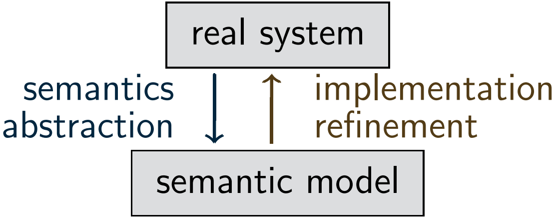
Transition systems $\widehat{=}$ extended digraphs



The semantic model yields a formal representation of:

- the states of the system $\longleftarrow$ **nodes**
- the stepwise behaviour $\longleftarrow$ **transitions**
- the initial states
- additional information on
  communication $\longleftarrow$ **actions**
  state properties $\longleftarrow$ **atomic proposition**

Transition system (TS)

A transition system is a tuple

$$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$$

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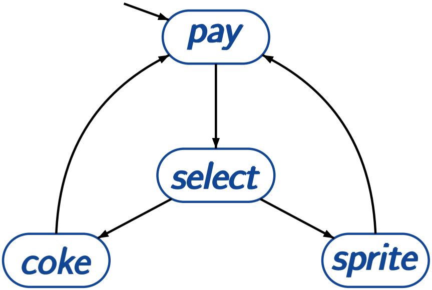
- $S_0 \subseteq S$ the set of initial states,
- AP a set of atomic propositions,
- $L : S \rightarrow 2^{AP}$ the labeling function

Transition system for beverage machine



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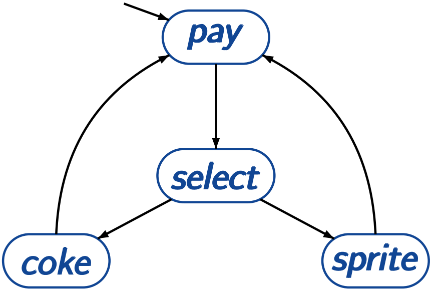

state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

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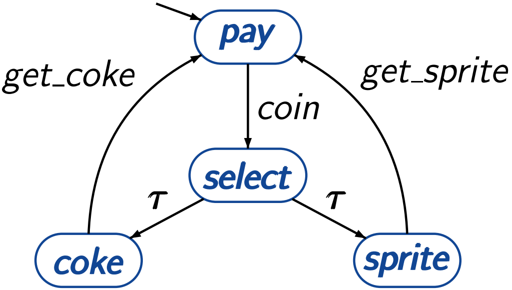

actions:
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get_sprite
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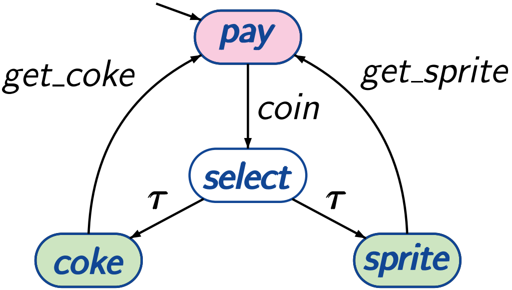

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state space $S = \{pay, select, coke, sprite\}$

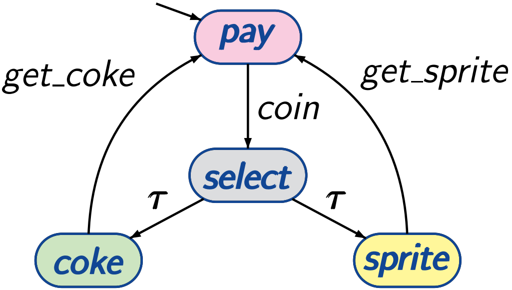
set of initial states: $S_0 = \{pay\}$

set of atomic propositions: $AP = \{pay, drink\}$

labeling function: $L(coke) = L(sprite) = \{drink\}$

$L(pay) = \{pay\}, \; L(select) = \emptyset$

Transition system for beverage machine



actions:
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state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

set of atomic propositions: $AP = S$

labeling function: $L(s) = \{s\}$ for each state s

"Behaviour" of transition systems

possible behaviours of a TS result from:

```
select nondeterministically an initial state s ∈ S_0
WHILE  s is non-terminal DO
      select nondeterministically a transition s --α--> s'
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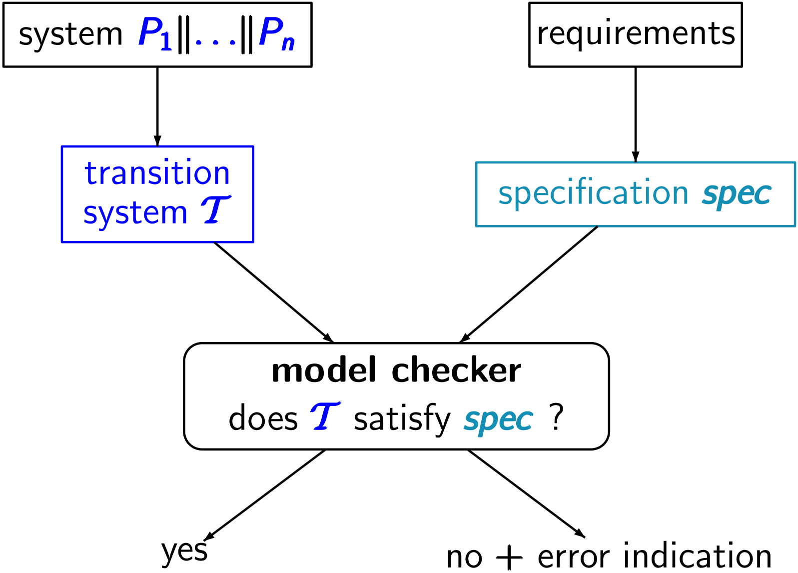
reachable fragment:

$\mathit{Reach}(\mathcal{T})$ = set of all states that are reachable from an initial state through some execution

Model checking



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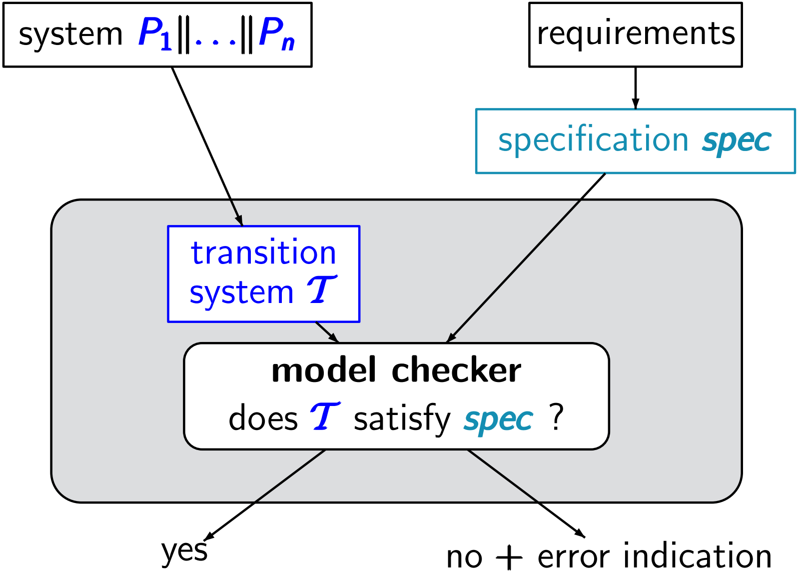

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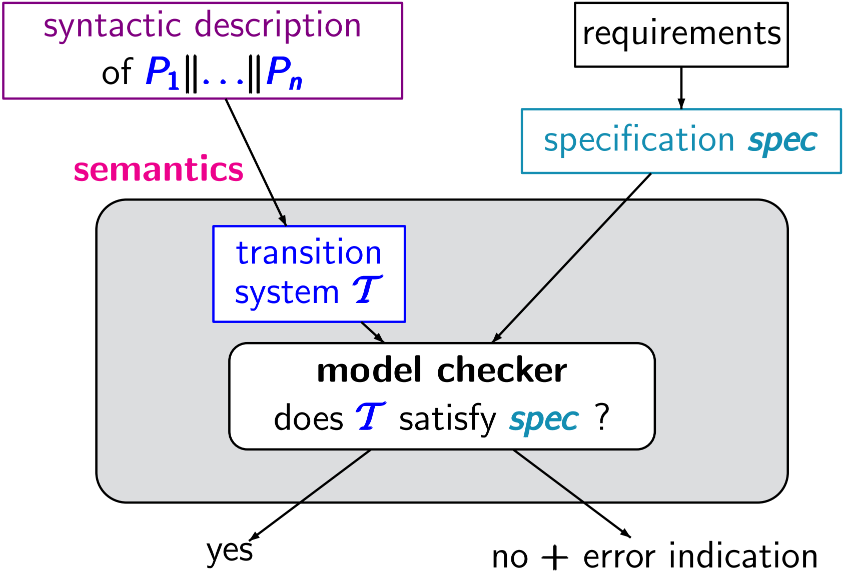
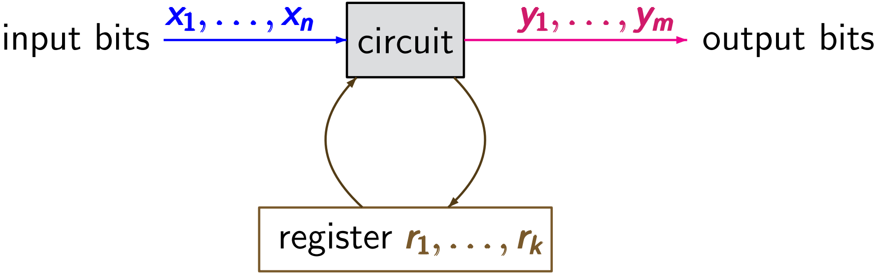


Modelling of sequential circuits by TS



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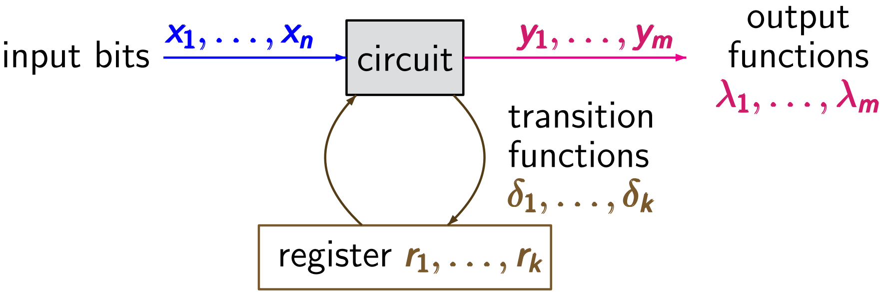

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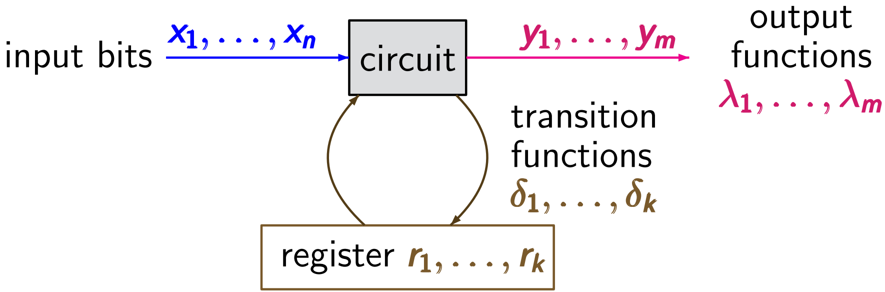

$$\delta_j, \lambda_i \mathrel{\widehat{=}} \text{switching functions } \{0,1\}^n \times \{0,1\}^k \longrightarrow \{0,1\}$$

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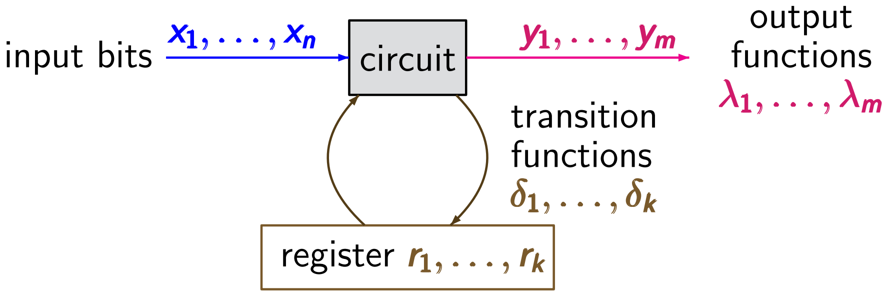
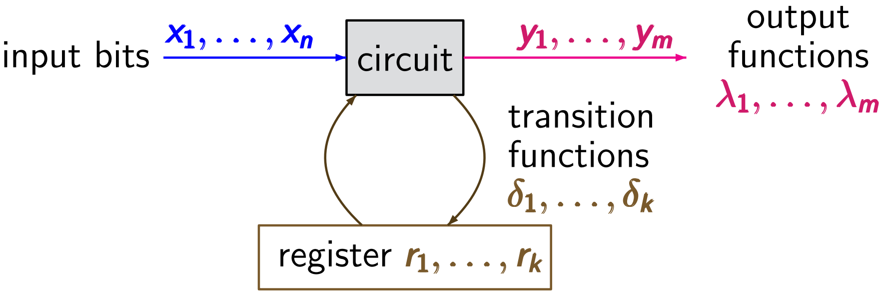


$\delta_j, \lambda_i \mathrel{\widehat{=}}$ switching functions $\{0,1\}^n \times \{0,1\}^k \longrightarrow \{0,1\}$

input values $a_1, \ldots, a_n$ for the input variables + current values $c_1, \ldots, c_k$ of the registers	$\mapsto$	output value $\lambda_i(\ldots)$ for output variable y_i; next value $\delta_j(\ldots)$ for register r_j

Modelling of sequential circuits by TS



initial register evaluation $[r_1 = c_{01}, \ldots, r_k = c_{0k}]$

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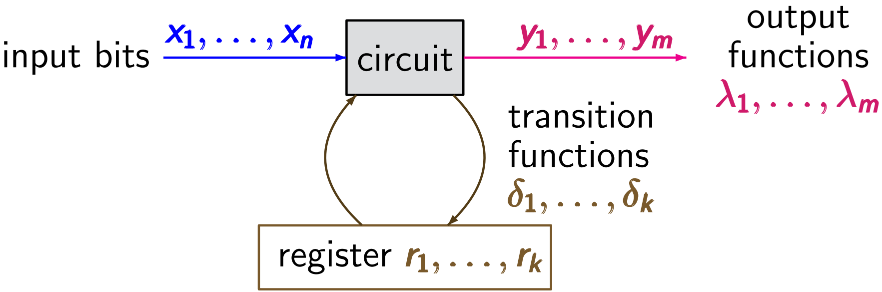

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transition system:

- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$

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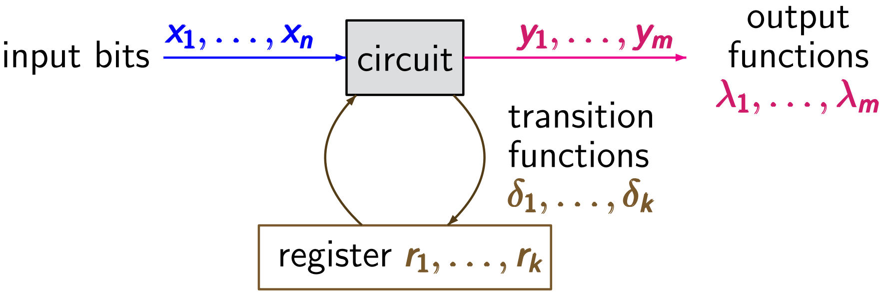

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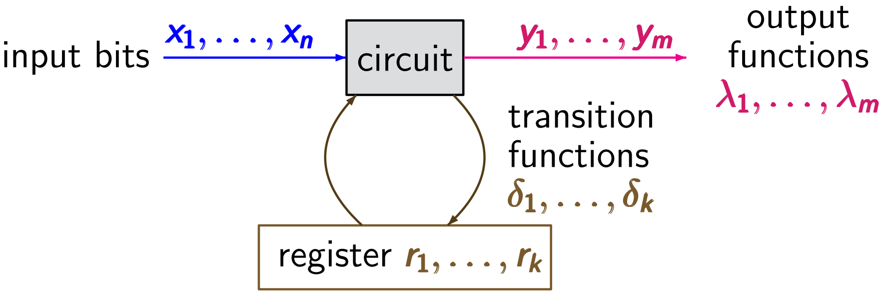

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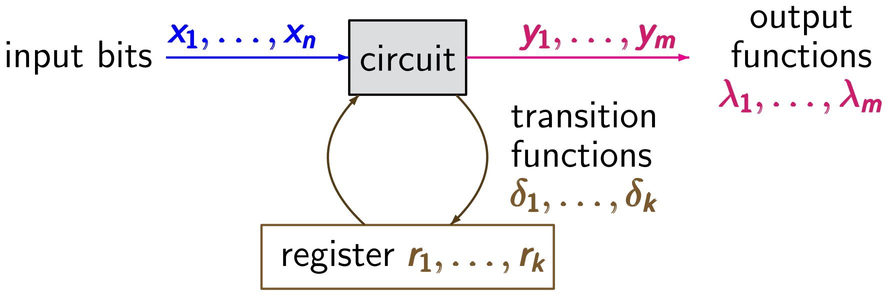

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transition system:

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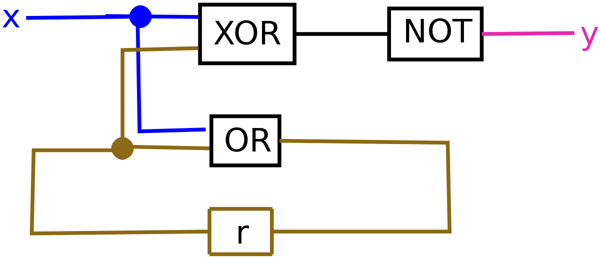
- transitions represent the stepwise behavior
- values of input bits change nondeterministically
- atomic propositions: $x_1, \ldots, x_n, y_1, \ldots, y_m, r_1, \ldots, r_k$

Example: sequential circuit



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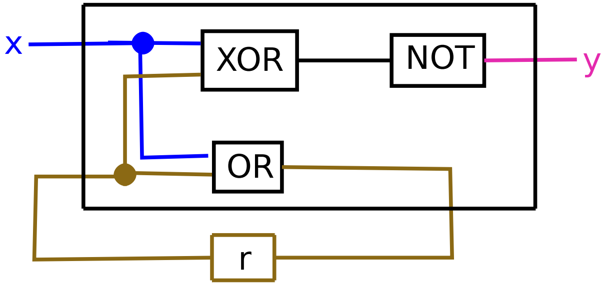

output function: $\lambda_y = \neg(x \oplus r)$

transition function: $\delta_r = x \vee r$

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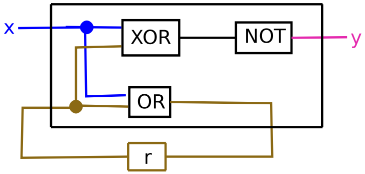

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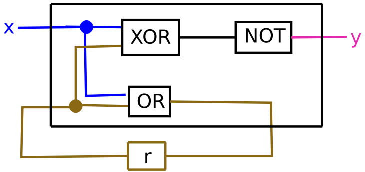

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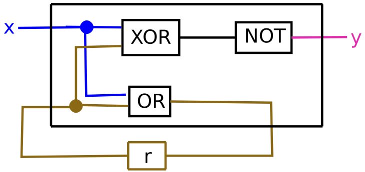

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x=0 r=0

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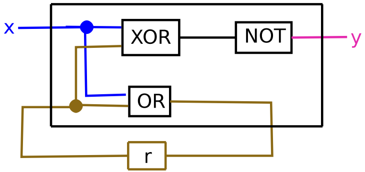

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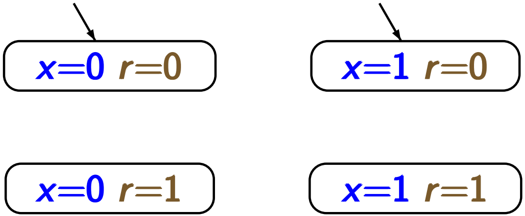

initial register evaluation: r=0

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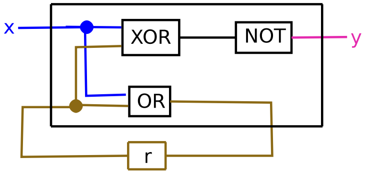

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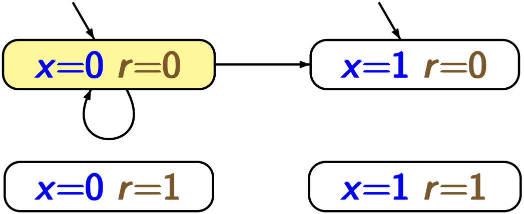

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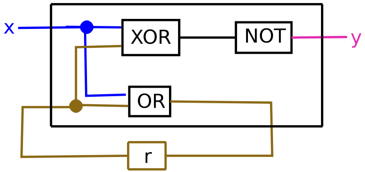

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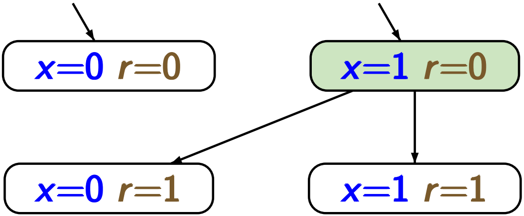

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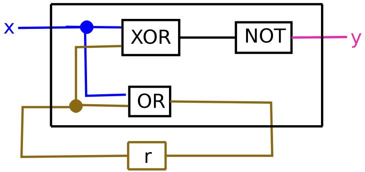

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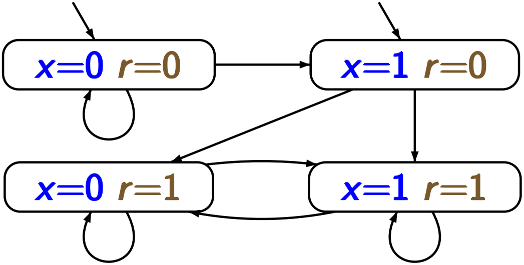

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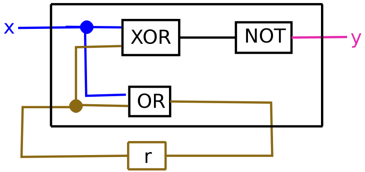

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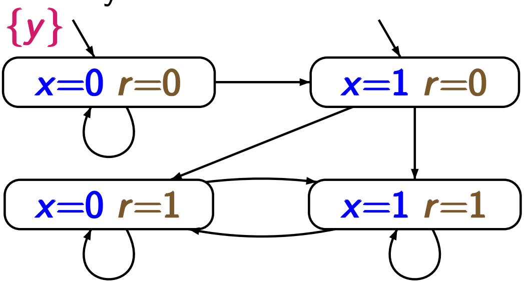

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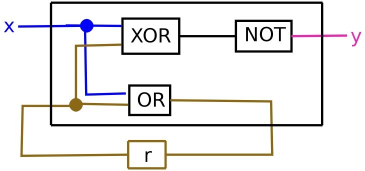

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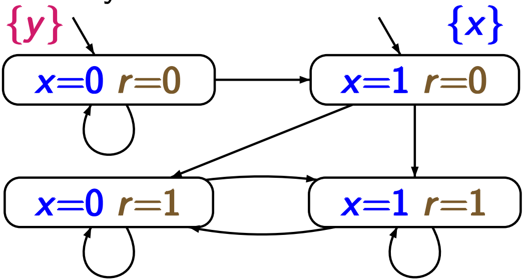

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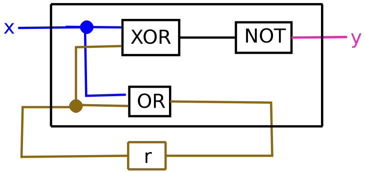

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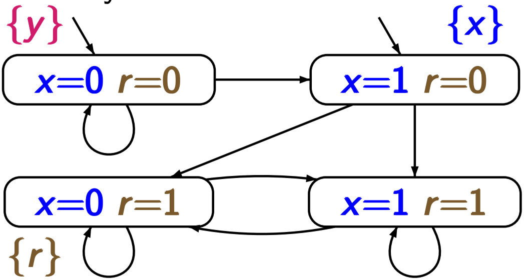

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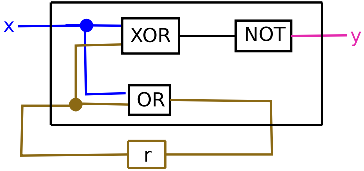

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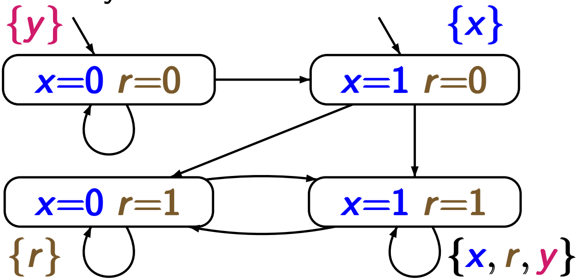

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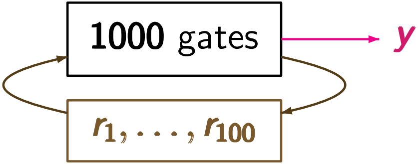
. . . has the transition system for a circuit of the form?



1 output bit
no input
100 registers

How many states . . .

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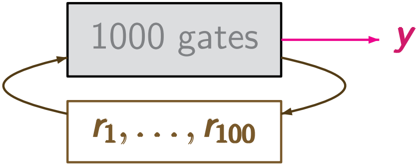

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answer: 2^{100}

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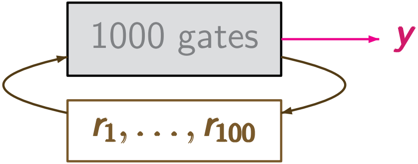

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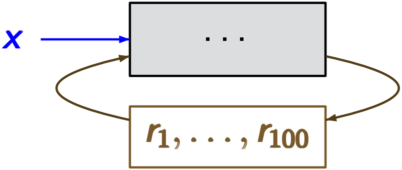

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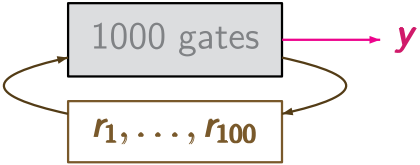

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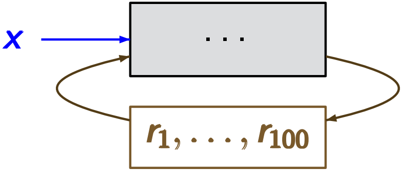

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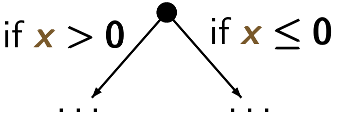
answer: $2^{100} * 2^1 = 2^{101}$

problem: TS-representation of conditional branchings **?**



Data-dependent systems

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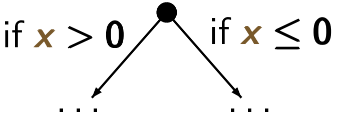

example: sequential program

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WHILE  x > 0 DO
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Data-dependent systems

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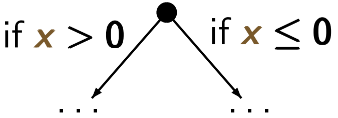

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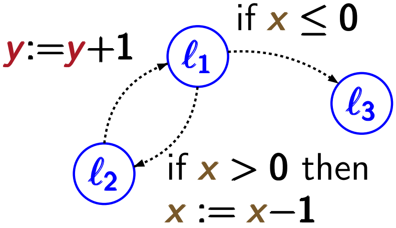

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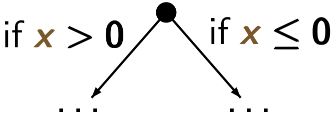

example: sequential program

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WHILE  x > 0 DO
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         y := y+1
OD
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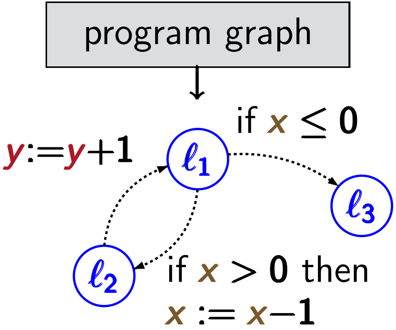

Data-dependent systems

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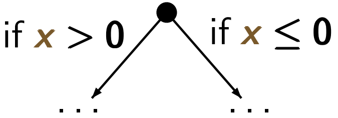

example: sequential program

$\ell_1 \rightarrow$ WHILE $x > 0$ DO
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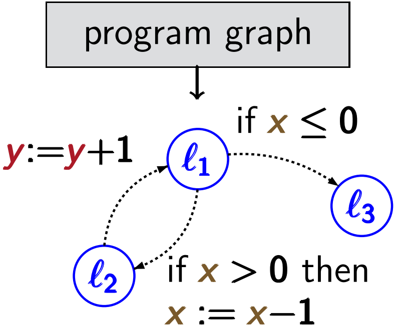

ℓ_1, ℓ_2, ℓ_3 are locations,
i.e., control states

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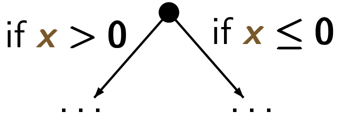

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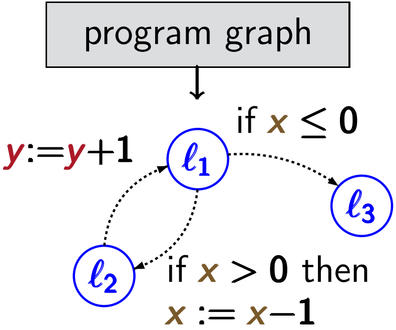

states of the transition system:

locations + relevant data (*here:* values for x and y)

Example: TS for sequential program

initially: $x = 2$, $y = 0$

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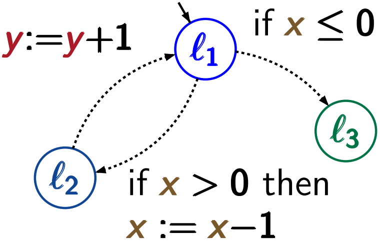
program graph



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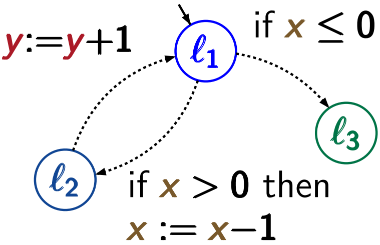

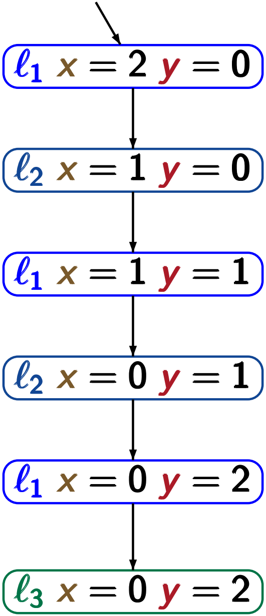

Example: TS for sequential program

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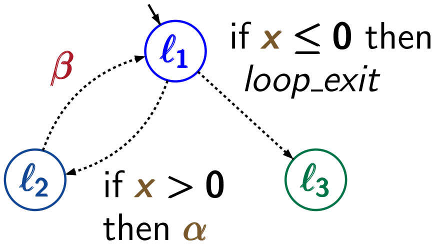

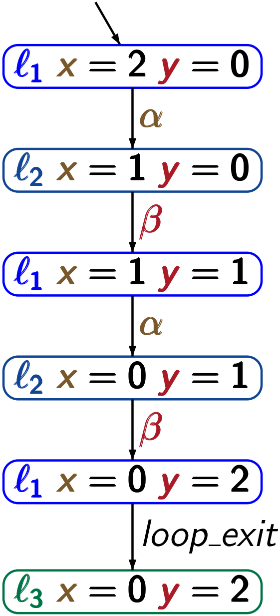

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Notation: $Eval(Var)$ = set of evaluations for Var

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Example:

$$[x{=}0,\ y{=}3,\ z{=}6] \models \neg x \wedge y{<}z$$

$$[x{=}0,\ y{=}3,\ z{=}6] \not\models x \vee y{=}z$$

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Let Var be a set of typed variables.

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example: if α is the assignment $x{:=}x{+}y$ then

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$$\Downarrow$$

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states in $\mathcal{T}_{\mathcal{P}}$ have the form

$\langle \ell, \eta \rangle$

location ↗ ↖ variable evaluation

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$$\frac{premise}{conclusion}$$

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It means that $\longrightarrow$ is the smallest relation such that:

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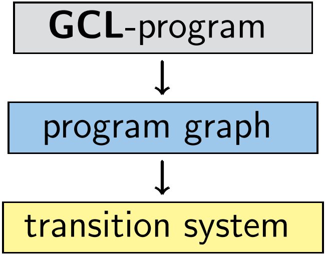

Guarded Command Language (GCL)

guarded command $g \Rightarrow$ ***stmt***

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conditional command:

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FI

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← IF g THEN stmt_1 ELSE stmt_2 FI

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← IF g THEN $stmt_1$
ELSE $stmt_2$
FI

symbol :: stands for the nondeterministic choice between enabled guarded commands

Guarded Command Language (GCL)

modeling language with nondeterministic choice

$$
\begin{aligned}
stmt \stackrel{\text{def}}{=} \;& x := expr \quad \big| \quad stmt_1 ; stmt_2 \quad \big| \\
& \texttt{DO} :: g_1 \Rightarrow stmt_1 \;\; \ldots \;\; :: g_n \Rightarrow stmt_n \; \texttt{OD} \\
& \texttt{IF} :: g_1 \Rightarrow stmt_1 \;\; \ldots \;\; :: g_n \Rightarrow stmt_n \; \texttt{FI} \\
& \vdots
\end{aligned}
$$

where x is a typed variable and $expr$ an expression of the same type

Guarded Command Language (GCL)

modeling language with nondeterministic choice

$$
\begin{aligned}
\mathit{stmt} \stackrel{\text{def}}{=} \; & x := \mathit{expr} \quad \big| \quad \mathit{stmt}_1 ; \mathit{stmt}_2 \quad \big| \\
& \texttt{DO} :: g_1 \Rightarrow \mathit{stmt}_1 \; \ldots \; :: g_n \Rightarrow \mathit{stmt}_n \; \texttt{OD} \\
& \texttt{IF} :: g_1 \Rightarrow \mathit{stmt}_1 \; \ldots \; :: g_n \Rightarrow \mathit{stmt}_n \; \texttt{FI} \\
& \vdots
\end{aligned}
$$

where x is a typed variable and *expr* an expression of the same type

semantics of a **GCL**-program: program graph

GCL-program for beverage machine

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uses two variables $\#sprite, \#coke \in \{0, 1, \ldots, max\}$
for the number of available drinks (sprite or coke)

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uses the following actions:

	enabled	effect
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get_sprite	if $\#sprite > 0$	$\#sprite := \#sprite - 1$

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refill	any time	$\#sprite := max$ $\#coke := max$

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insert_coin	any time	no effect on variables

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refill	any time	$\#sprite := max$ $\#coke := max$
insert_coin	any time	no effect on variables
return_coin	if machine is empty and user has entered a coin (no effect on variables)	

GCL-program for beverage machine

$$
\begin{array}{lll}
\texttt{DO} & :: & \text{true} \Rightarrow \text{insert\_coin}; \\
& & \texttt{IF} \; :: \; \#sprite = \#coke = 0 \Rightarrow \text{return\_coin} \\
& & \phantom{\texttt{IF}} \; :: \; \#coke > 0 \Rightarrow \#coke := \#coke - 1 \\
& & \phantom{\texttt{IF}} \; :: \; \#sprite > 0 \Rightarrow \#sprite := \#sprite - 1 \\
& & \texttt{FI} \\
& :: & \text{true} \Rightarrow \#sprite := \mathbf{\mathit{max}}; \#coke := \mathbf{\mathit{max}} \\
\texttt{OD} & &
\end{array}
$$

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin

         :: #coke > 0 ⇒ #coke := #coke − 1

         :: #sprite > 0 ⇒ #sprite := #sprite−1

      FI
   :: true ⇒ #sprite := max; #coke := max
OD
```

GCL-program for beverage machine


```
DO  ::  true ⇒ insert_coin; (* user inserts a coin *)
        IF  ::  #sprite = #coke = 0 ⇒ return_coin
                                  (* no beverage available *)
            ::  #coke > 0 ⇒ #coke := #coke − 1

            ::  #sprite > 0 ⇒ #sprite := #sprite−1
        FI
    ::  true ⇒ #sprite := max; #coke := max
OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin
                                   (* no beverage available *)
         :: #coke > 0 ⇒ #coke := #coke − 1
                                   (* user selects coke *)
         :: #sprite > 0 ⇒ #sprite := #sprite−1
                                   (* user selects sprite *)
      FI
   :: true ⇒ #sprite := max; #coke := max
                                   (* refilling of the machine *)
OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin
                                  (* no beverage available *)
         :: #coke > 0 ⇒ get_coke
                                  (* user selects coke *)
         :: #sprite > 0 ⇒ get_sprite
                                  (* user selects sprite *)
      FI
   :: true ⇒ refill
                                  (* refilling of the machine *)
OD
```

GCL-program for beverage machine


```
DO  ::  true ⇒ insert_coin;
        IF  ::  #sprite = #coke = 0 ⇒ return_coin

            ::  #coke > 0 ⇒ get_coke

            ::  #sprite > 0 ⇒ get_sprite

        FI
    ::  true ⇒ refill

OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin;
      IF :: #sprite = #coke = 0
                      ⇒ return_coin
         :: #coke   > 0 ⇒ get_coke
         :: #sprite > 0 ⇒ get_sprite
      FI
   :: true ⇒ refill
OD
```

... yields a program graph with

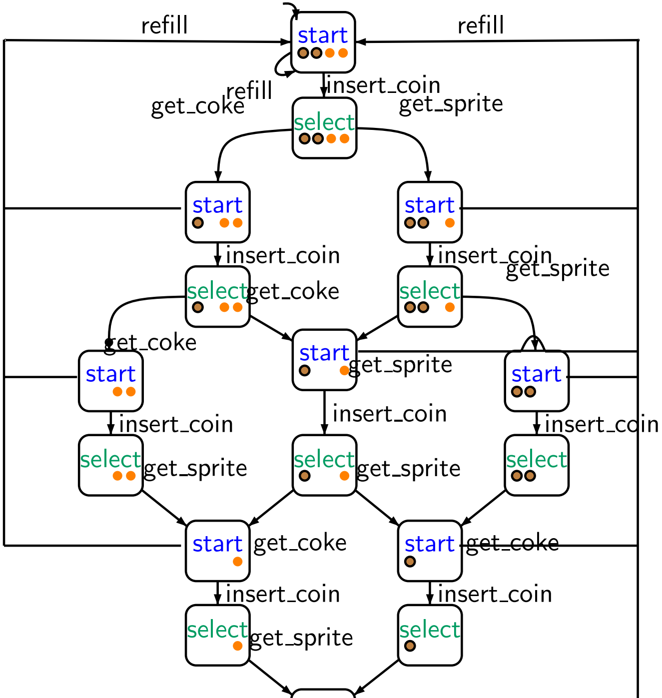
- two variables $\#sprite, \#coke \in \{0, 1, \ldots, max\}$

GCL-program for beverage machine


```
start  → DO :: true ⇒ insert_coin;
select →          IF :: #sprite = #coke = 0
                                  ⇒ return_coin
                     :: #coke   > 0 ⇒ get_coke
                     :: #sprite > 0 ⇒ get_sprite
                  FI
            :: true ⇒ refill
         OD
```

... yields a program graph with

- two variables $\#sprite, \#coke \in \{0, 1, \ldots, max\}$
- two locations *start* and *select*


refill
start
refill
insert_coin
refill
get_coke
select
get_sprite
start
start
insert_coin
insert_coin
select
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select
get_sprite
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