Overview

CTL model checking

given: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
CTL formula Φ over AP

question: does $\mathcal{T} \models \Phi$ hold ?

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idea:

- compute $Sat(\Phi) = \{s \in S : s \models \Phi\}$
- check whether $S_0 \subseteq Sat(\Phi)$

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FOR ALL subformulas Ψ of Φ DO
    compute Sat(Ψ)

OD
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FOR ALL subformulas Ψ of Φ DO
    compute Sat(Ψ)
    replace Ψ by a new atomic proposition a_Ψ
    FOR ALL s ∈ Sat(Ψ) DO add a_Ψ to L(s) OD
OD
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OD
IF S0 ⊆ Sat(Φ) THEN output "yes"
               ELSE output "no"
FI
```

Example: CTL model checking

$$\Phi = \quad \exists\bigcirc a \ \vee \exists(b \,\mathsf{U}\, \neg c)$$

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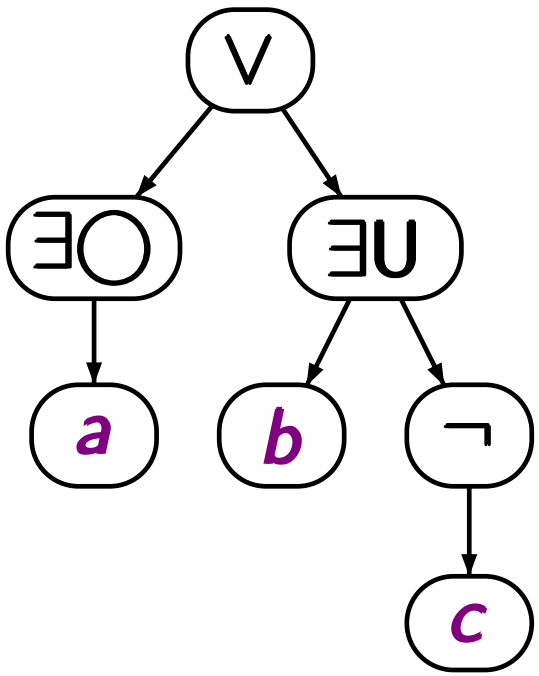
syntax tree for Φ



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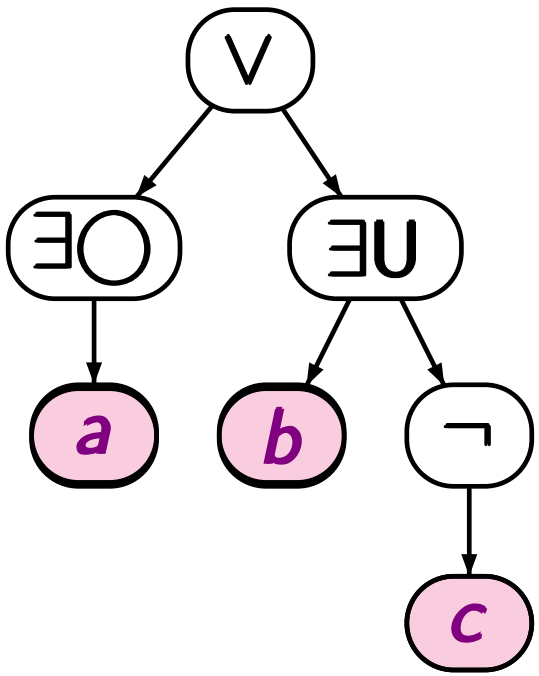

compute $Sat(a), Sat(b), Sat(c)$

processed in
bottom-up fashion

Example: CTL model checking

$$\Phi = \underbrace{\exists \bigcirc a}_{\Phi_1} \vee \exists (b \, \mathsf{U} \, \neg c)$$

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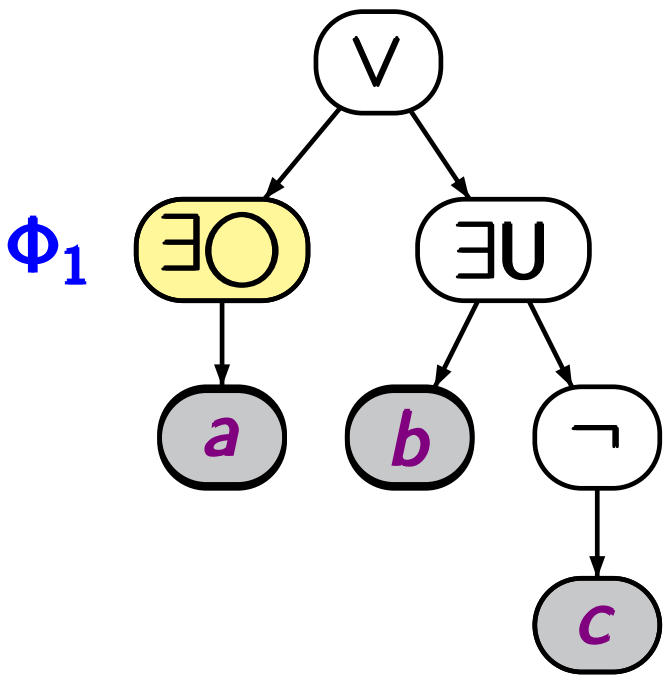

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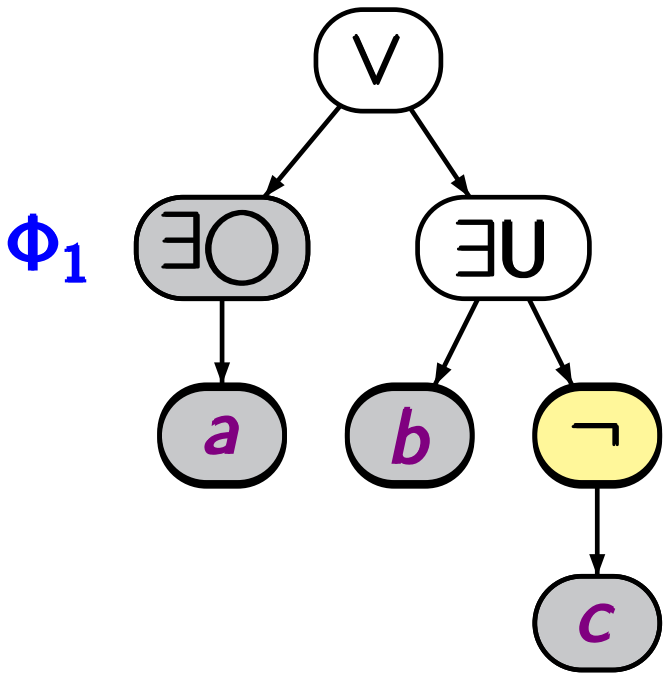

compute $Sat(a), Sat(b), Sat(c)$

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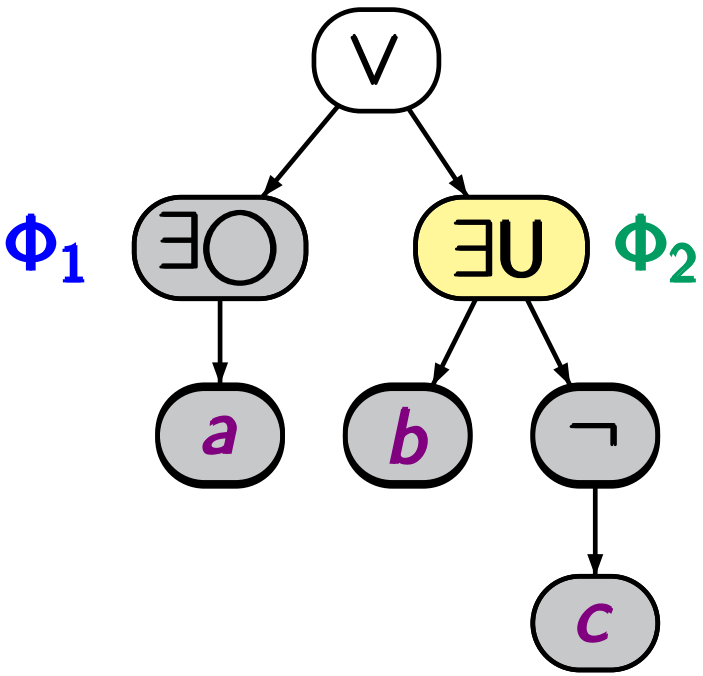
Example: CTL model checking

$$\Phi = \underbrace{\exists \bigcirc a}_{\Phi_1} \vee \underbrace{\exists (b \,\mathsf{U}\, \neg c)}_{\Phi_2}$$

syntax tree for Φ



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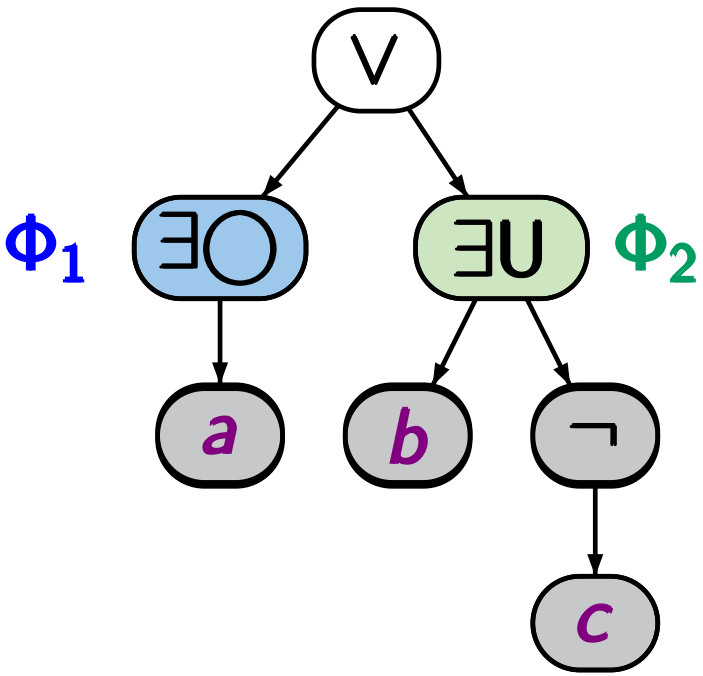

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replace Φ_1 with a_1

replace Φ_2 with a_2

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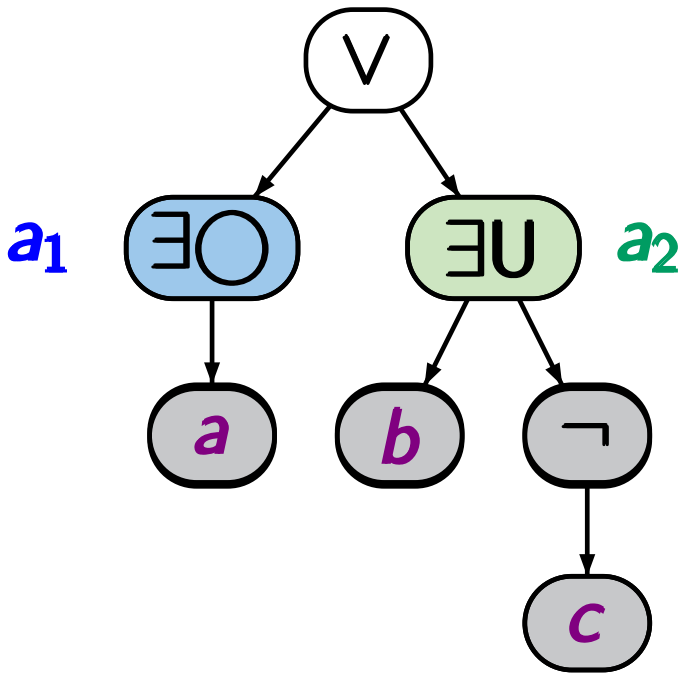
syntax tree for Φ



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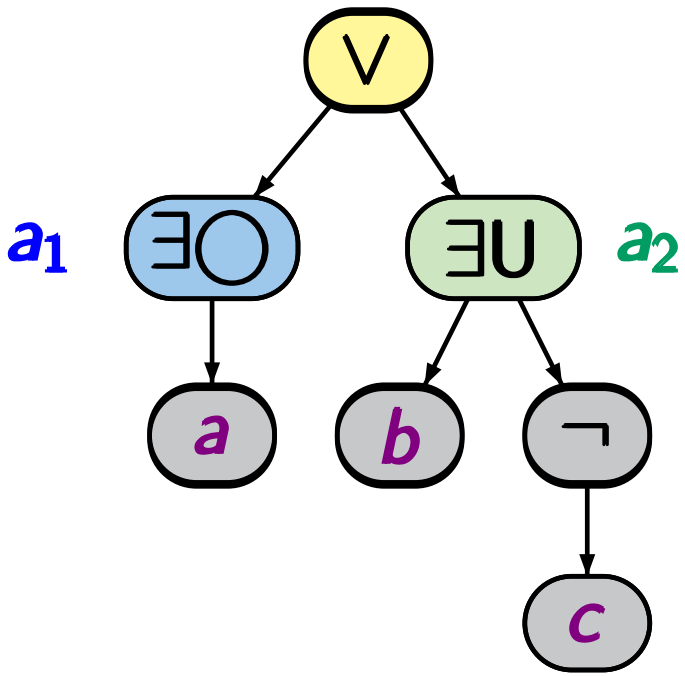

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$Sat(\Phi) = Sat(a_1) \cup Sat(a_2)$

CTL model checking

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method: regard in bottom-up manner all subformulas Ψ of Φ and compute their satisfaction sets

$$Sat(\Psi) = \{s \in S : s \models \Psi\}$$

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here: explanations for the case that Φ is in existential normal form

analogous algorithms can be designed for standard CTL (and the derived operators)

Recall: Existential normal form for CTL

For each **CTL** formula there is an equivalent formula in **$\exists$-normal form**, i.e., a **CTL** formula with the basis modalities $\exists\bigcirc$, $\exists\mathbf{U}$, $\exists\Box$.

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CTL formulas in ∃-normal form:

$$\Psi \;::=\; \mathit{true} \mid a \mid \neg\Psi \mid \Psi_1 \wedge \Psi_2 \mid \exists\bigcirc\Psi \mid \exists(\Psi_1 \,\mathbf{U}\, \Psi_2) \mid \exists\Box\Psi$$

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CTL formula $\rightsquigarrow$ **CTL** formula in ∃-normal form

$$\forall\bigcirc\Phi \rightsquigarrow \neg\exists\bigcirc\neg\Phi$$

$$\forall(\Phi_1 \,\mathbf{U}\, \Phi_2) \rightsquigarrow \neg\exists(\neg\Phi_2 \,\mathbf{U}(\neg\Phi_1 \wedge \neg\Phi_2)) \wedge \neg\exists\Box\neg\Phi_2$$

Recursive computation of the satisfaction sets

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$$Sat(true) = S$$

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\begin{aligned}
Sat(true) &= S \\
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Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) &= \ldots \\
Sat(\exists \Box \Phi) &= \ldots
\end{aligned}
$$

treatment of $\exists \mathsf{U}$ and $\exists \Box$:

via fixed point computation

Recall: expansion law for $\exists \mathsf{U}$

$$\exists(\Phi_1 \,\mathsf{U}\, \Phi_2) \;\equiv\; \Phi_2 \vee \big(\Phi_1 \wedge \exists\bigcirc\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)\big)$$

Recall: expansion law for $\exists \mathsf{U}$

$$\exists(\Phi_1 \,\mathsf{U}\, \Phi_2) \;\equiv\; \Phi_2 \vee \big(\Phi_1 \wedge \exists\bigcirc\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)\big)$$

$$\mathit{Sat}(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) = \mathit{Sat}(\Phi_2) \cup$$

$$\big\{s \in \mathit{Sat}(\Phi_1) : \mathit{Post}(s) \cap \mathit{Sat}(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) \neq \varnothing\big\}$$

Fixed point characterization of $\exists \mathsf{U}$

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i.e., the set $T = Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$ is a **fixed point** of the higher-order function $\Omega : 2^S \to 2^S$ given by:

$$\Omega(T) \;=\; Sat(\Phi_2) \cup \big\{ s \in Sat(\Phi_1) : Post(s) \cap T \neq \varnothing \big\}$$

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satisfies the following conditions:

(1) $\mathit{Sat}(\Phi_2) \subseteq \mathit{Sat}(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$

(2) If $s \in \mathit{Sat}(\Phi_1)$ and $\mathit{Post}(s) \cap \mathit{Sat}(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) \neq \varnothing$ then $s \in \mathit{Sat}(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$

Least fixed point characterization of $\exists \mathsf{U}$

$$\exists(\Phi_1 \,\mathsf{U}\, \Phi_2) \;\equiv\; \Phi_2 \vee \big(\Phi_1 \wedge \exists\bigcirc\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)\big)$$

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satisfies the following conditions:

(1) $Sat(\Phi_2) \subseteq Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$

(2) If $s \in Sat(\Phi_1)$ and $Post(s) \cap Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) \neq \varnothing$ then $s \in Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$

$Sat(\exists(\Phi_1 \mathsf{U} \Phi_2))$ is the smallest set s.t. (1) and (2) hold

The always operator

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$$Sat(\exists\Box\Phi) = \text{greatest set } V \text{ of states s.t.}$$

$$V \subseteq \{s \in Sat(\Phi) : Post(s) \cap V \neq \varnothing\}$$

Greatest fixed point characterization of $\exists\Box$

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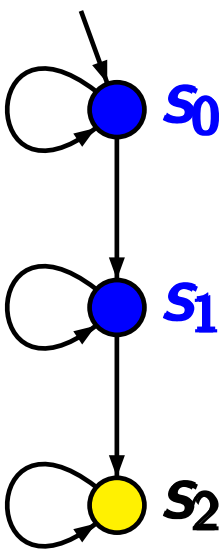

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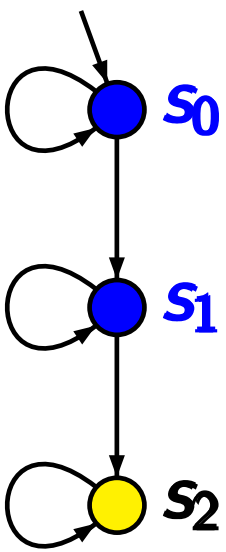

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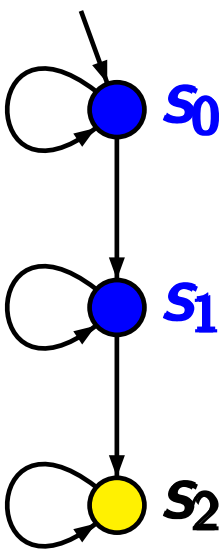

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$V \subsetneq Sat(\exists\Box a) = \{s_0, s_1\}$

Until versus weak until

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The formulas $\Psi = \exists(\Phi_1 \,\mathsf{U}\, \Phi_2)$ and $\Psi = \exists(\Phi_1 \,\mathsf{W}\, \Phi_2)$ fulfill the expansion law

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until: $\mathit{Sat}(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) =$ smallest set T of states s.t.

$$\mathit{Sat}(\Phi_2) \cup \{s \in \mathit{Sat}(\Phi_1) : \mathit{Post}(s) \cap T \neq \varnothing\} \subseteq T$$

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weak until: $\mathit{Sat}(\exists(\Phi_1 \,\mathsf{W}\, \Phi_2)) =$ greatest set V s.t.

$$\mathit{Sat}(\Phi_2) \cup \{s \in \mathit{Sat}(\Phi_1) : \mathit{Post}(s) \cap V \neq \varnothing\} \supseteq V$$

Fixed point equations for ∃U

$$Sat(\exists(a \,\mathsf{U}\, b)) = \text{smallest set of states } T \text{ s.t.}$$

$$(*) \quad Sat(b) \cup \{s \in Sat(a) : Post(s) \cap T \neq \varnothing\} \subseteq T$$

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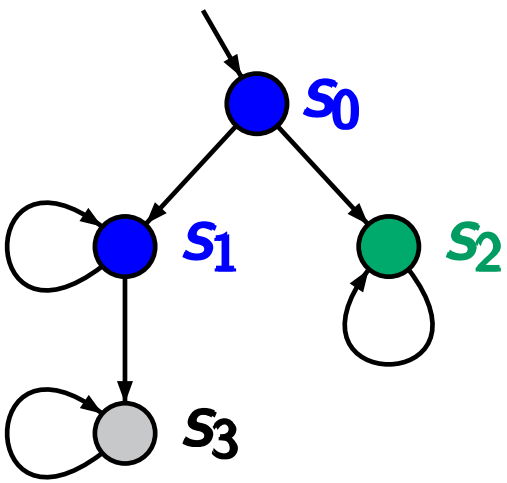

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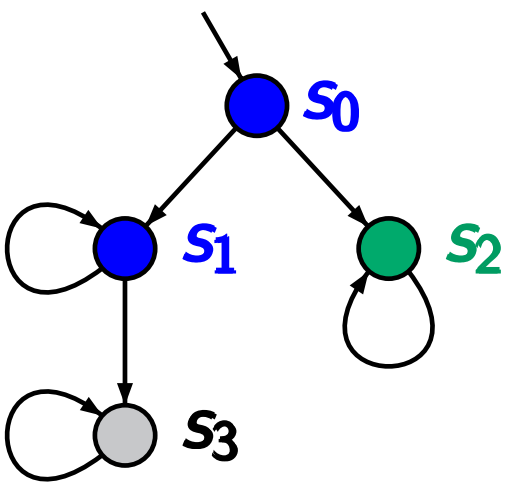

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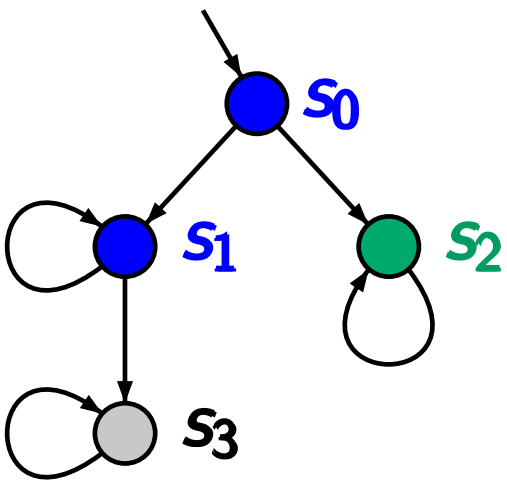

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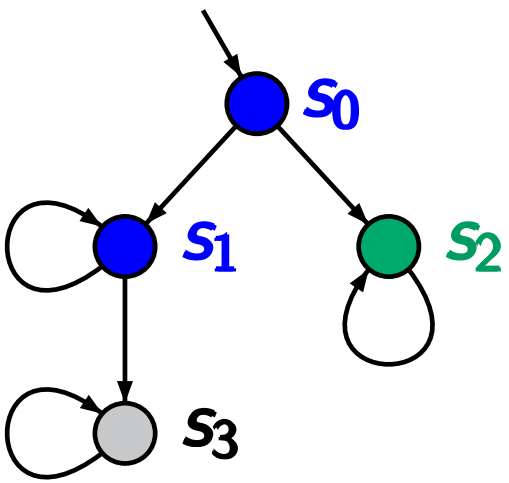

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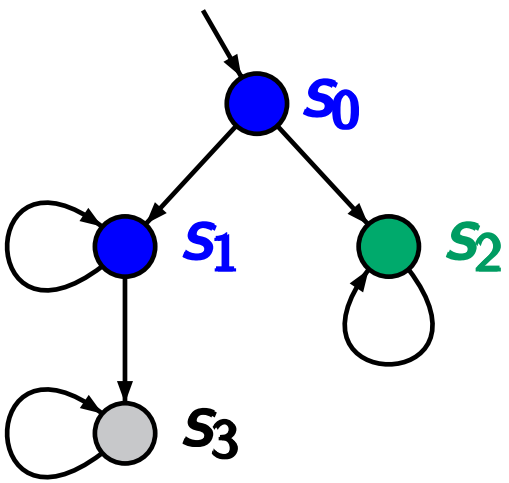

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$V = \{s_0, s_2\}$ satisfies $(**)$

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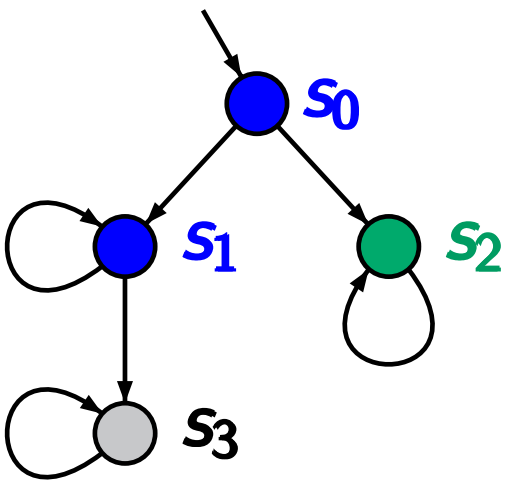
$(**)\quad V \subseteq Sat(b) \cup \{s \in Sat(a) : Post(s) \cap V \neq \emptyset\}$



$T = \{s_0, s_1, s_2\}$ satisfies $(*)$

$Sat(\exists(a\,\mathsf{U}\,b)) = \{s_0, s_2\} \subsetneq T$

$V = \{s_0, s_2\}$ satisfies $(**)$, but

$V \subsetneq Sat(\exists(a\,\mathsf{W}\,b)) = \{s_0, s_1, s_2\}$

Universally quantified formulas

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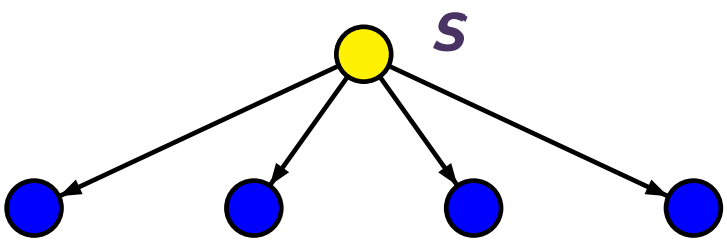

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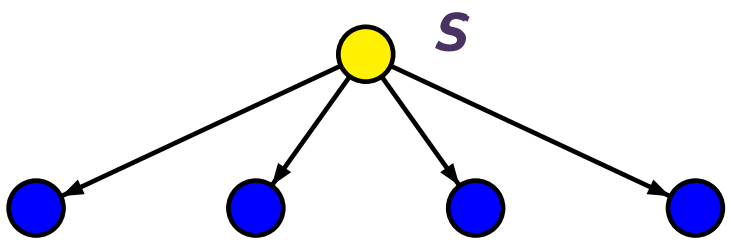

$Sat(\forall\Box a)$ = greatest set T of states s.t.

$$T \subseteq \{s \in Sat(a) : Post(s) \subseteq T\}$$

Universally quantified formulas

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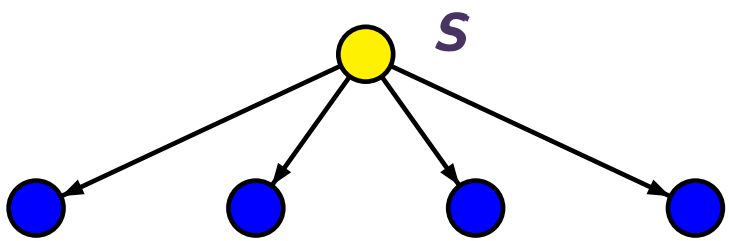

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$Sat(\forall(a \,\mathsf{U}\, b))$ = smallest set T of states s.t.

$$Sat(b) \cup \{s \in Sat(a) : Post(s) \subseteq T\} \subseteq T$$

Recursive computation of *Sat*(...)

Recursive computation of $Sat(\ldots)$

$$Sat(\Phi_1 \wedge \Phi_2) = Sat(\Phi_1) \cap Sat(\Phi_2)$$

$$Sat(\neg\Phi) = S \setminus Sat(\Phi)$$

$$Sat(\exists\bigcirc\Phi) = \{s \in S : Post(s) \cap Sat(\Phi) \neq \varnothing\}$$

$Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$ = smallest set T of states s.t.

- $Sat(\Phi_2) \subseteq T$
- $s \in Sat(\Phi_1)$ and $Post(s) \cap T \neq \varnothing \implies s \in T$

$Sat(\exists\Box\Phi)$ = greatest set V of states s.t.

- $V \subseteq Sat(\Phi)$
- $s \in V \implies Post(s) \cap V \neq \varnothing$

CTL model checking: until operator

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$$\exists(\Phi_1 \,\mathsf{U}\, \Phi_2) \;\equiv\; \Phi_2 \vee (\Phi_1 \wedge \exists\bigcirc\exists(\Phi_1 \,\mathsf{U}\, \Phi_2))$$

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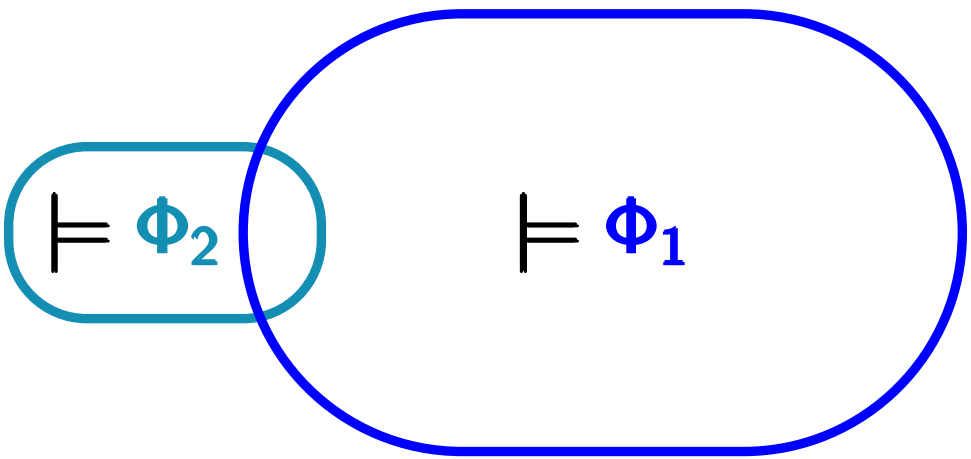

$$T_0 := Sat(\Phi_2)$$

CTL model checking: until operator

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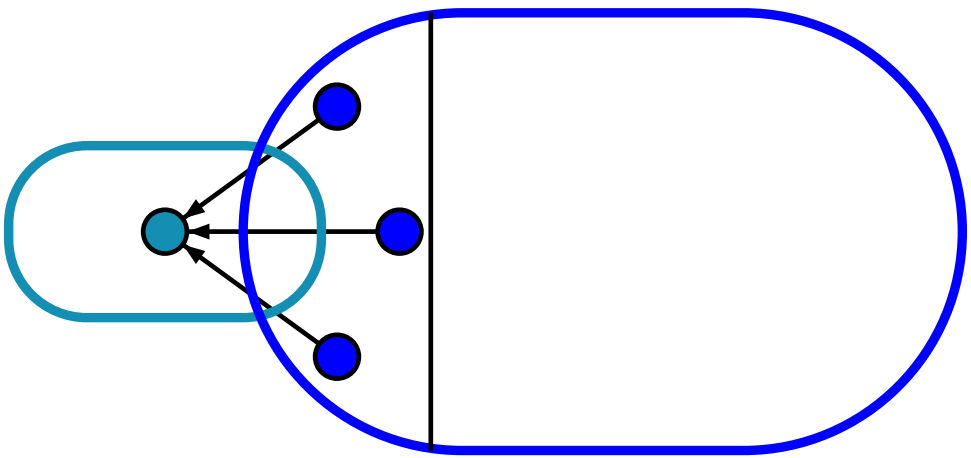

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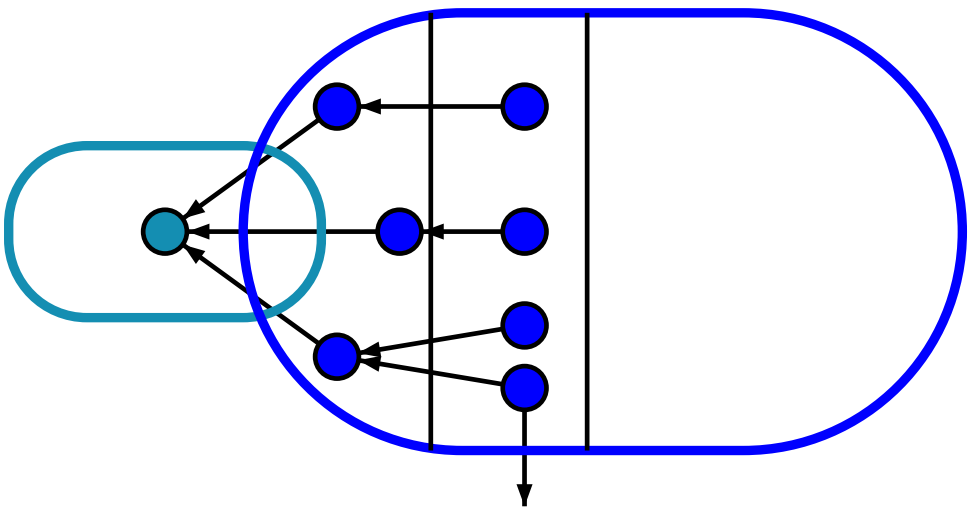

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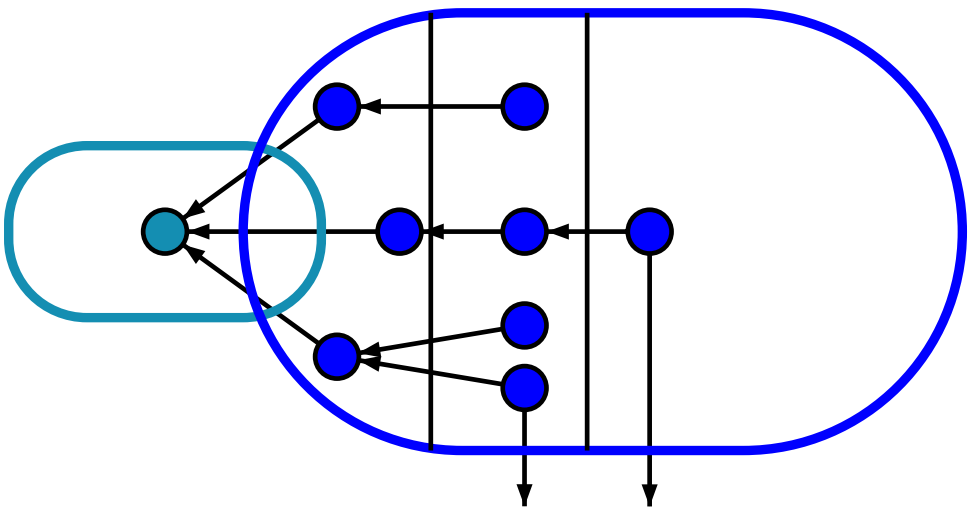

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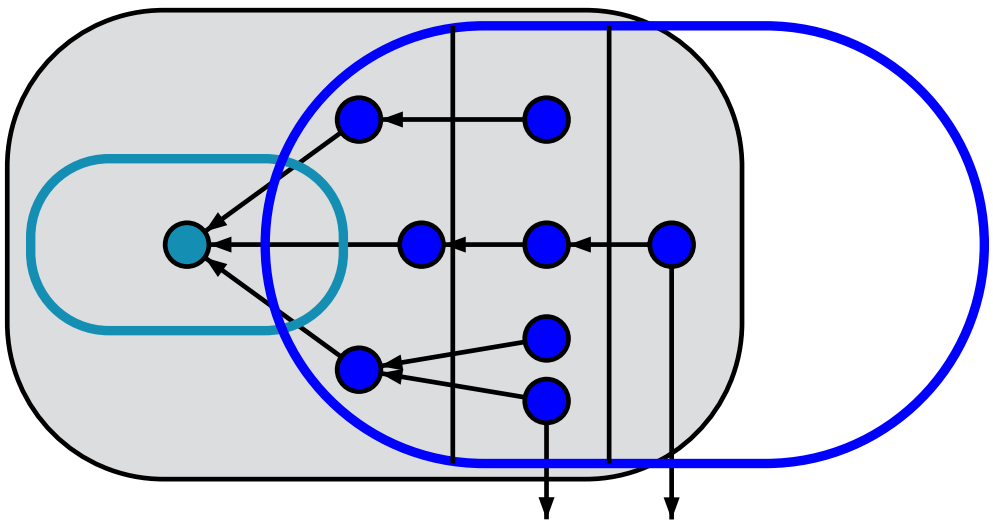

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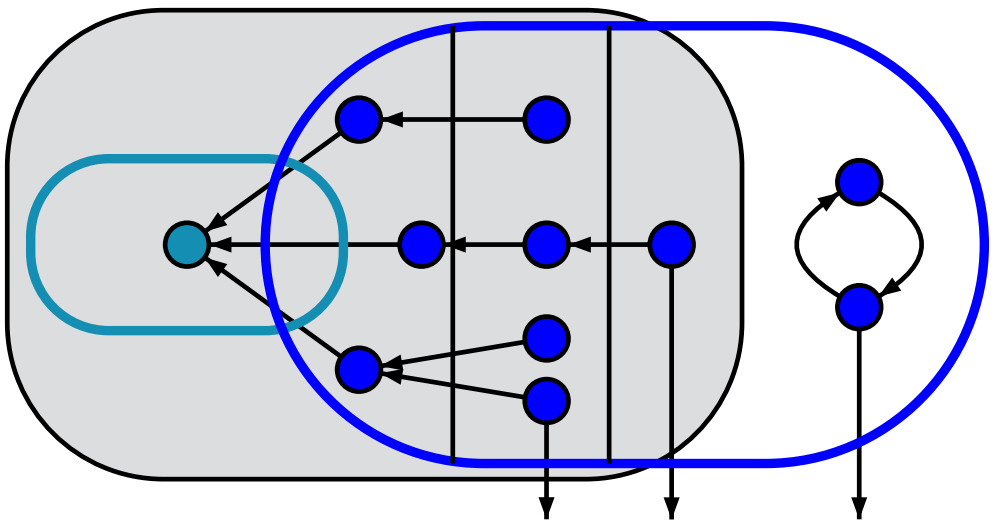

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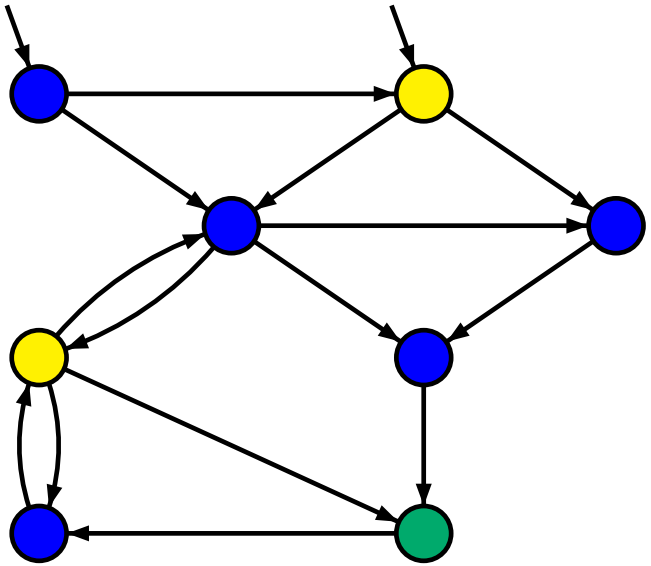
Example: until operator



● = $\{a\}$

● = $\{b\}$

● = $\varnothing$

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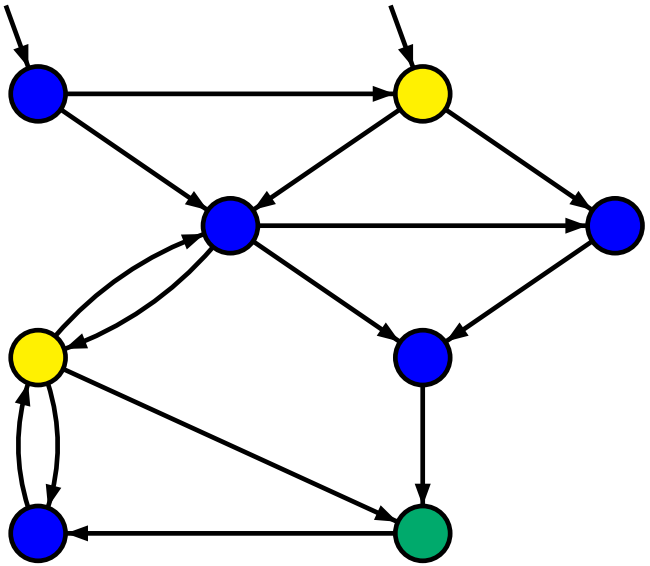

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computation of $Sat(\exists(a \,\mathsf{U}\, b))$

add all states $s \in Sat(b)$ to T
as long as there are unprocessed states in T:

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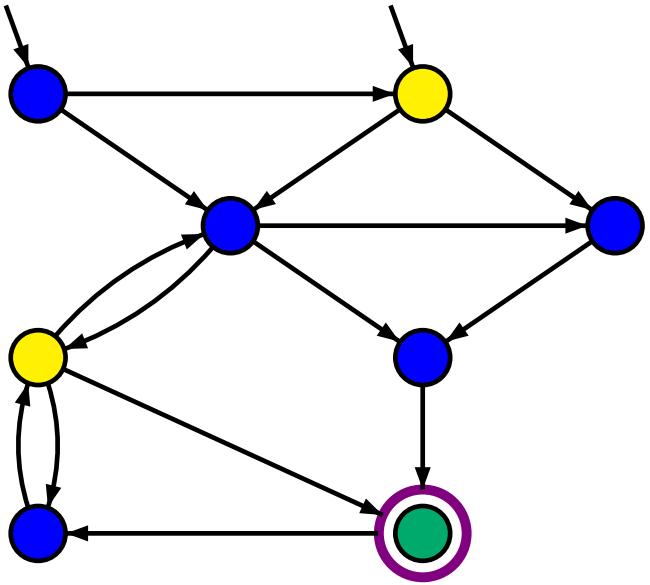

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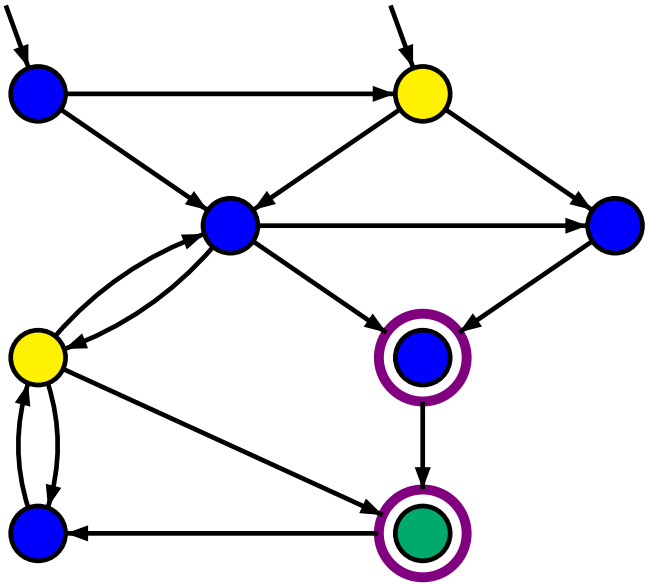

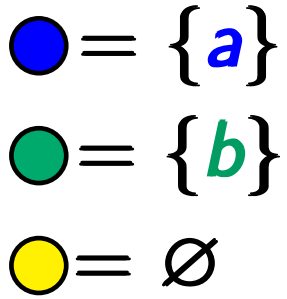

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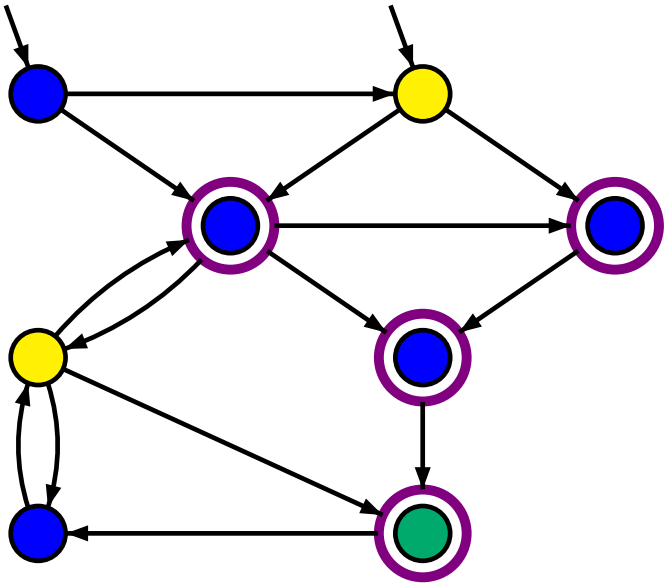

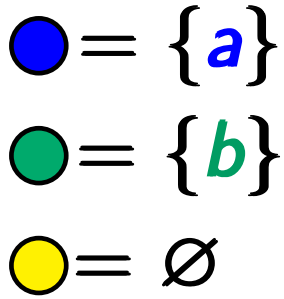

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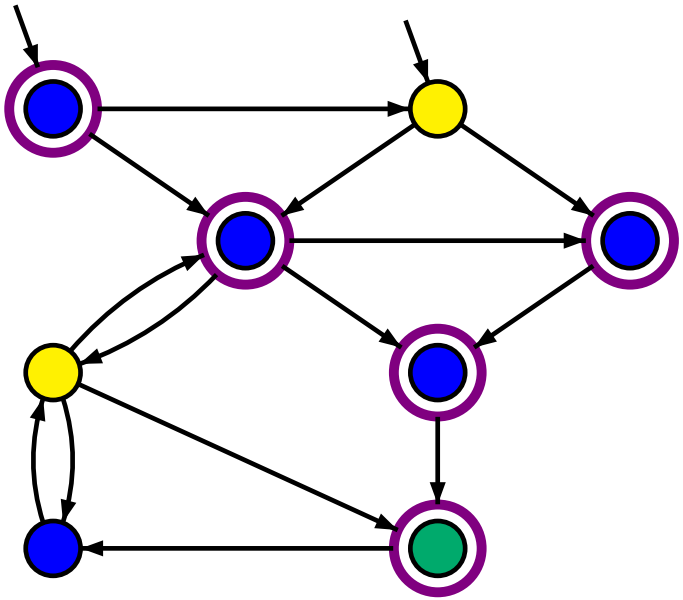

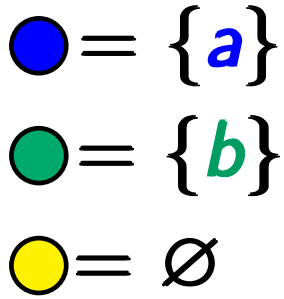

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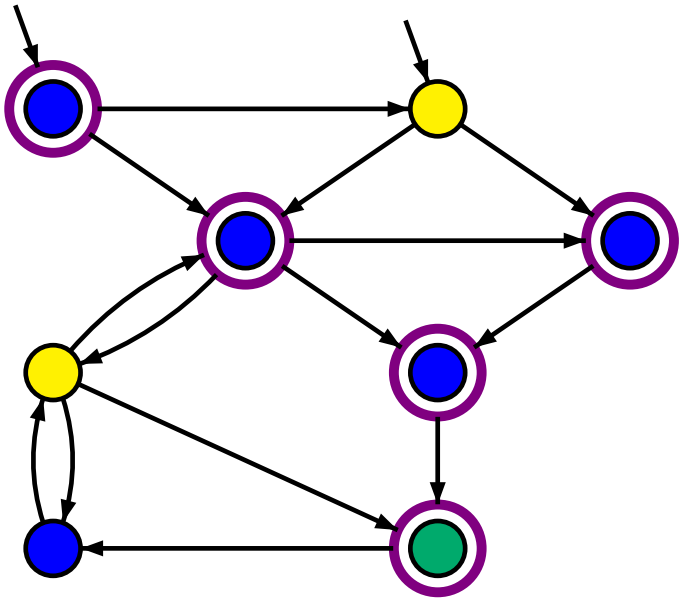

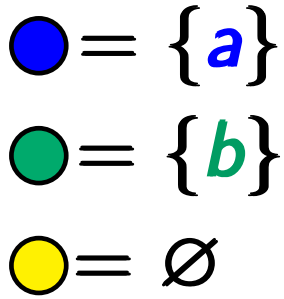

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`WHILE` $E \neq \varnothing$ `DO`

CTL model checking: treatment of $\exists U$

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complexity: $\mathcal{O}(size(\mathcal{T}))$

CTL model checking: always operator

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expansion law: $\exists\Box\Phi \equiv \Phi \wedge \exists\bigcirc\exists\Box\Phi$

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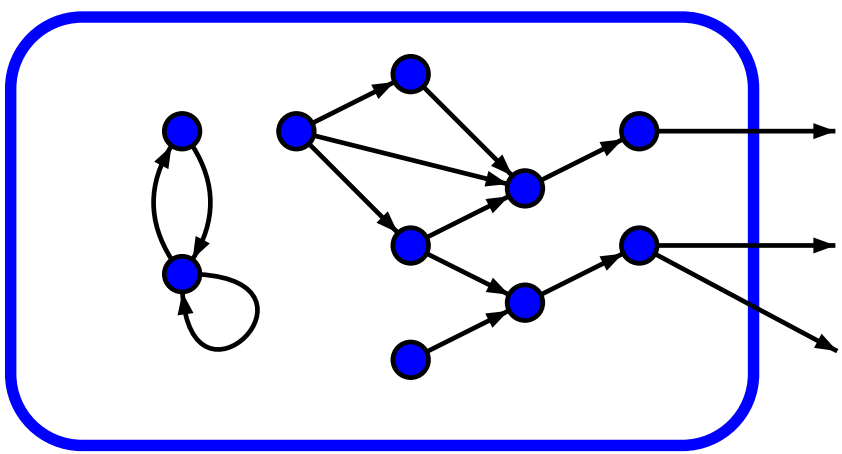
$Sat(\Phi)$



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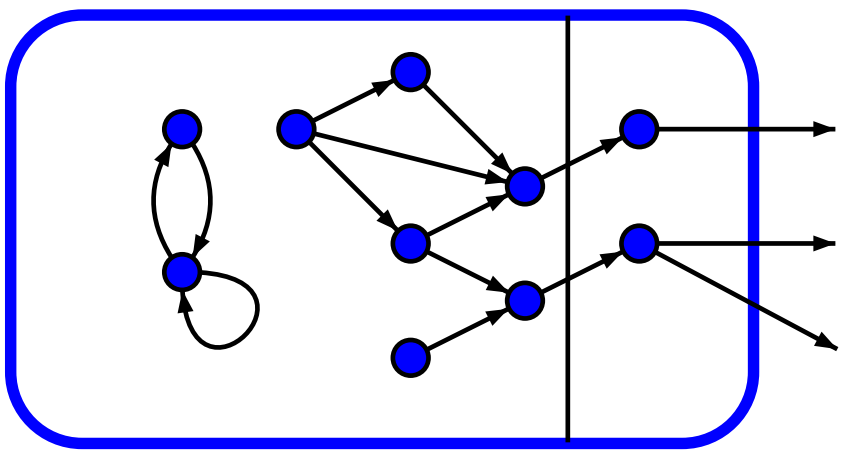

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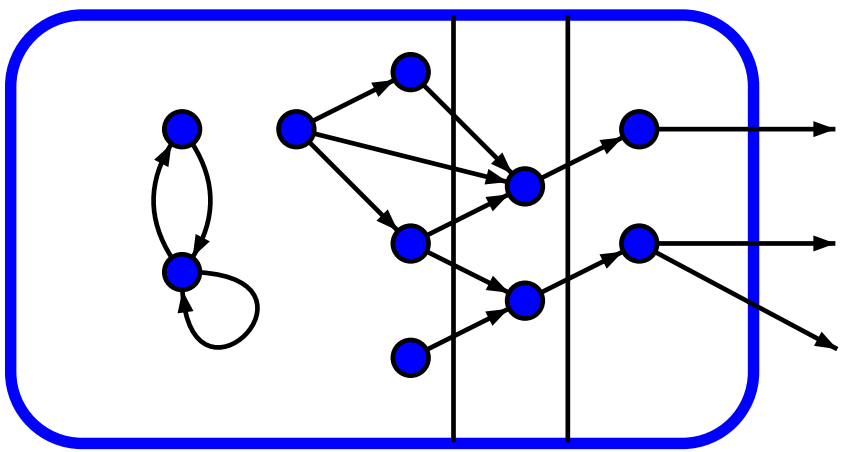

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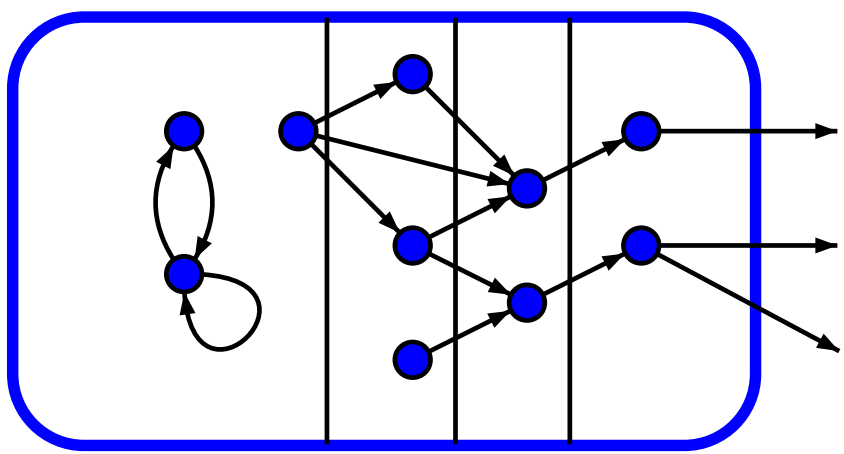

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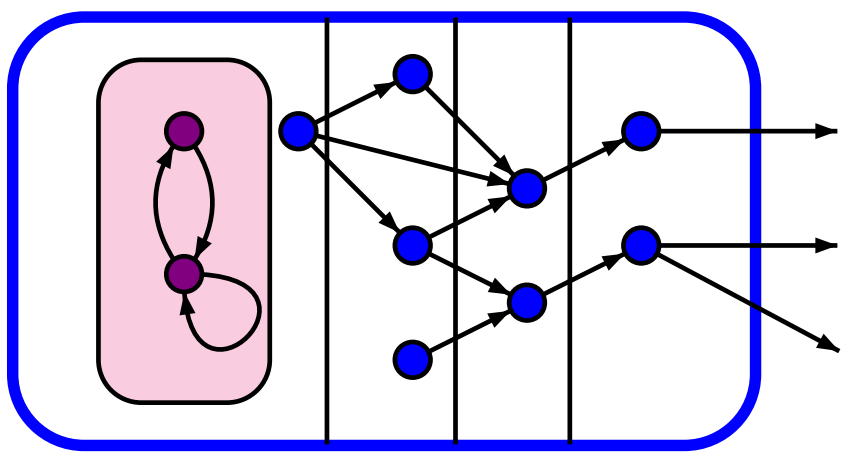

Computation of $Sat(\exists\Box\Phi)$

$T := Sat(\Phi) \leftarrow$ organizes the candidates for $s \models \exists\Box\Phi$

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WHILE $E \neq \varnothing$ DO

pick a state $s' \in E$ and remove s' from E

OD

Computation of $Sat(\exists\Box\Phi)$

$T := Sat(\Phi)$ ← organizes the candidates for $s \models \exists\Box\Phi$

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WHILE E ≠ ∅ DO
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OD
```

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WHILE E ≠ ∅ DO
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   FOR ALL s ∈ Pre(s') DO
    IF s ∈ T and Post(s) ∩ T = ∅ THEN
         remove s from T and add s to E
    FI
OD
```

Computation of $Sat(\exists\Box\Phi)$

$T := Sat(\Phi)$ ← organizes the candidates for $s \models \exists\Box\Phi$

$E := S \setminus T$ ← set of states to be expanded

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WHILE E ≠ ∅ DO
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naïve implementation:
quadratic time complexity

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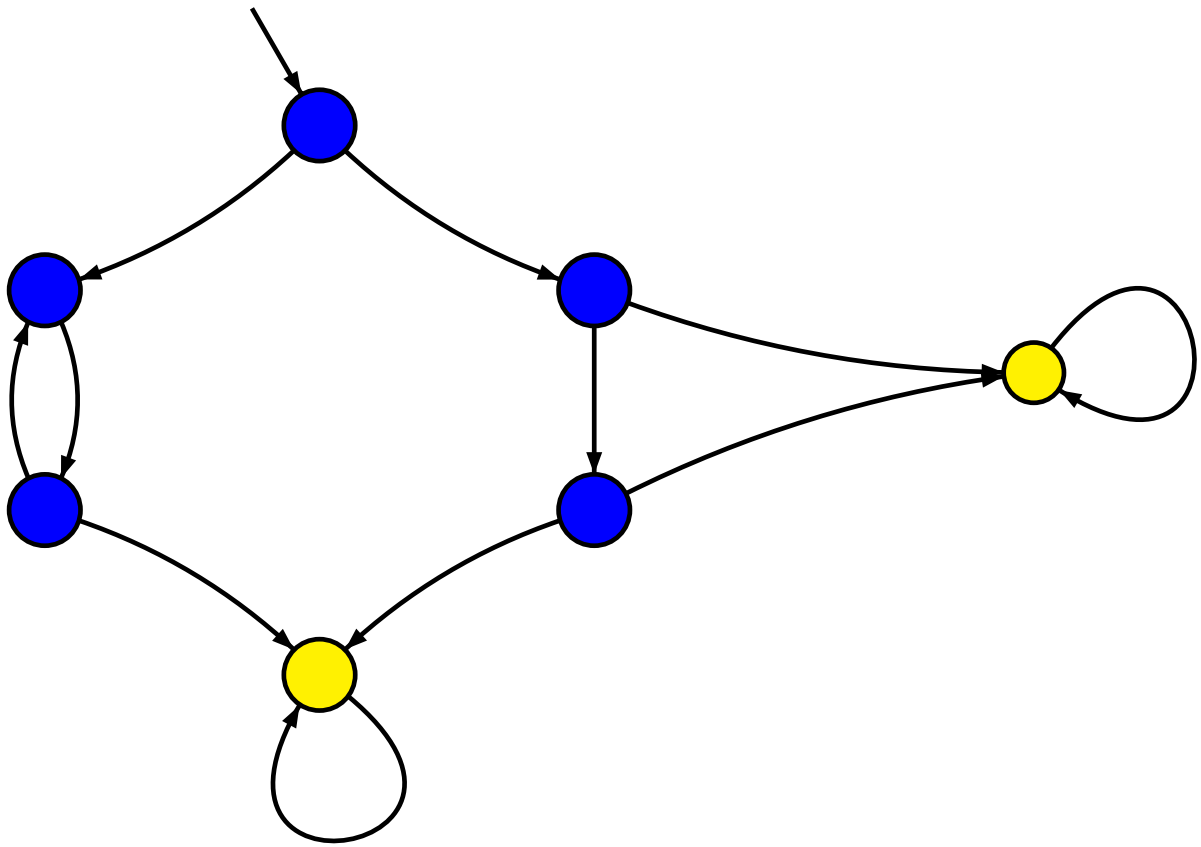
complexity:
$\mathcal{O}(size(\mathcal{T}))$

Example: CTL model checking for ∃□

computation of $T = Sat(\exists\Box blue)$



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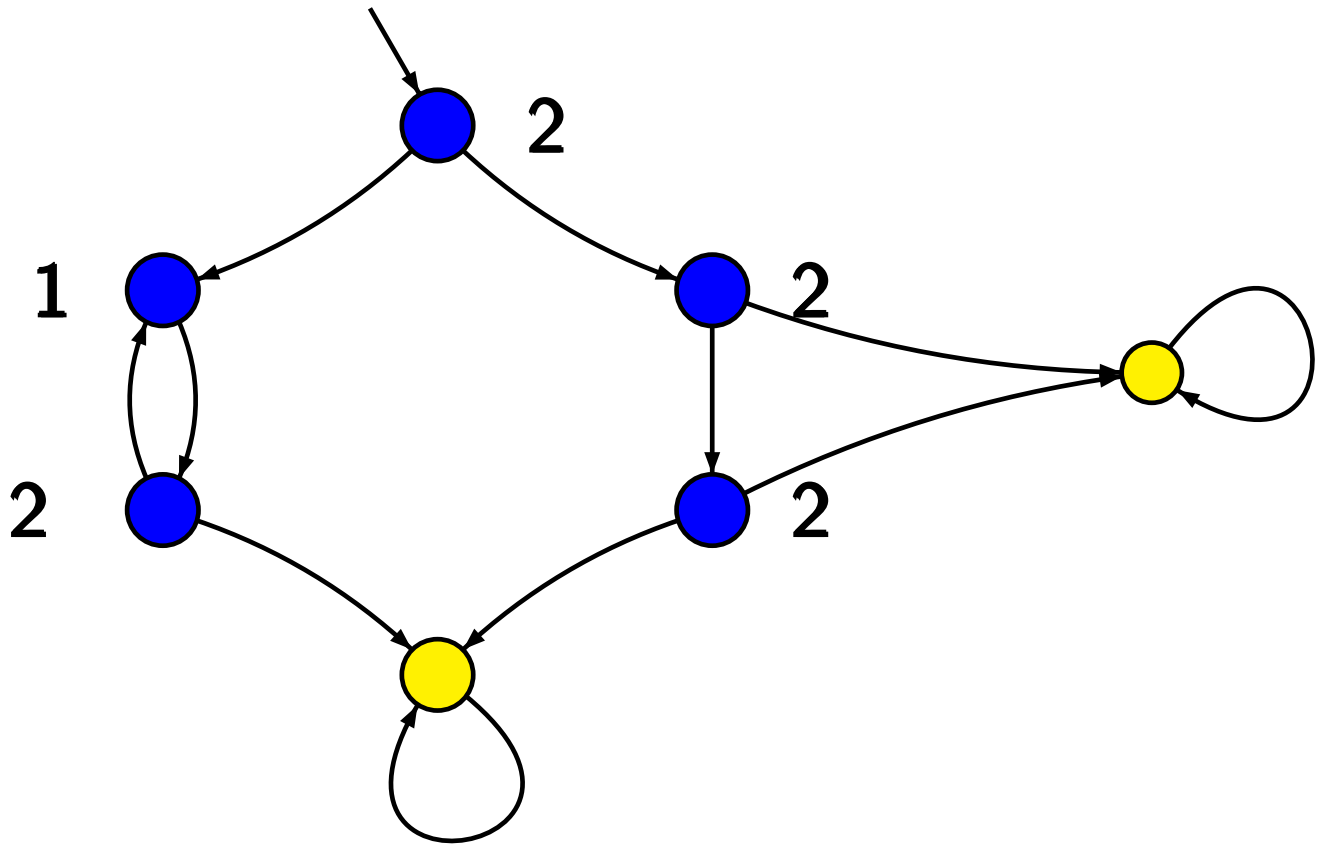

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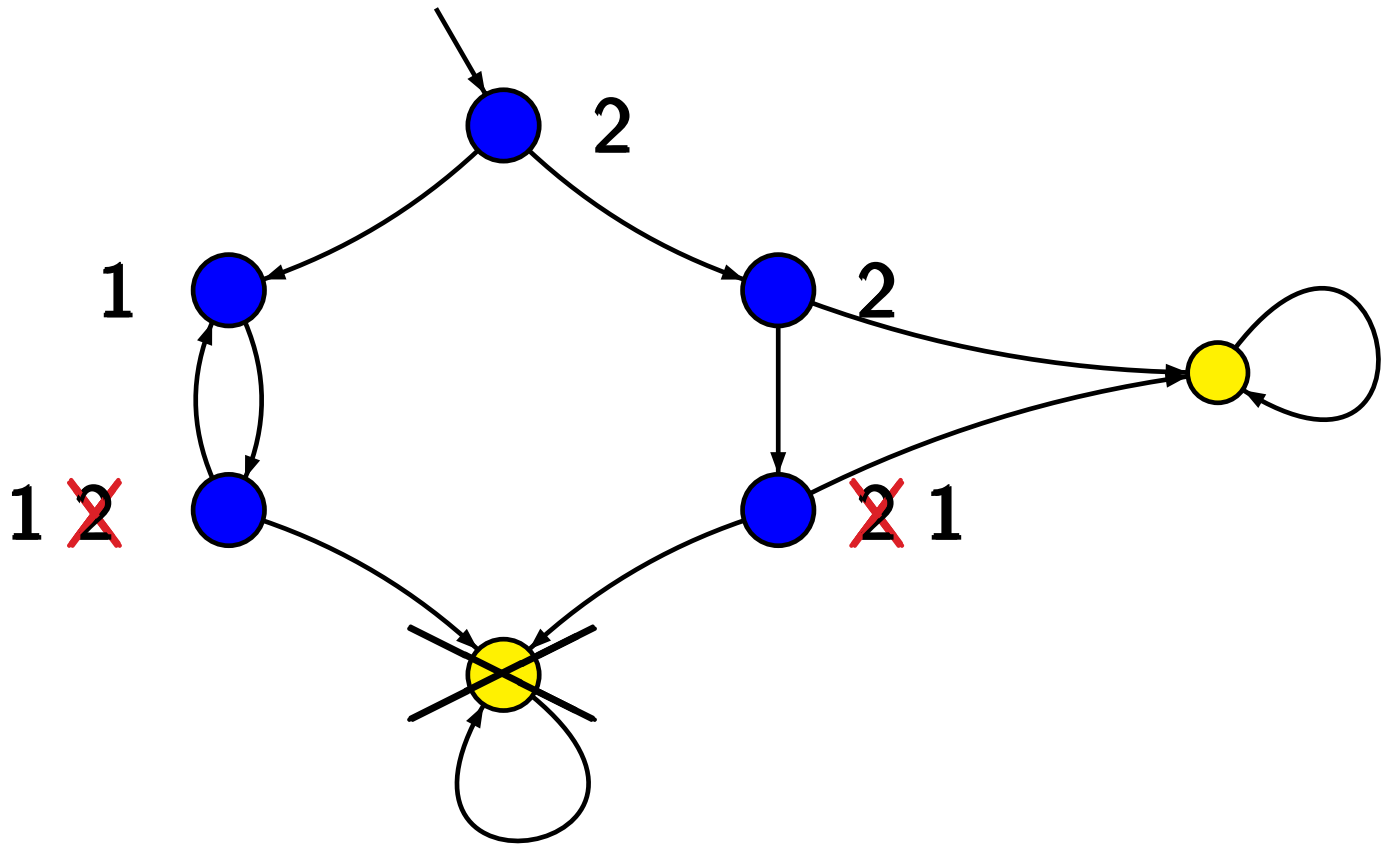

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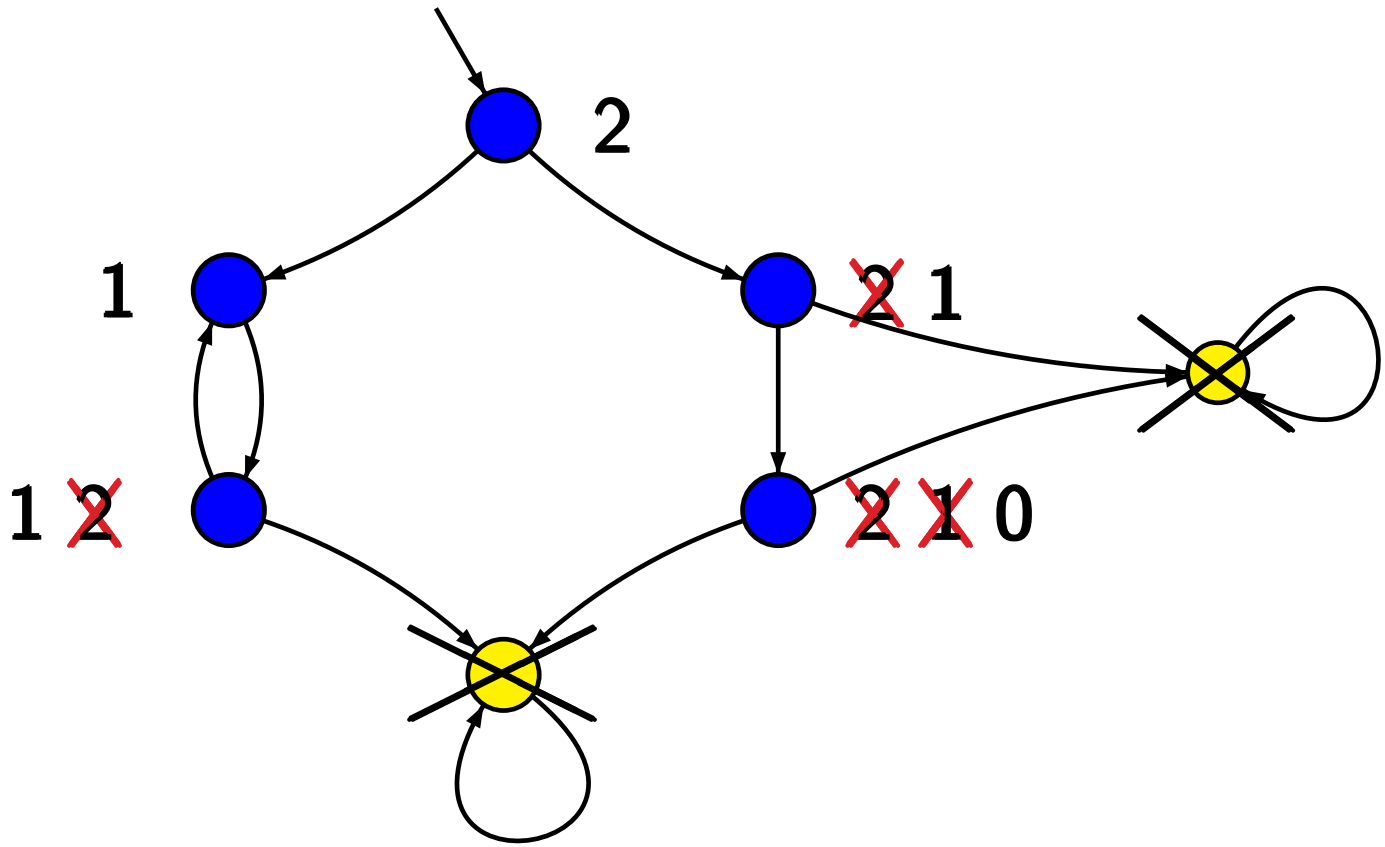

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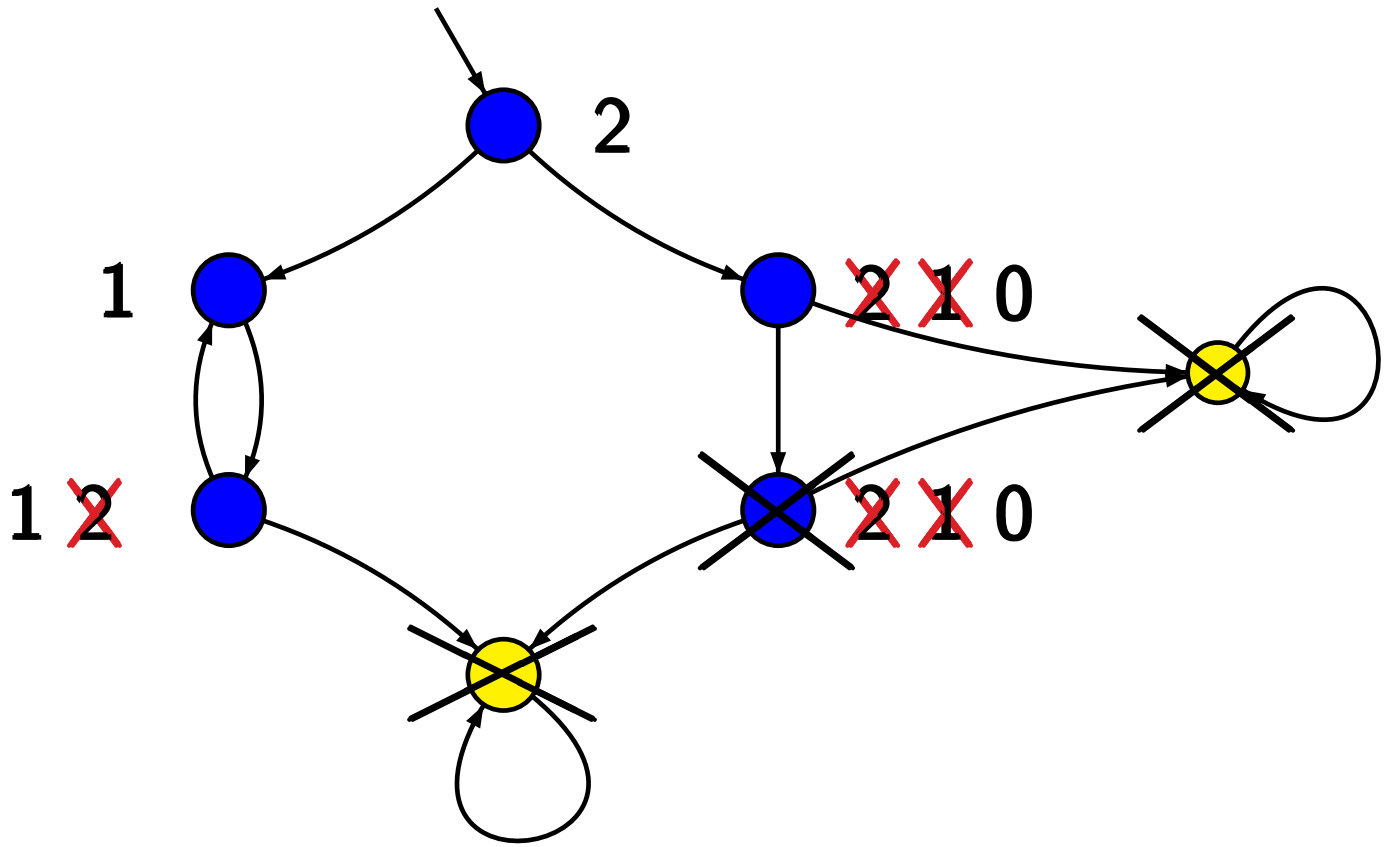

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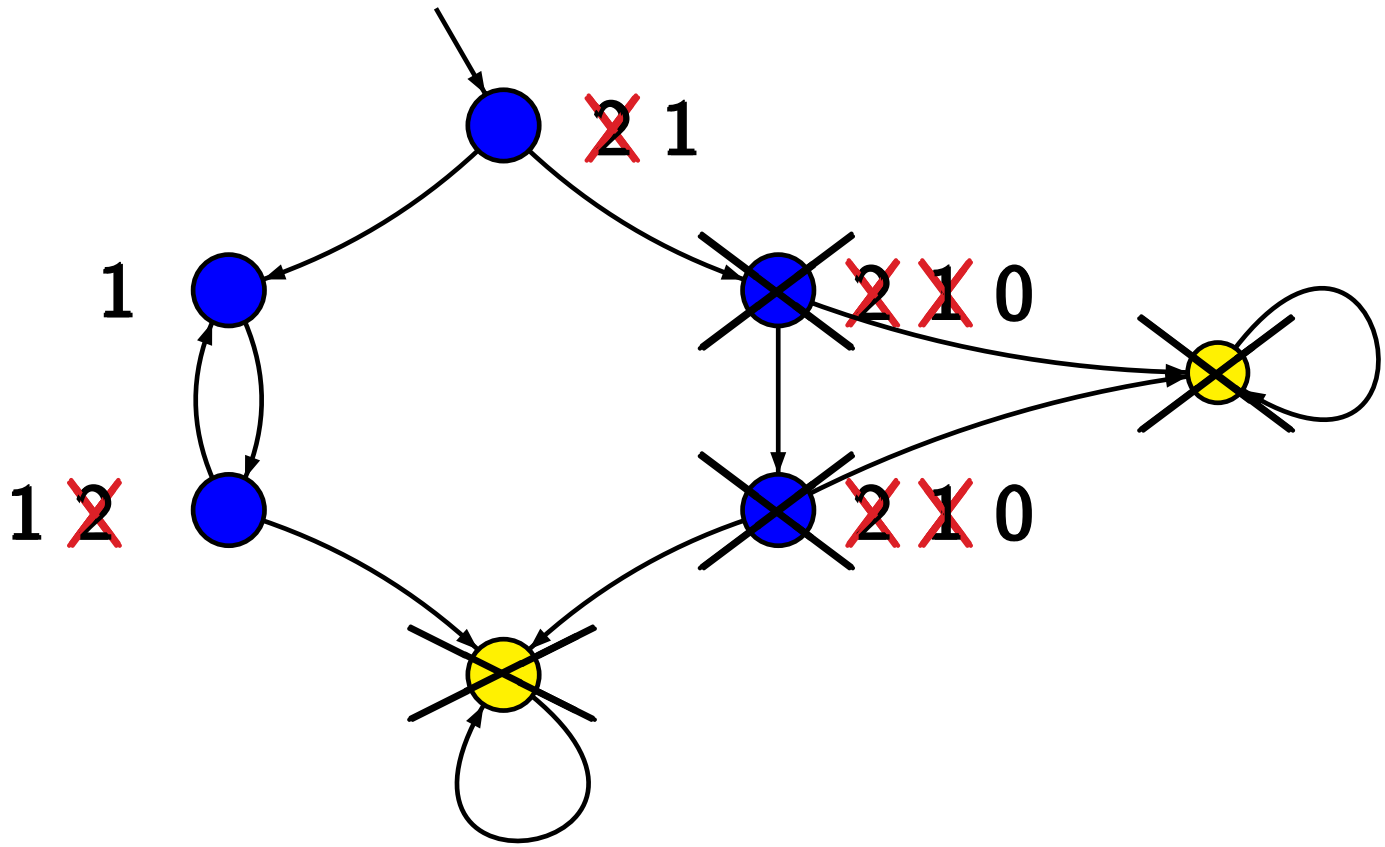

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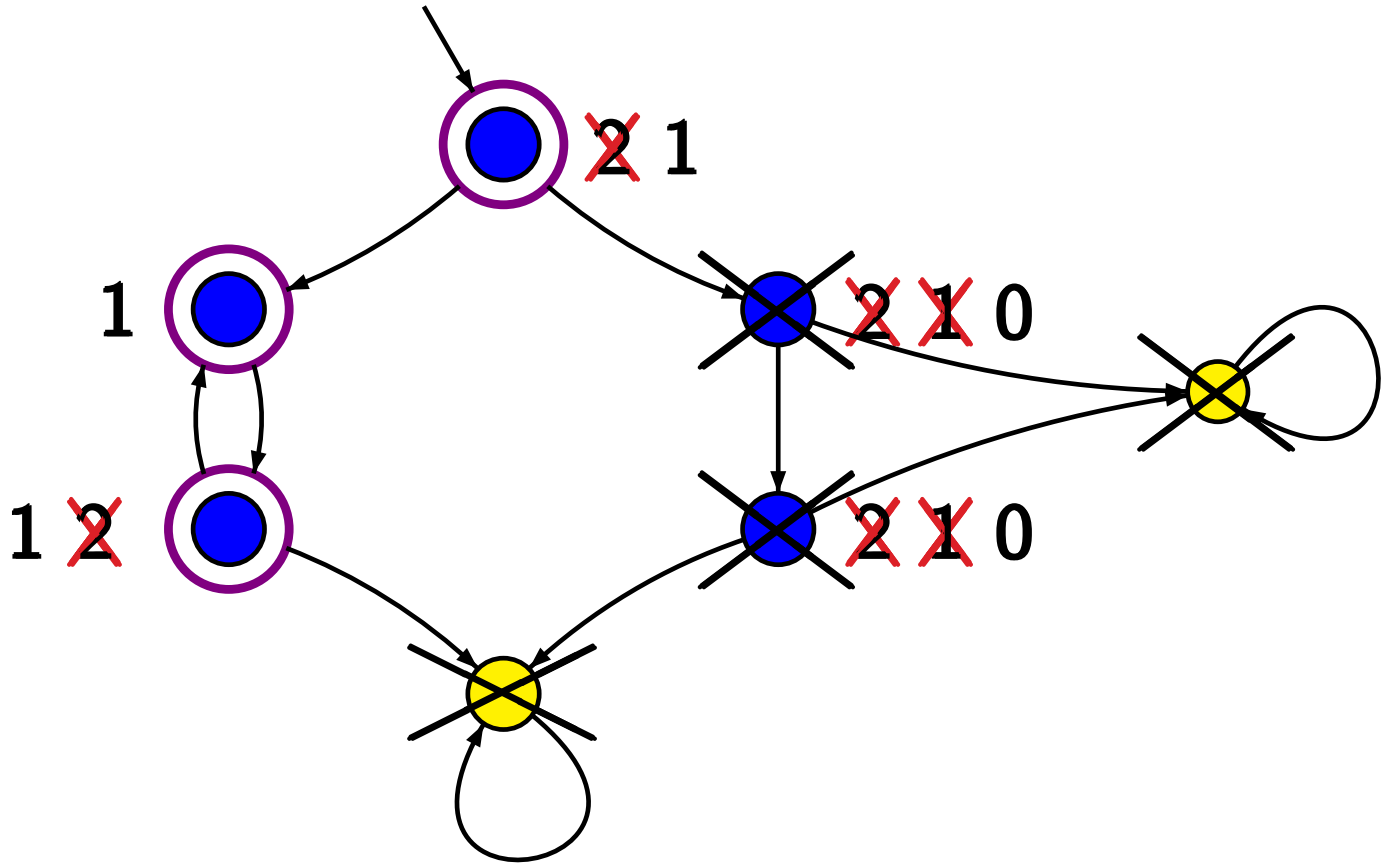

Recursive computation of satisfaction sets

case Φ is

true: return S

$a \in AP$: return $\{s \in S : a \in L(s)\}$

$\neg\Phi$: return $S \setminus Sat(\Phi)$

$\Phi_1 \wedge \Phi_2$: return $Sat(\Phi_1) \cap Sat(\Phi_2)$

$\exists\bigcirc\Phi$: return $\{s \in S : Post(s) \cap Sat(\Phi) \neq \varnothing\}$

$\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)$: ...

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time complexity: $\mathcal{O}(size(\mathcal{T}) \cdot |\Phi|)$

Recursive computation of $Sat(\ldots)$

$$Sat(\Phi_1 \wedge \Phi_2) = Sat(\Phi_1) \cap Sat(\Phi_2)$$

$$Sat(\neg\Phi) = S \setminus Sat(\Phi)$$

$$Sat(\exists\bigcirc\Phi) = \{s \in S : Post(s) \cap Sat(\Phi) \neq \varnothing\}$$

$$Sat(\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)) = \bigcup_{n\geq 0} T_n \text{ where}$$

$$T_0 = Sat(\Phi_2)$$

$$T_{n+1} = \{s \in Sat(\Phi_1) : Post(s) \cap T_n \neq \varnothing\}$$

$$Sat(\exists\Box\Phi) = \bigcap_{n\geq 0} V_n \text{ where}$$

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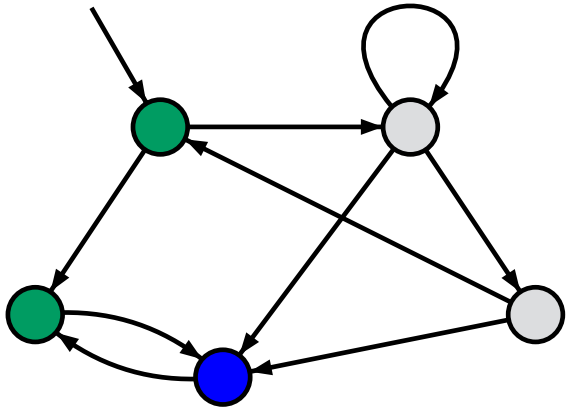

$$\Phi = \exists\Diamond\neg(\,\exists(a\,\mathsf{U}\,b)\ \vee\ \exists\Box\neg a\,)$$

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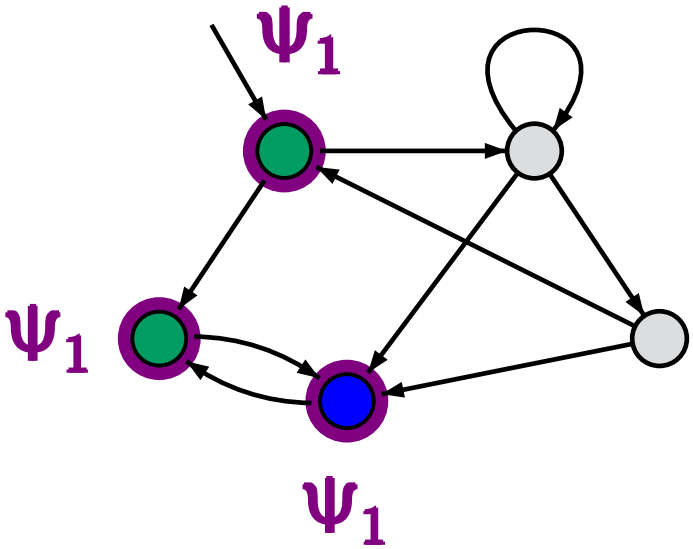

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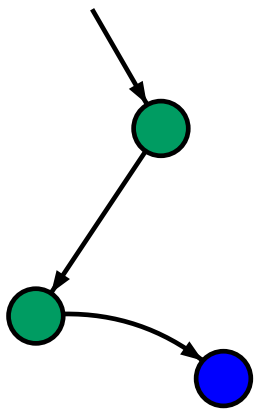

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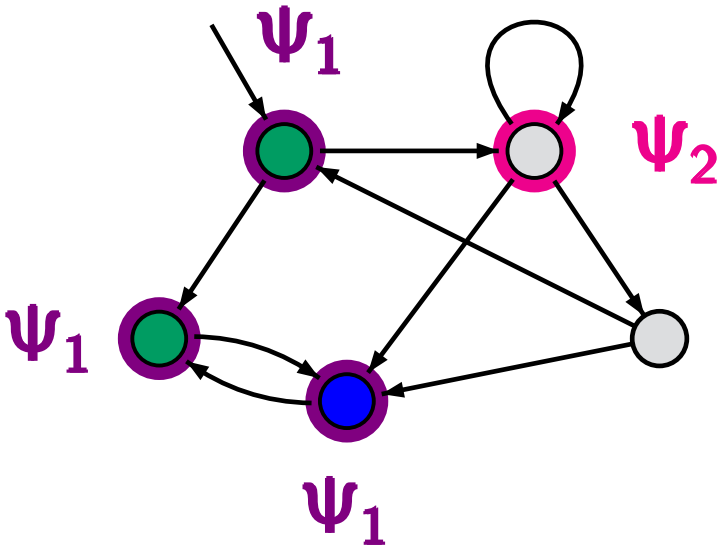

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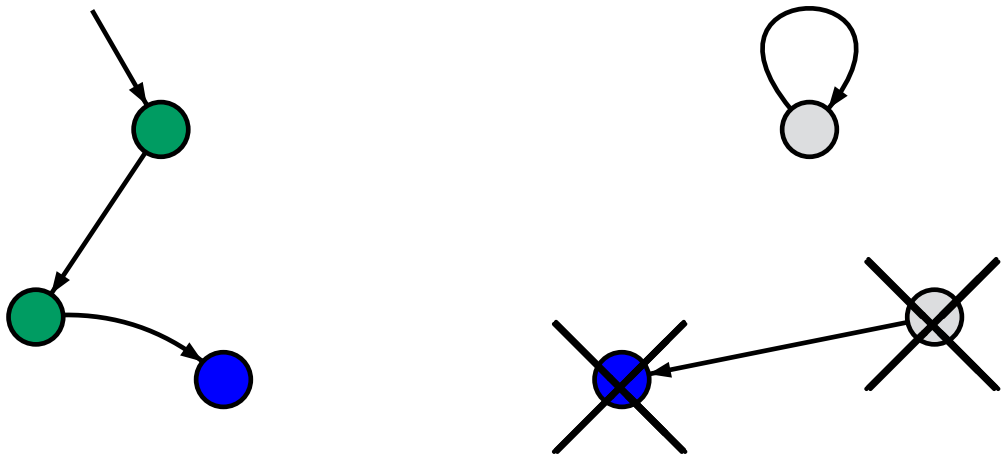

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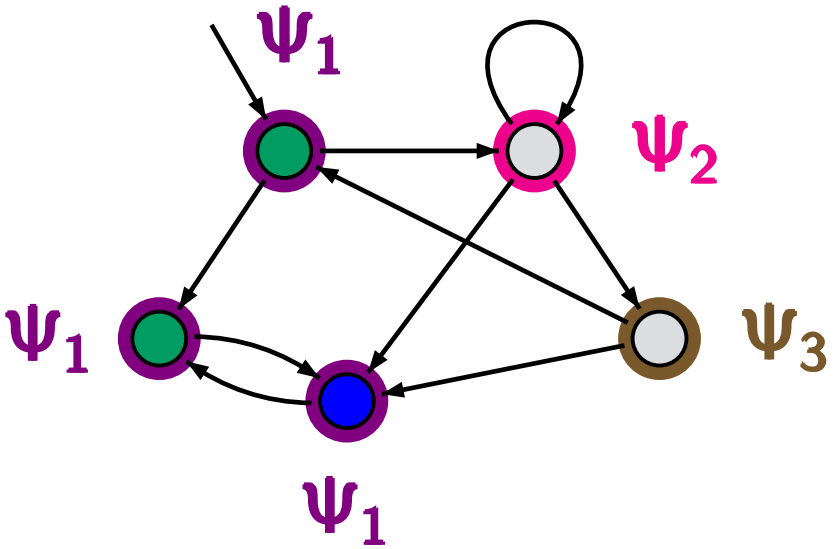

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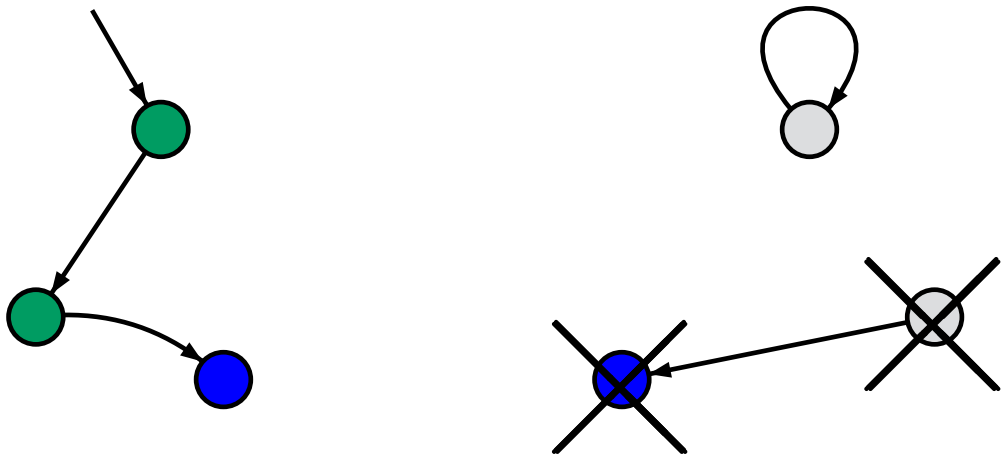

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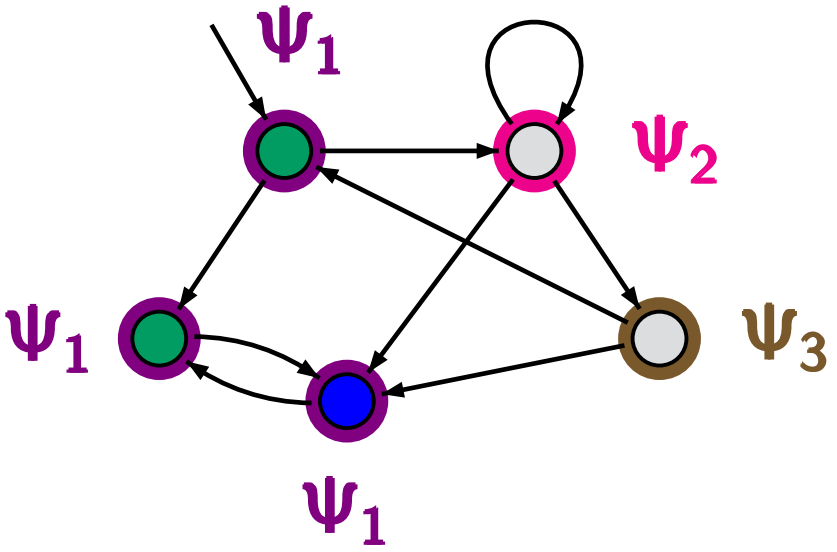

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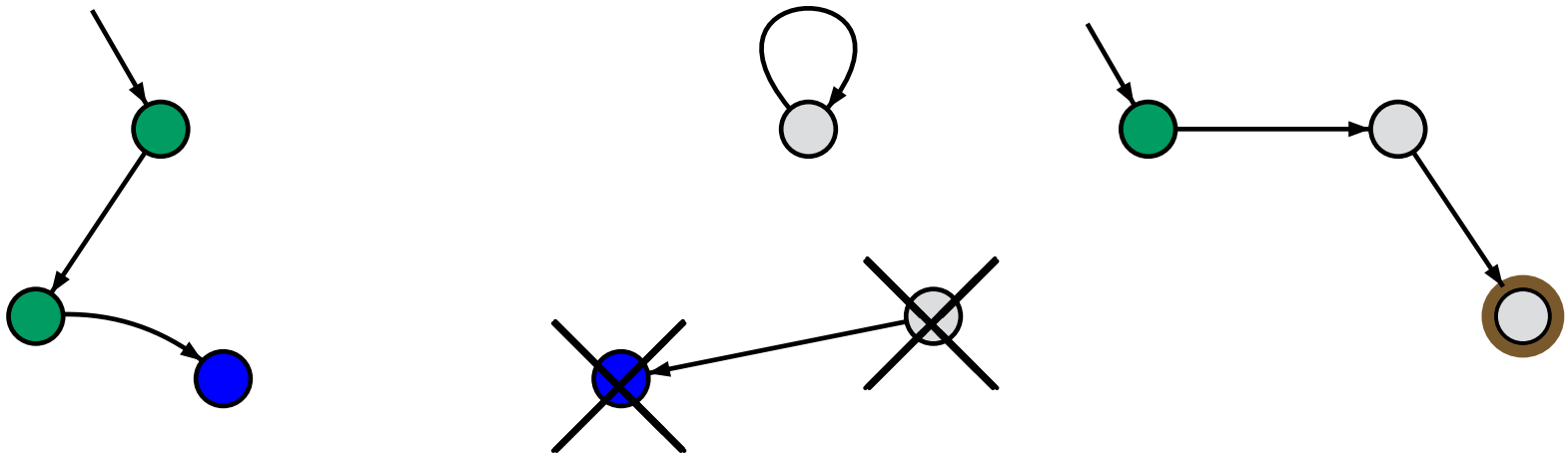

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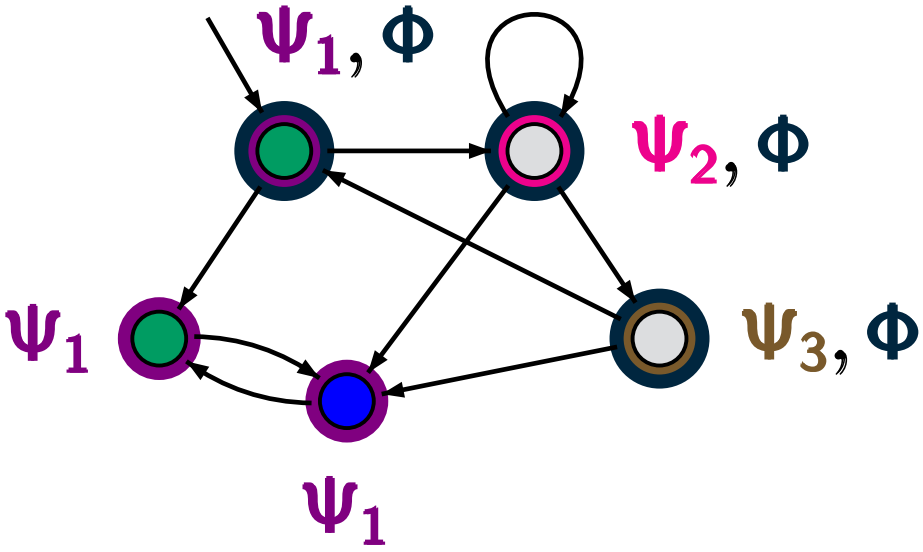

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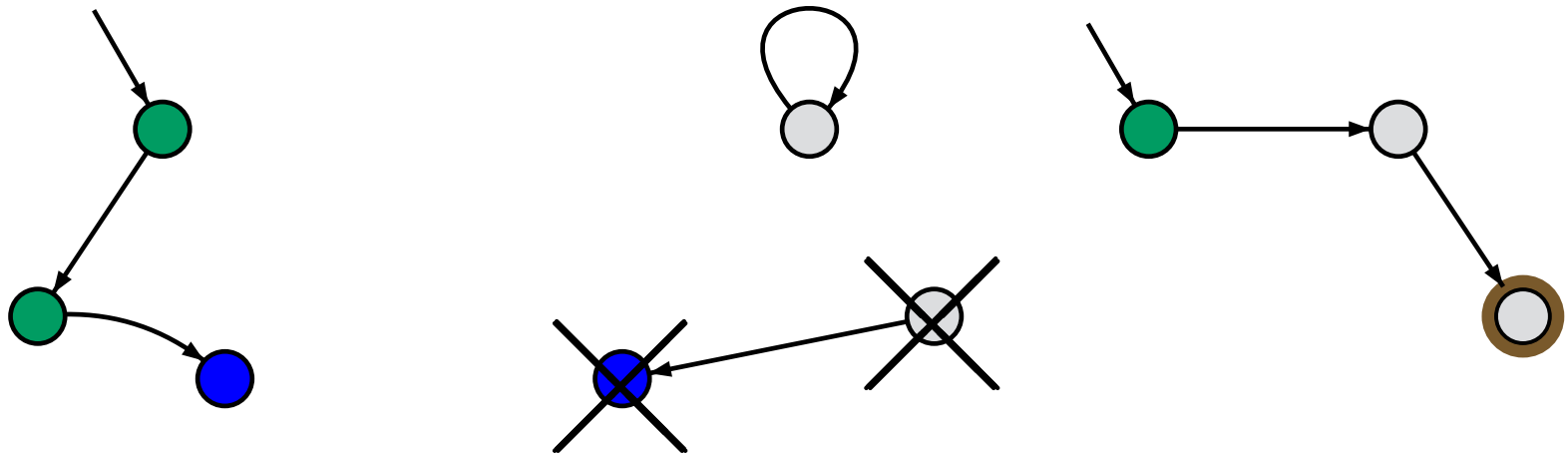

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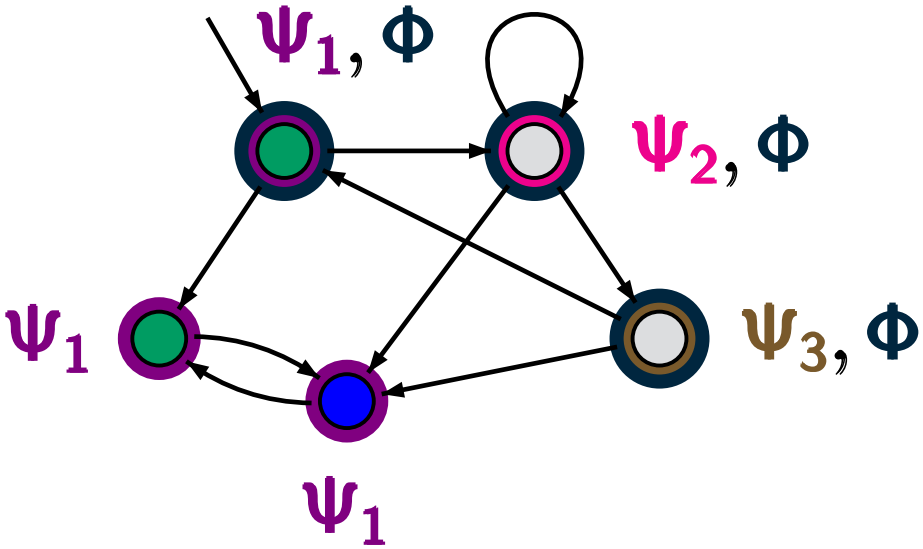

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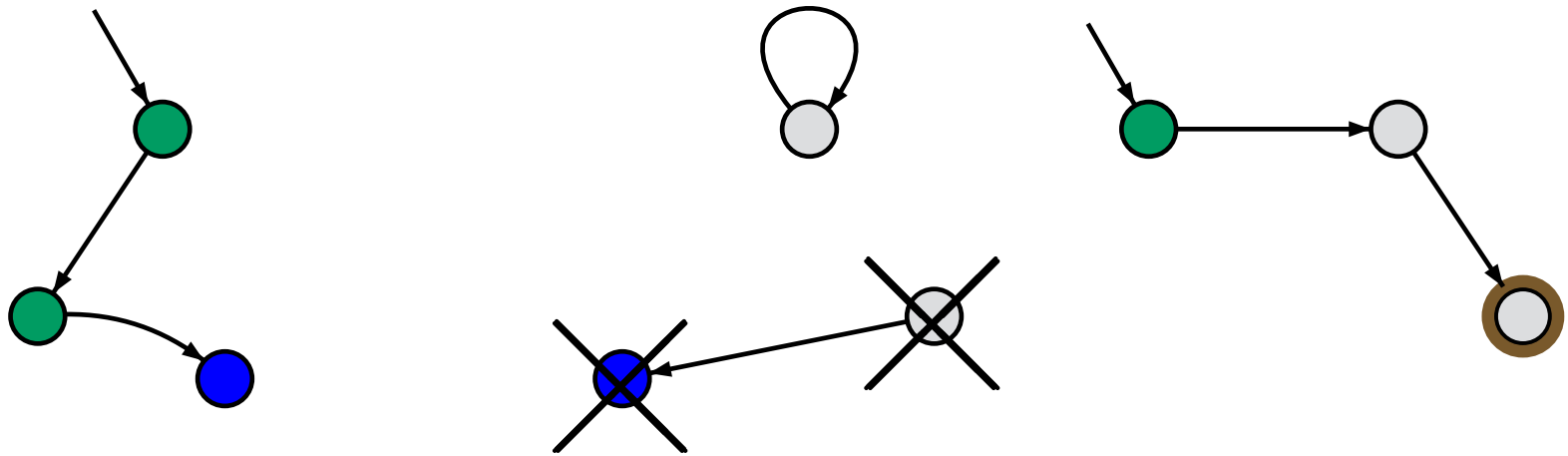

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Complexity of CTL/LTL model checking

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model complexity, i.e., for fixed specification:

CTL and **LTL**: $\mathcal{O}(\mathit{size}(\mathcal{T}))$

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model complexity, i.e., for fixed specification:

CTL and **LTL**: $\mathcal{O}(\mathit{size}(\mathcal{T}))$

If $\Phi \equiv \varphi$ then "often" we have: $|\Phi| = \exp(|\varphi|)$

CTL vs LTL

general observation:

CTL formulas are often "essentially longer" than equivalent **LTL** formulas, provided there is one.

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Recall: for each **CTL** formula Φ we have:

If Φ is equivalent to some **LTL** formula then:

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- $|\varphi_n| = \mathcal{O}(poly(n))$
- φ_n has an equivalent **CTL** formula
- there is no **CTL** formula of polynomial length that is equivalent to φ_n

LTL-encoding the Hamilton path problem

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digraph G with n nodes $\rightsquigarrow$ transition system $\mathcal{T}_G$ + LTL formula φ_n

s.t. G has a Hamilton path iff $\mathcal{T}_G \not\models \neg\varphi_n$

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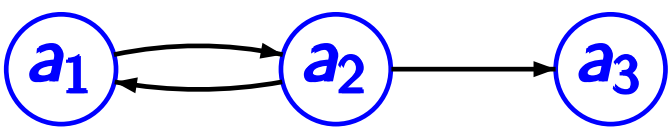
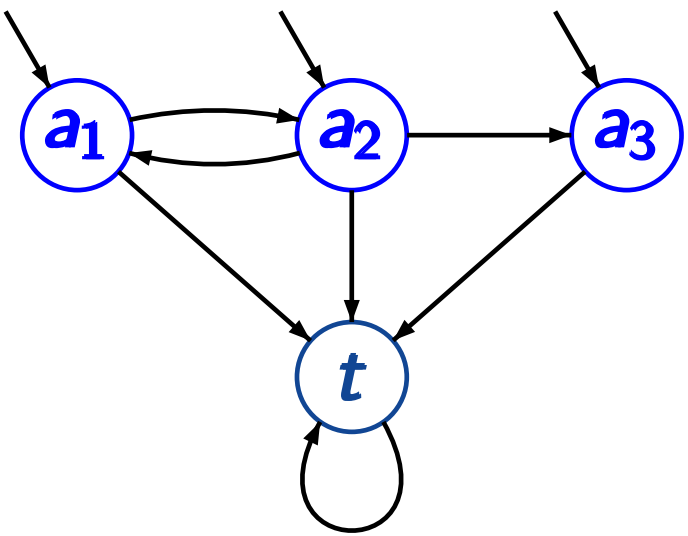
digraph G $\rightsquigarrow$ transition system $\mathcal{T}_G$





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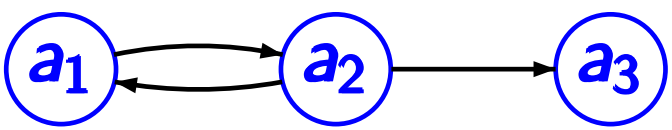

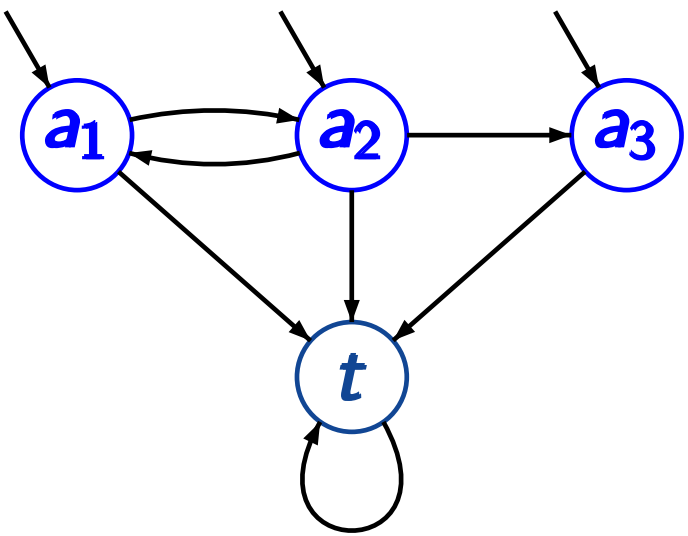

$AP = \{a_1, a_2, a_3\}$

LTL-encoding the Hamilton path problem

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s.t. G has a Hamilton path iff $\mathcal{T}_G \not\models \neg\varphi_n$

$$\varphi_n \;=\; \bigwedge_{1\leq i\leq n} \Big(\Diamond a_i \wedge \Box\big(a_i \longrightarrow \bigcirc\Box\neg a_i\big)\Big)$$

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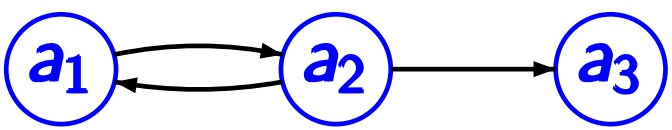

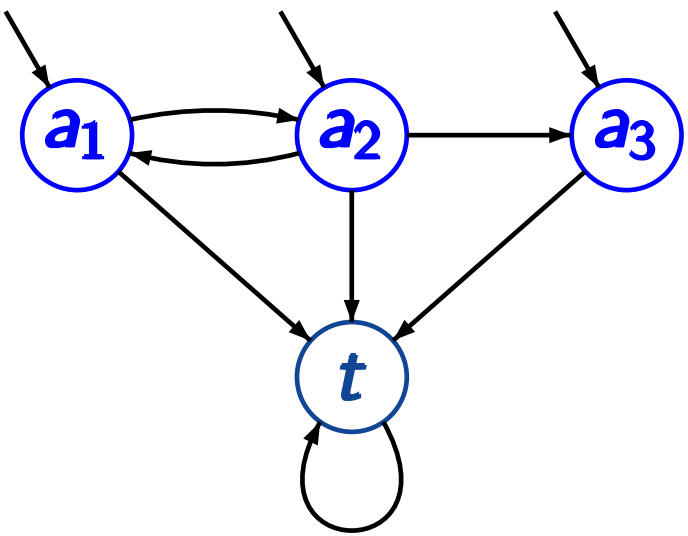

$AP = \{a_1, a_2, a_3\}$

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CTL formula Φ_n, e.g., for $n = 3$:

$$\begin{array}{l}(a_1 \wedge \exists\bigcirc(a_2 \wedge \exists\bigcirc a_3)) \vee (a_1 \wedge \exists\bigcirc(a_3 \wedge \exists\bigcirc a_2)) \vee \\ (a_2 \wedge \exists\bigcirc(a_1 \wedge \exists\bigcirc a_3)) \vee (a_2 \wedge \exists\bigcirc(a_3 \wedge \exists\bigcirc a_1)) \vee \\ (a_3 \wedge \exists\bigcirc(a_1 \wedge \exists\bigcirc a_2)) \vee (a_3 \wedge \exists\bigcirc(a_2 \wedge \exists\bigcirc a_1))\end{array}$$

Formulas φ'_n

LTL formula φ'_n such that $\mathit{Words}(\varphi'_n)$ is

$$\{\{a_{i_1}\}\ldots\{a_{i_n}\}\varnothing^\omega : (i_1,\ldots,i_n) \text{ permutation of } (1,\ldots,n)\}$$

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CTL vs LTL

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If $P \neq NP$ then there is a sequence $(\varphi_n)_{n \geq 0}$ of **LTL** formulas such that:

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