdot |\Phi| \cdot |fair|)$